ADVANCED TEXTBOOK

OF

ECTRICITY & MAGNETISM

VOLUME I.

MAGNETISM AND ELECTROSTATICS

BY

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PREFACE

The aim of this book is to give a clear and comprehensive account of the main principles of the subject based on accurate scientific definitions and embodying the distinctive results of modern research. Both the experimental and the theoretical sides of the work have been fully treated, and great care has been taken to deal adequately with the many difficulties which arise in connection with the theoretical explanations of the various phenomena.

The point of view of the writer is that of a teacher with many years' experience in dealing with senior students of Electricity both on the Engineering and on the pure Physics side. This will explain why, in a book the scope of which is that of the Final Degree Examinations of the Universities, certain of the elementary yet fundamental sections are treated at some length. Even the student who feels that he already has sufficiently grasped these fundamental principles will probably appreciate a revision on scientific lines.

The conception of *potential*, the visualisation of *tubes of force and induction*, the idea of the *electron*, and certain features and anticipations of the "New Physics" are introduced early, and kept in view throughout. Incidentally no apology is needed for the introduction at an early stage in the Magnetism of the ideas of permeability, hysteresis, etc., which are in most textbooks relegated to the later chapters; these terms are now in daily use, the student will repeatedly encounter them in his reading, and experience shows that an early elementary treatment is alike possible and desirable, minute details of theory and experiment being postponed to a later stage.

Throughout the book special attention has been devoted to the various units and systems of units, and their relationships have been indicated, so that students may experience no difficulty in connection with modern research work or general problems necessitating a change from one system to another. The book contains numerous fully worked examples illustrating important principles and applications, and a large number of problems of a similar character to be worked by the student. The more important symbols recommended by the International Electro-technical Commission have been adopted.

As already indicated, this work covers the ground of the Final Degree Examinations of the Universities. The author is greatly indebted to the Senate of London University and to the Controller of H.M. Stationery Office for the permission, freely given, to insert the examination questions contained in the book.

For this second edition several sections have been amended and extended, and a quantity of new matter has been added in connection with Magnetic Induction, Ships' Compass Errors, the Dielectric Constant, the Induction Coil, Telegraphy and Telephony, Electromagnetic Waves, Conduction through Gases, Radioactivity and the Electron Theory. In particular Chapters XXII. and XXV. have been much extended, and Chapters XXIII. and XXIV. have been re-written and considerably enlarged.

The best thanks of the author are due to Mr. C. T. R. Wilson for the kind loan of his photographs on the tracks of α and β particle, etc., and for permission to reproduce them in the book.

R. W. H.

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CHAPTER I.

MAGNETISM.—FUNDAMENTAL PHENOMENA.

1. Natural and Artificial Magnets.—The name *magnet* was applied at a very early date to pieces of a mineral found in Magnesia in Asia Minor. These specimens of what is now known as **magnetite** or magnetic iron ore (Fe_3O_4) were found to possess the following properties:—

(*a*) They attracted small pieces of iron or steel. If rolled in iron filings and then lifted out, the filings were found to cling to certain parts, other parts remaining bare. Frequently there were two regions where the filings mainly adhered, and between them was a region showing no attraction (Fig. 1). The regions of greatest attraction are called the *poles*, and the region midway between the poles where there is no attraction the *magnetic equator* or *neutral line*. All specimens do not exhibit definite poles.

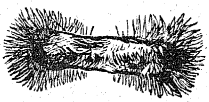


Fig. 1.

(*b*) If such a specimen as is shown in Fig. 1 were suspended so as to be free to turn in a horizontal plane, it came to rest in a definite direction nearly north and south, the one end always pointing towards the north and the other end towards the south. The utilisation of this property in navigation led to magnetite being referred to as *lodestone* (A.S. loedan = to lead). The pole which points towards the north is called the *north-seeking* or, briefly, the *north pole* of the magnet, and the pole which points towards the south the *south-seeking* or *south pole* of the magnet.

(*c*) If bars of iron or steel were rubbed from end to end with one end of such a specimen as is shown in Fig. 1, or if the bars were even held near the lodestone, they were converted into magnets, and possessed the same properties as the specimen itself.

Magnetite is found in abundance in Scandinavia (*e.g.* Dannemora), Finland, the Urals, the States of New York, New Jersey, and Pennsylvania, and in Canada; it is also widely distributed in smaller amounts, occurring as grains in certain rocks, *e.g.* granite, and its presence leads to the magnetic properties of many basalts, of haematite, etc.

Magnetite is often spoken of as a *natural magnet*, whilst a bar of iron or steel which has been magnetised either by rubbing with the natural magnet, or by more powerful methods to be described later, is called an *artificial magnet.* Nickel, cobalt and manganese, although considerably inferior to iron, can also be magnetised. In the study of magnetism which follows, artificial magnets of iron or steel are always referred to.

2. Poles. Magnetic Attraction and Repulsion. The Earth a Magnet.—Permanent artificial magnets are made of hard steel (Art. 7) and are frequently horseshoe-shaped or in the form of bars. As in the case of the natural magnet of Fig. 1, if a bar magnet be placed in iron filings, the latter adhere mainly to regions near the ends, and if the bar be suspended (Fig. 2) so as to be free to turn in a horizontal plane, it comes to rest in a definite direction nearly north and south, the one end always pointing northwards and the other southwards. **The strongest parts of the magnet are therefore near the ends, and these are referred to as the "poles," the one which points towards the north being called the "north pole" and the one which points towards the south the "south pole" of the magnet.**

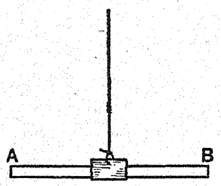


Fig. 2.

The filings experiment above, and others, more exact, to be described later, indicate that the bar magnet exhibits magnetism *in varying degree along its surface* from zero at its centre to a maximum in *regions* near the ends, but it may be stated that at some distance from the magnet the action is very nearly the same as if the magnetic effects

were due to *two points within the bar and near the ends*; these points may be regarded as the poles, and the straight line joining the poles is called the *magnetic axis*.

The longer the bar in comparison with its thickness, the nearer the poles are to the ends. An indefinitely thin magnet would exhibit no lateral magnetism, and the poles would be at the ends; such a magnet is called a *simple magnet*. The Robison **ball-ended magnet** consists of a magnetised steel rod on the ends of which balls of steel or iron are screwed, and it has been proved that this magnet acts almost like a simple magnet with poles situated at the centres of the balls.

Exp. Suspend a magnet by means of a thread as shown in Fig. 2. Bring the north pole of a second magnet *gradually* near the north pole of the suspended one: *repulsion* ensues. Remove the second magnet, bring the suspended one to rest and then bring the south pole of the second magnet near the north pole of the suspended one: *attraction* ensues. Similarly, bring the north pole of the second near the south pole of the first and the result is *attraction*. Bring the south poles *gradually* together and the result is *repulsion*.

The above establishes the fundamental law in magnetism, viz. **like poles repel and unlike poles attract.**

The setting of a suspended magnet in a definite direction nearly north and south is merely another illustration of the law. The earth itself is a magnet having its magnetic "poles" not so very far removed from its geographical poles. The magnetic pole in the northern hemisphere is called the "north magnetic pole of the earth," but it must be remembered that as it attracts the *north* pole of a magnet the two are unlike poles, i.e. *the north magnetic pole of the earth is like the south pole of a magnet*. Similarly, the magnetic pole in the southern hemisphere is called the "south magnetic pole of the earth"; but it attracts the *south* pole of a magnet, and therefore *the south magnetic pole of the earth is like the north pole of a magnet*.

The vertical plane in which the magnetic axis of a suspended magnet or a compass comes to rest is called the *magnetic meridian of the earth* at that particular place.

The property of magnets (and of the earth) that they

possesses poles with different characteristics is called "**polarity.**"

The fact that a suspended magnet or compass does not, in general, point geographically north was noted by Adsiger in 1269, and again by Columbus in 1492, the latter also observing that the deviation from the true north was different at different places.

In 1576 Norman discovered that if a magnet were suitably suspended—"freely suspended at its centre of gravity" as it is often worded—it came to rest in the magnetic meridian but with its north pole "dipping" in London, the angle between the horizontal and the magnetic axis of the magnet being (then) 71° 50′. In general, in the northern hemisphere the north pole of the above magnet dips downwards, and in the southern hemisphere the south pole dips downwards, the dip angle being different at different places; in the vicinity of the equator the magnet lies horizontally in the magnetic meridian; at the north magnetic pole it sets vertically, the north pole being downwards; at the south magnetic pole it sets vertically, the south pole being downwards (Fig. 3) The line drawn through places where the above magnet lies horizontally *i.e.* where the dip is zero gives the earth's *magnetic equator*. Further details of the earth's magnetism appear in subsequent pages. Needless to say, Fig. 3 is diagrammatic only; it is inserted to assist the student to grasp the general idea.

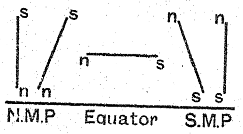


Fig. 3.

3. Magnetic Induction. Magnetic Induction by the Earth. Magnetic Difference between Iron and Steel.

Exp. Let the magnet *NS* (Fig. 4) be fixed vertically in a clamp, and let a short piece of steel be brought near the lower end; it is attracted by the magnet, and clings to it as shown at (a). Let another short piece of steel be now brought near the lower end of the piece clinging to the magnet; this second piece is also attracted, and, if the magnet be strong enough, it will remain hanging to the first piece as shown at (b). If the first piece be now gently detached from the magnet, it will be found that the lower piece still clings to it, and, if the two pieces of steel be tested, each will be found to be a magnet with poles as shown at (c). It will be noticed that these poles are arranged relative to one another and to the magnet *NS*, so that dissimilar poles are in contact, and therefore exert mutual attraction.

If this experiment be modified, as indicated in Fig. 5, by arranging the magnet and the pieces of steel on a table so that they are

not allowed to come into contact with one another, it will be found that the result is exactly the same as before, only that the magnetisation produced in the pieces of steel is somewhat more feeble

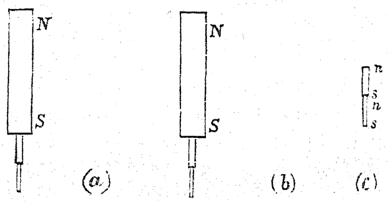


Fig. 4.

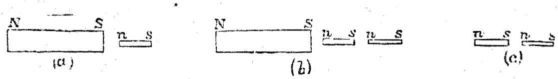


Fig. 5.

than in the first case. On removing the magnet NS, each piece of steel will be found to possess all the properties of a magnet as shown at (c).

The phenomena indicated in the above experiments, viz. the making of a piece of unmagnetised steel into a magnet by the influence of another magnet, with or without actual contact, are referred to as "magnetic induction," and the steel is said to be *magnetised by induction.* The "law of induced polarity" is readily derived from the experiments. Thus in Fig. 5 (a) the end of the steel opposite the south pole of the inducing magnet is found to be a north pole. If

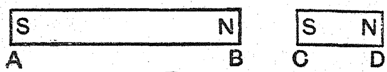


Fig. 6.

the experiment be repeated, and a piece of unmagnetised steel CD (Fig. 6) be placed near the north pole of the inducing magnet AB, the end C of the steel opposite the north pole is found to be a south pole. Hence *when a bar is magnetised by induction, the end of it nearest the inducing pole acquires polarity opposite to that of the inducing pole.*

The experiments also indicate why a magnet attracts an unmagnetised piece of iron or steel. When brought near, the iron or steel is no longer unmagnetised, but is magnetised by induction, and by the law of induced polarity we have the conditions necessary for attraction. Thus *induction always precedes the attraction between a magnet and a (previously) unmagnetised body.* Further, if the north pole of a powerful magnet be brought near the north pole of a weaker one the two will repel, but the powerful magnet, acting inductively on the weaker one, may reverse its polarity, and the repulsion change to attraction.

Exp. If it be required to test whether a given bar AB of iron or steel is a magnet, and if so to find its polarity, bring one end A of it gradually near the north pole of a suspended magnet or compass. If attraction ensues, the bar may be a magnet with the end A a south pole, or it may be unmagnetised originally and only magnetised inductively at the time, the north pole of the suspended magnet inducing a south pole at A. To settle the question bring A near the south pole of the suspended magnet; if repulsion ensues the bar is a magnet, the end A being the south pole, but if attraction again takes place the bar is originally unmagnetised, being only magnetised by induction. Repeat with the end B and verify. Clearly *repulsion is the only sure test for polarity.*

A vertical bar of iron (or steel) in the northern hemisphere will be found to be magnetised by the earth's inductive action, *the bottom end being a north pole and the top end a south pole.* A horizontal bar of iron laid parallel to the direction in which a suspended magnet or a compass needle comes to rest will also be found to be magnetised, *the end pointing northwards being a north pole,* but the magnet will not be quite so strong as in the case of the vertical bar. A bar of iron held parallel to the direction in which a compass points, but inclined at an angle of about 67° to the horizontal, its lower end towards the north, will be even more strongly magnetised than the vertical bar, *the lower end pointing northwards being a north pole.*

All this is in agreement with the "law of induced polarity." The north magnetic pole of the earth is similar to the south pole of a magnet: it induces a *north* pole in

the end of the iron towards it, viz. the lower end of the vertical bar, and the ends pointing northwards of the other bars. In the southern hemisphere the vertical bar, for example, would for a like reason be magnetised by the earth's inductive action, *with the bottom end a south pole and the top end a north pole.*

Exp. Repeat the first experiment (Fig. 4), using pieces of soft iron of equal weight to the pieces of steel. Whilst only about two pieces of steel could be hung on the magnet, perhaps eight or nine pieces of soft iron can be attached one under another. This shows that the iron magnets are stronger, *i.e.* that *soft iron is more readily magnetised than hard steel.*

Detach the upper piece of iron from the magnet. Whilst the pieces of steel still hung together and showed polarity, the pieces of soft iron fall away from each other, and when tested show no signs of magnetism. This shows that *soft iron loses its magnetisation much more readily than hard steel.*

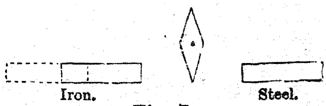


Fig. 7.

The first fact may also be shown thus :—Place equal pieces of iron and steel at equal distances from a compass needle, as shown in Fig. 7. The compass acts inductively on each bar, so that each tends to attract it, *but the compass is deflected towards the iron*, showing that this is the stronger. The iron must be moved to a greater distance (dotted lines) in order that the two attractions may counteract each other.

The power of retaining magnetisation when the inducing influence is removed is called "retentivity" and the magnetisation retained is called "permanent" or "residual" magnetism. The retentivity of steel is greater, therefore, than that of soft iron, and is increased by the presence of 5 to 8 per cent. of *tungsten* or about 4 per cent. of *molybdenum* (Arts. 7, 12). It may be mentioned that soft iron may indicate more residual magnetism than steel *if carefully protected from the least disturbing influence*, but very little disturbing effect will wipe out the magnetisation; in saying the retentivity of steel is the greater, reference is made to the more stable condition. Material which retains a good portion of its magnetisation *despite disturbing influence* is also said to have a large

coercivity; the coercivity of steel is larger, therefore, than that of iron. These terms will be more exactly defined and better understood later.

The stronger the poles developed in a material when under a given inductive influence the greater is said to be the "susceptibility" of the material to magnetisation; hence the susceptibility of soft iron is greater than that of steel. Clearly, then, if a magnet be required to acquire quickly strong magnetisation and to become demagnetised quickly, it must be made of soft iron; this is the case with *electromagnets* (Art. 7), and the material employed is frequently the best Swedish soft iron, well annealed.

4. Demagnetising Effect of Magnetic Poles.—The preceding explains the demagnetising effect of a magnet on itself. Thus if *NS* (Fig. 8) be a permanent magnet and *O* any point in it, then at *O* there will be a magnetic force acting in the direction from *N* towards *S*, due to the poles of the magnet. This tends to magnetise, by induction, the material at *O*, so that southern polarity is towards *N* and

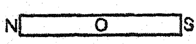


Fig. 8.

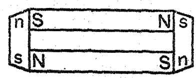


Fig. 9.

northern polarity towards *S*, *a distribution of magnetisation exactly opposite to that of the magnet.* This demagnetising effect is reduced by arranging bar magnets in pairs as shown in Fig. 9, with soft iron "keepers" across the poles. The north pole of the magnet, for example, induces a south pole in the part of the keeper in contact with it, and the effect of this at any point of the magnet is equal and opposite to the effect of the magnet pole. The function of keepers will be better understood after reading Art. 8.

Again let Fig. 10 represent a bar of iron lying off the north pole of a magnet, and therefore magnetised by induction as indicated. If an isolated *north* pole be **imagined**

free to move at the point O, it will be urged to the right by the inducing magnet, but the induced poles in the bar will tend to urge it towards the left; *thus the force on the north pole at O is less in the bar than at the same point O when the bar is not there*, owing to the demagnetising effect of the poles. This fact is of importance later.

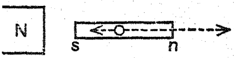


Fig. 10.

5. Magnetic Field. Lines of Force and Induction, Tubes of Force and Induction.—The magnetic field of a magnet is the whole space round about the magnet where its influence is felt, *i.e.* the whole space in which magnetic force due to the magnet can be detected. The *extent* of a magnetic field actually observed in practice depends, therefore, on the sensitiveness of our means of observation.

Consider a point P in the field of a bar magnet, and imagine an *isolated north pole perfectly free to move* to be situated at P. It will experience magnetic forces due to the magnet, being attracted by the south pole and repelled by the north pole, and will move in a certain definite direction, viz. the direction of the resultant force at P. As the pole changes its position relative to the magnet, the direction of the resultant force changes, and the direction of motion changes accordingly. Thus the pole will be urged along a curve starting from the north pole and terminating at the south pole of the magnet. *The line (or curve) along which an isolated north pole would travel if free to move in a magnetic field is called a line of force.* An isolated south pole would, of course, move in the opposite direction, but it is customary to speak of the direction in which a *north* pole would move; this is sometimes called the *positive direction* of the line of force.

In practice it is impossible to obtain a single pole (Art. 8); hence consider a small compass needle placed at the point P. Its north pole will tend to move along a line of force, and so also will its south pole, but the latter will tend to move the other way. The net result will be

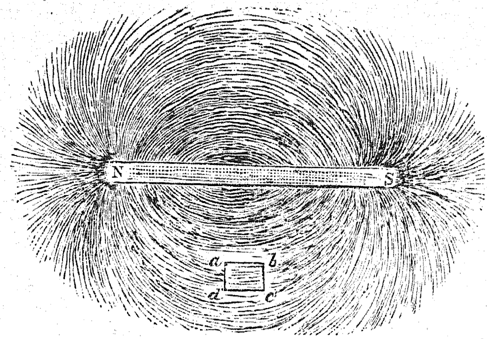


Fig. 11.

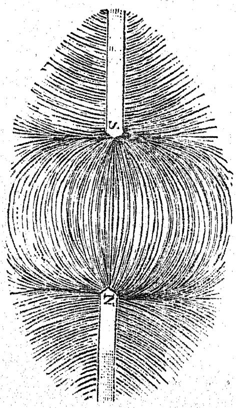


Fig. 12.

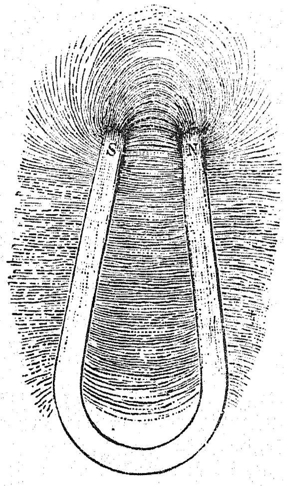


Fig. 13.

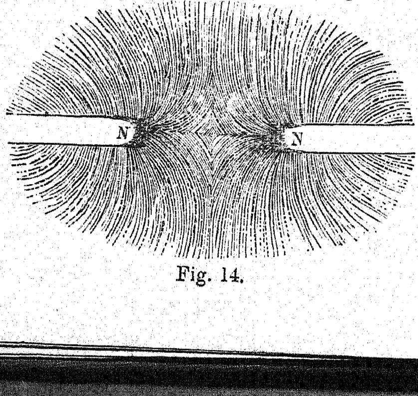


Fig. 14.

that the compass, being small, will set itself with its magnetic axis in the direction of the magnetic force at P, which in the case considered *will be a tangent to the line of force through the point P.* Clearly, if the lines of force in a field are straight, the compass needle will set itself *with its magnetic axis along the lines.* If the needle is a *very* small one, or if it is a piece of iron filing magnetised by induction, we may assume that it lies *on and along* the line of force whether straight or curved, and this furnishes a means of tracing out the lines of force in any particular case.

To summarise: **A line of force is a line along which an isolated north pole would travel if free to move in a magnetic field, and it is such that the tangent at any point gives the direction of the resultant force at that point.**

Exps. *To exhibit the lines of force in a magnetic field by means of iron filings.*—Let a sheet of thin glass or thin pasteboard be placed over a magnet, and let iron filings be dusted over it. It will be noticed that as the filings fall on the card, they at once come under the inductive action of the magnet, and each, becoming a magnet, sets itself parallel to the direction of the resultant force at the point where it is placed. On gently tapping the card, the filings thus arrange themselves in continuous curves, and roughly map out the lines of force in the plane of the card.

Fig. 11 shows the general arrangement of the filings over an ordinary bar magnet, and the student may readily verify the fact that the direction of the magnetic force at any point is tangential to the curve passing through that point. The lines of force are supposed to run from N to S, that is, in the direction in which a north pole would travel.

Fig. 12 shows the general direction of the lines of force between two dissimilar poles of two bar magnets, and Fig. 13, for a horseshoe magnet, illustrates the same thing.

The distribution of the lines of force between two similar poles is shown by Fig. 14. In the previous cases, the lines from a north pole, N, curve round and enter a south pole, S. In this case, the lines from one north pole do not run into the other north pole. The two sets repel each other and travel out at the sides, each set going back to the south pole of its own magnet. *Since lines of force cannot intersect each other* (*see page* 15), *Fig.* 14 *is not strictly correct* (*see Fig.* 16). Figs. 15 to 21 indicate the best method of sketching various cases of lines of force.

It must be remembered that the above figures only show the lines of force *in the plane of the card,* that is,

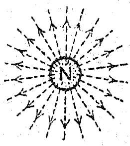


Fig. 15.

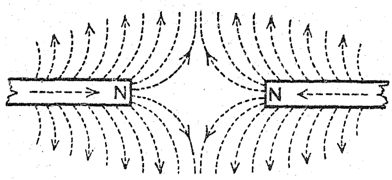


Fig. 16.

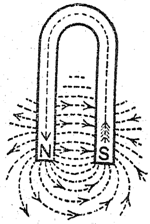


Fig. 17.

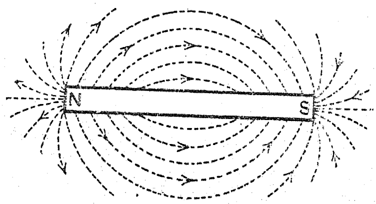


Fig. 18.

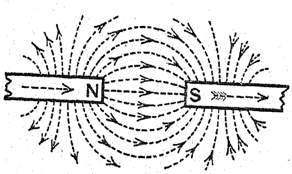


Fig. 19.

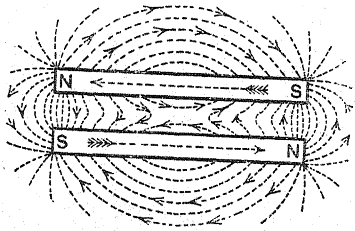


Fig. 20.

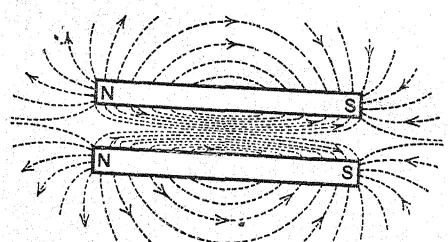


Fig. 21.

they represent longitudinal *sections* of the field. All longitudinal sections are, however, exactly similar, so that the figures completely represent the field if we remember that the lines of force surround the magnet on all sides, and that the above figures represent their disposition in *any* longitudinal section. It must further be remembered that lines of force have no real existence—they are merely graphical representations of the direction of the magnetic force at any point in a field of force, and the experiments with the filings are merely magnetic devices for indicating to the eye the general direction of these forces, just as a chain hanging vertically indicates the direction of the force of gravitation.

Exps. *To exhibit the lines of force in a magnetic field by means of a small compass needle.*—This is a much more sensitive and accurate method than that of iron filings. The general procedure is to place a small compass, *ns*, in the field, and to make a mark opposite each end (Fig. 22). The compass is then moved along until the end *s* is exactly over the mark which has just been made opposite *n*, and the new position of *n* is marked. This is repeated, and finally a curve is drawn through the various points thus obtained; this is the line of force required, and the whole field can be mapped out in this way by starting the compass from different positions. The following special cases should be traced:—

Fig. 22.

(*a*) Place a sheet of drawing-paper on the table, and trace the lines of force due to the earth's horizontal field. If no iron or steel or magnet be near to interfere with the result, a series of parallel straight lines will be obtained running nearly north and south. The positive direction of the lines, *i.e.* the direction in which the north pole is urged, is *from the south towards the north.*

(*b*) Using a large sheet of drawing-paper, place a magnet on the middle of the paper with its length in the magnetic meridian, its south pole pointing northwards, and trace the combined field of the earth and the magnet. Fig. 23 shows the result obtained. At points near the magnet the forces due to the magnet predominate, and the lines obtained are similar to those displayed by the filings in Fig. 11. At more remote points the earth's field predominates, and we get the earth's lines, distorted, however, by the presence of the magnet. Due magnetic north of the magnet the horizontal field of the magnet is in the opposite direction to the horizontal field of the earth; hence at a certain point X the two fields are equal and opposite, and the resultant horizontal field is therefore zero. X is called a **null** or **neutral point.** At points between X

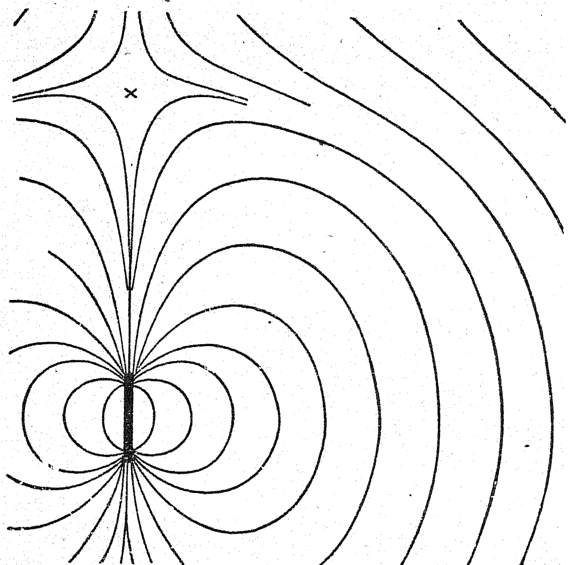


Fig. 23.

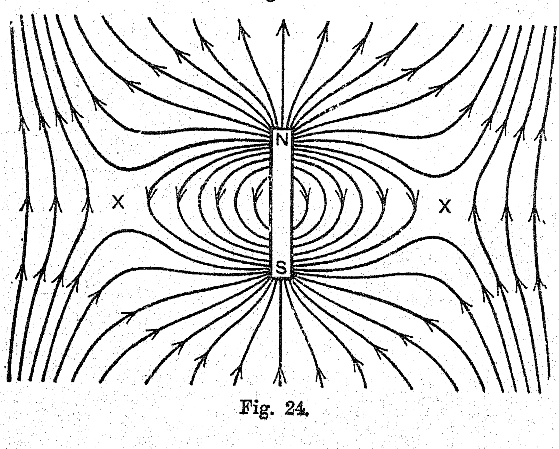


Fig. 24.

and the magnet pole the compass obeys the magnet and ***sets with its north pole pointing southwards***, towards the south pole of the magnet. Beyond X the compass obeys the earth, and ***sets with its north pole pointing northwards.*** At X the compass sets in any position. There will be a second null point off the other pole of the magnet and due magnetic south of it.

(*c*) Repeat the experiment with the magnet in the magnetic meridian as before, but with the north pole pointing northwards. Fig. 24 gives the result. In this case it will be readily seen that the earth's horizontal field and the magnet's horizontal field are in the same direction off the ends of the magnet, but they are in opposite directions off the sides of the magnet. The null points X where the two fields are equal and opposite are as indicated, *i.e.* on the line bisecting the axis of the magnet at right angles.

(*d*) Repeat the experiment with the bar magnet lying east and west at right angles to the meridian. Fig. 25 gives the result when the south pole of the magnet is towards the east. X is again a null

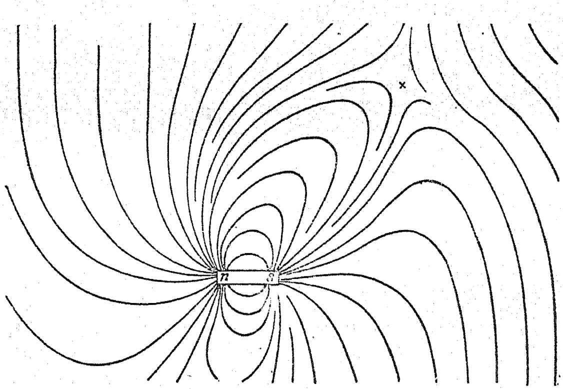


Fig. 25.

point, and there is another diagonally opposite to X. Repeat with the north pole of the magnet towards the east.

The above experiments are of importance, since certain magnetic measurements can be obtained from them.

From the definition of a line of force given above, it is clear that lines of force cannot intersect, for if two lines, for example, crossed each other, it would mean that at the

point of intersection the resultant force would act in two different directions, which is impossible.

So far we have concerned ourselves only with the space *outside* the magnet—the "external field" as it is called—and we have seen that the lines of force pass from the north pole through the field to the south pole, this being the *positive* direction, *i.e.* the direction in which a north pole would be urged. Further, these lines on reaching the south pole are continued through the substance of the magnet finally reaching the north pole again and forming closed curves. As will be seen later (Art. 37*a*), the parts

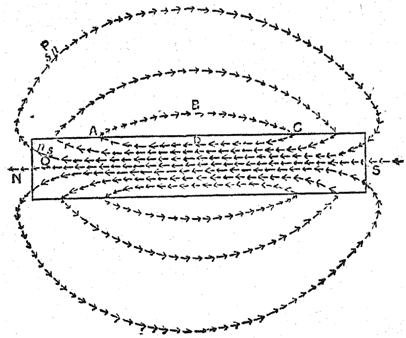


Fig. 26.

of these closed curves inside the magnet are, however, not lines of force as previously defined but what are called *lines of induction* (*in air* lines of force are also lines of induction—see next page). Thus when a line of induction *CDA* (Fig. 26) leaves the magnet at *A* it is continued into the line of force *ABC* (which is also a line of induction) in the air, and when this passes from the air to the magnet it is continued into the line of induction in the magnet. For simplicity we are, at this stage, making no reference to the earth's field in which the permanent magnet *NS* is lying, nor to the field in the magnet in the directions **N to** *S due to its own poles* (demagnetising effect). Again, polarity appears on those parts of the surface of a magnet where lines emerge from it into the air or pass

from the air into the magnet, northern polarity appearing at the former regions and southern polarity at the latter. In the case of a simple magnet the lines emerge from, and pass into, the ends only, and there is no lateral magnetism (Fig. 27).

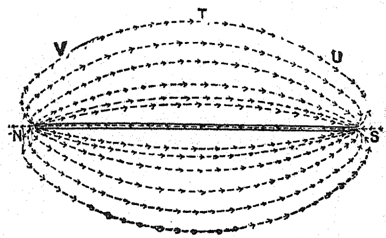


Fig. 27.

It may now be mentioned that *if we fix our attention on the idea of induction* which takes place in a magnetic field, and define a **line of induction as the direction along which induction takes place,** we are concerning ourselves more with the idea of *some alteration in the medium due to the magnetic force,* and the closed curves we have been dealing with, both the parts in air and the parts in the magnet, are "lines of induction" throughout. *In air* they are also lines of force. These ideas will, however, receive fuller treatment later; for the present we are dealing with the external field of a permanent magnet in air and *the direction of the forces in that field,* and therefore with the lines of force as previously defined, and as shown directly by the previous experiments.

Faraday, whose attention was directed to the influence of the medium between two magnetic poles rather than to the action of the magnetic poles on each other at a distance, pictured the magnetic effects which take place in a magnetic field as being due to a condition of strain in the medium, consisting of a tension in the direction indicated by the lines of force and a pressure at right angles to that direction. The effect, in fact, is the same as if each line of force tended to contract in the direction of its length (just

as a stretched elastic cord would tend to contract), whilst lines running in the same direction tended to repel each other. Thus longitudinal contraction accounts for the attraction between the unlike poles of Fig. 12, whilst lateral repulsion accounts for the repulsion between the like poles of Fig. 14, and similar remarks apply in all other cases of attraction and repulsion between magnetic poles. Further, when a magnet is deflected from its position of rest in a magnetic field, the lines of force will be distorted, and, tending to shorten, will exert a stress upon the magnet tending to bring it back to its original position of rest in the field.

In the mathematics of lines of force it is usual to conceive them gathered together in such a way as to form tubular spaces (which will, for example, be cylindrical if the lines of force are straight and parallel, conical if the lines of force are straight and not parallel), such tubes touching each other laterally and filling the entire field. These are called **tubes of force**, and when conceived on a definite plan so that a definite number emanate from any given pole they are called **unit tubes of force** (Art. 18). The stronger the magnetic pole the more unit tubes of force are conceived to emerge from it, and the stronger a magnetic field the more unit tubes pass through a given area; in fact, as will be seen later, the number of unit tubes of force per unit area taken at right angles to the direction of the force at any point in a magnetic field is, by convention, numerically equal to the strength of the field at that point.

If these tubes be endowed with the property that they tend to contract in the direction of their length and to expand laterally, we have an explanation of the fundamental facts in magnetism. Carrying the conception further, we may say, in the case of a permanent magnet in air, that tubes of force pass from the north pole through the external field to the south pole and are then continued through the magnet as tubes of induction from the south pole to the north pole. Further, as mentioned above in the case of lines of force and induction of a magnet in air, *if we concentrate on the idea of induction due to magnetic*

force, the closed circuits are tubes of induction throughout. In air they are also tubes of force.

Two warning notes may be given at this stage. Many text-books use the expression "lines" in quantitative work where "unit tubes," *conceived on a definite plan* to be given later, is really intended, and in electrical engineering it is, in fact, the custom. The importance of this statement will be understood after reading Chapter II. In the sections which follow "lines" is used in a general sense and "unit tubes" whenever numerical relations are referred to.

The reader will probably have grasped the distinction between "lines or tubes of force" and "lines or tubes of induction" from the preceding: *they refer to different ways of looking at the actions in a field.* When we speak of the magnetic force in a magnetic field we are thinking of the action of the medium *on something other than itself*; in fact, the intensity of a field is measured *by the force* it exerts *on a certain pole put in it.* When we speak of induction we are thinking of *some change in the medium itself.* The student of mechanics will see that the former corresponds to "stress" and the latter to "strain," and just as

$$\text{Strain} \propto \text{Stress},$$

so

$$\text{Induction} \propto \text{Intensity},$$

i.e.

$$B \propto H.$$

The unit of induction, however, is so taken that in air (strictly *in vacuo*) $B = H$, and lines or tubes of induction in air coincide with lines or tubes of force. In a magnetisable medium (*e.g.* iron in a magnetic field) B and H have different values, B being greater, and the lines or tubes of induction exceed the lines or tubes of force; in crystalline media the lines or tubes of force and induction do not even necessarily coincide in direction. All these points will, however, be discussed later.

6. Methods of Making Magnets.—Given one magnet (or more) any number can be made by utilising the property of induction.

Exps. *To make a magnet by the methods of* (1) *single touch,* (2) *divided touch,* (3) *double touch.*

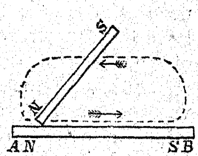


Fig. 28.

(1) *Single Touch.*—Place the bar AB to be magnetised on the table. Hold a permanent magnet NS in the position indicated (Fig. 28), and rub the bar from end to end several times with one pole—the north in Fig. 28—of the magnet. The magnet must be drawn over the bar always in the same direction, A to B, and each time the end B is

reached it must be lifted well clear of the bar in carrying it back to the end *A*. This rubbing process must be repeated on the other side of the bar, which will be magnetised as shown, the end *B* being a south pole and *A* a north pole. It will be noted that *the end of the bar where the rubbing magnet leaves it is of opposite polarity to the rubbing pole.*

(2) *Divided Touch.*—Two bar magnets are placed in line on the table with unlike poles facing, and the bar to be magnetised is placed with its extremities resting on these two poles (Fig. 29). It is then rubbed from the middle outwards by the dissimilar poles of two other magnets, the rubbing ends of which follow the paths

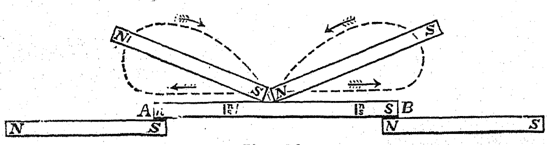


Fig. 29.

indicated by the dotted lines and arrows. The end of the bar on the left rests on a south pole, and that half of the bar is rubbed by a south pole; the other end rests on a north pole, and that half is rubbed by a north pole. By the rule given for "single touch" the end of the bar on the left is a north pole and that on the right a south pole, and the effect of the bottom magnets is also to magnetise the bar in this direction by induction.

(3) *Double Touch.*—The bar and magnets are arranged as above, a piece of wood or cork being placed between the poles of the rubbing magnets. These are then moved together from the middle to either end, back to the other end, and so on backwards and forwards, finishing at the middle. The rule for polarity is as before.

Except for the occasional magnetisation of knitting-needles, etc., for laboratory purposes, the above methods have been superseded by methods based on the utilisation of an electric current, and known as **electromagnetic methods.** Thus if a coil of insulated copper wire be wrapped round, say, a cylinder of cardboard (forming what is called a *solenoid*), the bar to be magnetised inserted in the cylinder, and a strong electric current passed through the coil, the bar will be found to be magnetised (Fig. 30). If it be of steel, when the current is stopped and the bar removed it will be found to be a

permanent magnet; if it be of soft iron it will be a strong magnet while the current is passing, but will lose the bulk of its magnetisation when the current ceases. The details of these electrical methods will be given later, but the following "rules for polarity" may be noted:—

(*a*) *Looking at the end of the bar, if the current in the coil is counter-clockwise in direction that end will be a north pole; if it be clockwise that end will be a south pole.* (End Rule.)

(*b*) *Hold the thumb of the right hand at right angles to the fingers; place the hand on the wires with the palm facing the bar and the fingers pointing in the direction of the current. The thumb will point towards the north pole of the bar.* (Hand Rule.)

(*c*) *Imagine a man swimming in the circuit in the direction of the current, with his face to the bar; his left hand will point towards the north pole of the bar.* (Ampère's Rule.)

From the above it follows that to convert a horseshoe-shaped piece of iron or steel into a magnet by this method, one limb must be wound in one direction and the other in the opposite direction, so that the ends may be north and south poles respectively (Fig. 30).

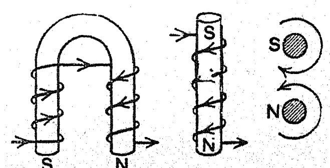


Fig. 30.

Electromagnets consist of cores of soft iron wrapped round with coils of insulated copper wire. The cores become powerful magnets while the current is passing, and bars of steel can be converted into magnets by rubbing them over the poles of these electromagnets after the manner, for example, of single touch. Mere contact, in fact, with a pole of a powerful electromagnet will often convert a bar of iron or steel into a strong magnet.

7. Practical Points.—From the results of numerous experiments the following practical points may be summarised.

For permanent magnets steel should be used containing

from 5 to 8 per cent. of tungsten and from ·4 to ·6 per cent. of carbon. Mme Curie's experiments seem to indicate that the presence in the steel of about 4 per cent. of molybdenum is even more effective than tungsten in producing permanence of magnetisation. Owing to the demagnetising action of the poles of a magnet on itself, long bars have greater powers of retaining their magnetisation than short bars, for the poles are further apart, and therefore exert less demagnetising effects on the more central portions of the bar; hence for permanent magnets the ratio of the length to the diameter—the "**dimension ratio**" as it is called—should be as large as possible.

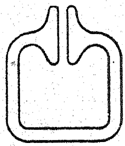


Fig. 31.

The more nearly the magnet is formed into a closed circuit the more permanent is its magnetism. Hence, in the case of horseshoe magnets, the nearer the poles are brought together the better is the result from this point of view; it has also been shown that in this case, if the magnet be provided with enlarged pole pieces (Fig. 31), the demagnetising effect is reduced and greater constancy results.

The magnet should be forged at as low a temperature as is convenient in practice. Before hardening, it is advisable to heat it to about 900° C., allow it to cool to about 750° C., maintain it at this temperature for a time, and then allow it to cool; this tends to make the material normal and homogeneous. In hardening it should be heated to about 950° C., allowed to cool to about 700° C., and then quenched at this temperature in oil at about 20° C. The magnet should then be matured by steaming for several hours, allowing it to cool once or twice during the process. It should be magnetised by some electromagnetic method, being gently tapped by a wooden mallet during the process. The current should not be stopped suddenly, but should be reduced gradually to zero.

Early experiments showed that in the case of thick steel bars, only the surface layers were magnetised; thus, if such a magnet were placed for a short time in strong nitric acid and the surface layers eaten off, the magnet was

found to have lost practically all its magnetisation. This led to the introduction of *compound* magnets, in which several thin bars of the required length were separately magnetised and bound together. Fig. 32 shows the simple forms of a compound bar magnet and a compound horseshoe magnet. All the north poles of the component magnets are at N., and constitute the north pole of the compound magnet. Similarly, all the south poles of the component magnets at S constitute the south pole of the compound magnet. Electromagnets are also often built up of thin strips or wires. Magnets built up in this way are also called **laminated** magnets. Modern fine-grained steel and modern methods of magnetising have practically done away with the necessity for compound *permanent* magnets.

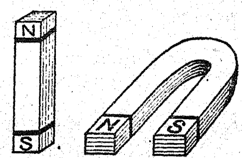


Fig. 32.

In the case of an electromagnet the material and shape depend on the purpose for which the magnet is to be employed. Being usually intended to quickly take up and quickly lose magnetisation, the cores should be of pure Swedish iron thoroughly annealed. Since the demagnetising effect of short magnets is very pronounced, the cores are often short. If the electromagnet is to act upon an armature at a fair distance, the poles should not be too near together, otherwise the lines of force will pass from pole to pole without traversing the armature. If the electromagnet is to exert a maximum pull over a short distance, the magnetic circuit of the electromagnet should be as complete as possible, and the pole area must be kept small, *i.e.* the surfaces of contact of the magnet and its armature should be rounded off. Theory shows that the pull (P) between two magnet faces of area A square centimetres is given by the expression

$$P = \frac{B^2 A}{8\pi} \text{ dynes (Art. 276)},$$

where B is the *number of unit tubes per square centimetre*

at the surfaces of contact. Rounding off the poles will decrease the area of contact A, but the resulting increase in B, and therefore B^2, will considerably more than compensate.

The strength of an electromagnet also depends, of course, upon the strength of the current and upon the number of times it is carried round, *i.e.* upon the number of turns in the wires, but the magnetisation is only proportional to these two factors when the core is far from being completely magnetised, *i.e.* far from saturation. **Fröhlich's law** of the electromagnet is

$$B = \frac{I}{a + \beta I},$$

where B has the same meaning as before, I is the current, and a and β are constants. Thompson gives the formula

$$N = Y \frac{I}{I + I'},$$

where N is the *total number of unit tubes* emanating from the pole of the magnet, Y is the maximum number of unit tubes if the current were indefinitely increased, and I' is the current necessary to half saturate the iron, or, as it is called, the **diacritical current.** These formulae need not be learned; they are merely inserted here for reference.

8. The Molecular Theory of Magnetisation.—According to this theory—

(1) The molecules, not only of a magnet but also of a piece of unmagnetised iron or steel, are complete magnets.

(2) Before the material is magnetised, these molecular magnets under the influence of their own mutual forces

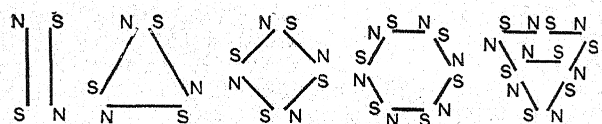


Fig. 33.

arrange themselves with their magnetic axes in various directions so as to form closed circuits amongst themselves, in which case no lines pass out into the external space and the material exhibits no signs of magnetisation. The

actual "irregular" arrangement of the molecules of an unmagnetised bar of iron is not known, but it is possible that the particles arrange themselves in neutralising groups of two or more molecules, stable under ordinary circumstances, somewhat as indicated in Fig. 33.

(3) The process of magnetisation consists in rotating these molecular magnets so that their axes point more or less in the direction of the magnetising force (*i.e.* the influence producing magnetisation), their north poles thus pointing one way and their south poles the other. The restraining influence which prevents them all pointing one way on the application of the least magnetising force is due to the action of each molecule on its neighbour. As the magnetising force increases so more and more molecules become rotated, until, when all point in the proper direction, the material is magnetised to saturation, and no further increase in magnetising force can increase the magnetisation.

The conditions are now somewhat as indicated in Fig. 34. Throughout the interior of the material each molecular pole touches an unlike molecular pole and the two neutralise, so that there is no free polarity. At the end on the

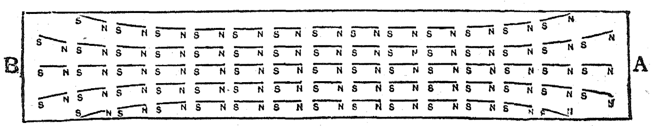


Fig. 34.

right we have a number of free molecular north poles and at the end on the left a number of free molecular south poles, so that the former end of the material exhibits north polarity, the latter end south polarity, lines of force pass through the external field, and the material is, in fact, now a magnet.

The figure indicates the existence of lateral magnetism; this bending of the molecular chains towards the sides as they approach the ends may be partially explained by the existence of lateral repulsion between adjacent chains, in

conjunction with the tendency of lines of force to contract longitudinally. Fig. 35 would depict an ideal case of a bar magnetised to saturation with no lateral magnetism and poles only at the ends.

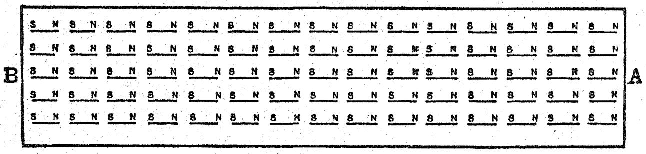


Fig. 35.

There are many facts met with in elementary magnetism which point to the truth of, or are readily explained by, the molecular theory.

(*a*) If a magnet be broken into two parts each will be found to be a magnet, a north pole appearing at one side of the break and a south pole at the other (Fig. 36). If the parts be again broken the same result holds, and, in fact, *it is impossible to obtain a magnet with only one pole.* This is in agreement with the molecular theory, for since it is impossible to cut through a molecule the fracture must take place between them, and an examination of Fig. 34 will show that this must leave a north pole at one side of the fracture and a south pole at the other.

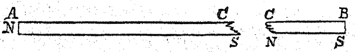


Fig. 36.

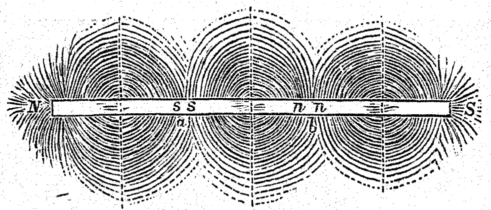


Fig. 37.

(*b*) When a magnet is irregularly magnetised it may exhibit what are known as *consequent poles*, that is to say, poles lying between the terminal ones of the magnet. Fig. 37 shows a magnet

with consequent poles at *n* and *s*. The magnet practically consists of three smaller magnets joined end to end, so that similar poles are united at *n* and *s*. Magnets with consequent poles may be produced either by accidental irregularity in magnetisation or purposely. For example, by touching a bar of steel at several points with the poles of a strong magnet, consequent poles of polarity opposite to that of the magnet pole are produced at the points touched. In the left hand portion (Fig. 37) the molecules have their north poles to the left, their south poles to the right; in the middle portion they have their south poles to the left, their north poles to the right; and in the right hand portion they have their north poles to the left, their south poles to the right.

(*c*) Reference has been made to the fact that in the case of thick bars only the surface layers are magnetised (Art. 7). This is explained by saying that the magnetising force is only capable of rotating the surface layer of molecular magnets, the interior molecular magnets being still in the form of neutralising groups forming complete circuits amongst themselves. Further, the rule for polarity given in connection with "single touch" is quite in accord with the molecular theory.

(*d*) Experiment shows that there is a limit to the magnetisation of a magnetic material beyond which it is impossible to go however great the magnetising force, and the material is then said to be saturated. On the molecular theory magnetic saturation would, as previously indicated, correspond to the setting of all the molecules in a definite direction.

(*e*) When a bar is suddenly magnetised by a strong electric current a sharp click is heard, which indicates some molecular rearrangement, probably accounted for by the molecular theory.

(*f*) When a bar of iron is magnetised it becomes very slightly longer; as will be explained in Art. 13, the effect is complicated, but, broadly speaking, it indicates molecular rearrangement as above.

(*g*) Heat is produced when a bar is rapidly magnetised and demagnetised; this points to the same conclusion and supports the molecular theory. When a molecular magnet swings round into some new position, consequent upon the action of a magnetising or demagnetising force, it acquires kinetic energy and oscillates about its new position until that energy of motion is gradually converted into heat.

(*h*) A twisted bar tends to untwist during magnetisation. Here again the effect is complex, but in a general way it supports the theory.

(*i*) It has been indicated that soft iron is more readily magnetised than hard steel, but that it loses its magnetisation much more readily than hard steel. This is explained by the fact that the **molecular rigidity** (*i.e.* the opposition to any molecular movement) of hard

steel is greater than that of soft iron; the molecular magnets in steel are therefore more difficult to rotate into a definite direction than those of iron, but once there they are more difficult to rotate back again.

(*j*) If a magnet be made red hot and then be allowed to cool it will be found to have become demagnetised. The heat increases the molecular motion and diminishes the molecular rigidity, so that the molecular magnets obey their natural tendency, *i.e.* swing round and form complete circuits amongst themselves. For a like reason a red hot ball of iron suspended opposite the pole of a magnet is not attracted, but if allowed to cool below a certain temperature known as the *critical temperature* (about 750° C., differing for different specimens) induction takes place and attraction follows (Art. 14).

(*k*) If a bar during the process of magnetisation be gently tapped with a wooden mallet the magnetisation is assisted, for the vibration (to put the matter simply) loosens the molecules, and the magnetising force is better able to set them in a definite direction. For a somewhat like reason, tapping a permanent magnet in a weak field or throwing it about will tend to demagnetise it.

(*l*) The action of a "keeper" in reducing the demagnetising effect of a magnet on itself has already been mentioned (Art. 4) and their general effect in increasing the retentivity of a magnet will now be realised. Fig. 38 shows a horseshoe magnet fitted with a keeper of soft iron across its poles. Without the keeper the end molecules are more or less unstable and easily disturbed, so that they tend to rotate obeying their mutual attraction and repulsion, thereby producing partial demagnetisation. With the keeper in position, however, it is magnetised by induction, and each pole of the magnet being faced by an unlike pole in the keeper the two tend to hold and strengthen each other. Further, the molecular magnetic chains of the magnet are continued in the keeper, forming a closed circuit which has very little tendency to break and rearrange itself; hence the magnet retains its magnetism.

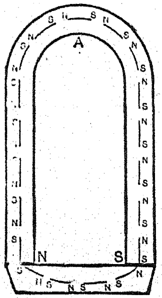


Fig. 38.

The reader may be warned, however, that keepers are not the "be all" and "end all," for experience shows that whilst on the whole their employment is beneficial the result is a compromise, for every time a keeper is suddenly put on the magnet is weakened, and every time it is suddenly pulled off the magnet is strengthened. The effect produced on the external field of a horseshoe magnet by placing a keeper across its poles should, by iron filings, be compared with the case without a keeper shown in Fig. 13. When the keeper is on there is, as we should expect, little external field.

(*m*) A ring of iron may be magnetised *circumferentially* by drawing a magnet round it several times, or by means of an electric current (Fig. 39). The molecules are set in a definite direction, but there are no free poles and no signs of magnetisation. If the ring be cut, however, a north pole appears at one side and a south pole at the other (Fig. 40). This is in accord with the molecular theory.

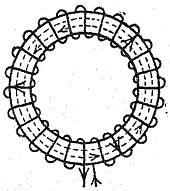


Fig. 39.

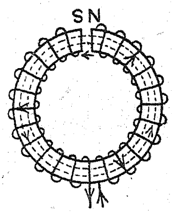


Fig. 40.

If the molecules of a bar undergoing magnetisation moved with perfect freedom, the smallest magnetising force would be sufficient to produce saturation, and this, of course, does not hold in practice. *Weber*, to whom the molecular theory is really due, suggested that each molecule was under the influence of a force towards its neutral position, tending, therefore, to maintain it in its original position, the actual position assumed by the molecule at any stage being such that its magnetic axis was in the direction of the resultant of this force and the magnetising force at that stage.

This, however, does not account for the existence of residual magnetism, and *Maxwell* suggested that the phenomenon was somewhat similar to the strain in a solid. In the latter case, if the strain does not exceed the elastic limit, recovery is more or less perfect, but if this limit be exceeded recovery is not perfect; hence Maxwell assumed that if the rotation of the axis of the molecule was below a certain value it would return to its original position when the magnetising force was removed; but if the rotation exceeded this value the molecule would not return when the magnetising force was removed, and there would be a permanent set. *Wiedemann* suggested that the opposition to rotation was of a frictional character, *i.e.* that some

sort of friction must be overcome in setting the molecules into a definite direction, which friction must be again overcome in demagnetisation.

Ewing has shown, however, that it is quite unnecessary to assume either Weber's directive force, Maxwell's strain analogy and permanent set, or Wiedemann's frictional resistance, and that the mutual attractions and repulsions between the molecular magnets themselves are sufficient to account for all the observed phenomena.

Consider four molecules in equilibrium in the absence of any magnetising force, and arranged as shown in Fig. 41 (*a*): the group forms a closed magnetic circuit, and there is no external field. Let now a small magnetising force be applied in the direction indicated. The molecules are slightly rotated as shown in Fig. 41 (*b*), their north poles moving in the direction of the magnetising force. There is now a small resultant magnetisation in the direction of the force and proportional to it, but if the magnetising force be removed at this stage the molecules will merely return to their initial positions, and there will be no residual magnetism.

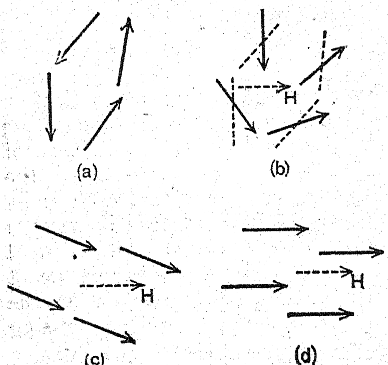


Fig. 41.

As the magnetising force is increased a point is reached where the arrangement becomes unstable; the molecules then swing rapidly round into new stable positions, their directions approximating to that of the magnetising force (Fig. 41 (*c*)). This rearrangement is completed within a very small range of the magnetising force, the magnetisation and the external field rapidly increase, and if the magnetising force be removed the arrangement is but

slightly disturbed, and there is, therefore, residual magnetism. If the magnetising force is still further increased the molecules tend to set still more in the direction of the force (Fig. 41 (*d*)), and the saturation condition is reached.

Fig. 42 (*a*) gives the "magnetisation curve" for this group and Fig. 42 (*b*) gives the curve for a sample of iron actually obtained in practice. The exact meaning and construction of such curves will be given later; for the present the reader may concern himself only with the general similarity, which is further evidence of the molecular theory. In Fig. 42 (*a*) the part *A* corresponds to the first stage in the process of magnetisation (Fig. 41 (*b*)). *B′* corresponds to the second stage, the sudden swing round (Fig. 41 (*c*)), and *C* corresponds to the third stage (Fig. 41 (*d*)). It is quite clear that in the case of an actual bar of iron undergoing magnetisation there will be a large number of such groups as the one considered, together with others, and, as these will not all pass from one stage to another at the same time, the sharp-cornered curve of Fig. 42 (*a*) will become the smooth curve of Fig. 42 (*b*) in practice.

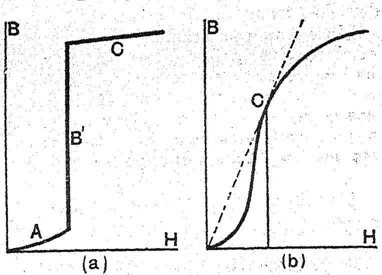


Fig. 42.

Although the molecular theory explains the fundamental facts in magnetism and magnetisation, *it does not explain why the molecules are magnets.* Ampère assumed that the magnetism of the molecules was due to currents of electricity flowing round them in perfectly conducting channels, in which case, once started, they would continue to flow without dissipation of energy, and the molecules would be magnetised in accordance with the rules of Art. 6.

The modern "electron" theory may be looked upon as an extension of the Ampèrian current idea above. The existence of particles more minute than any previously known in science has

now been clearly established, the mass of each being about $\frac{1}{2000}$ part of the mass of a hydrogen atom, and each being always associated with a definite (negative) charge of electricity; in fact the "mass" is probably due entirely to the "charge." These are called **electrons.** An atom probably consists of a nucleus of positive electricity or **protons** surrounded by a number of electrons, the latter revolving in orbits round the nucleus. The nucleus may also contain electrons but the protons are always in excess so that the nucleus is on the whole positive, and this positive is balanced by the external electrons when the atom is neutral. Electrons may be detached from or added to an atom thus resulting in charged atmos.

Since a molecule is an aggregation of atoms, and a moving charge of electricity has been proved to produce the same magnetic effects as a current of electricity, **the modern electron theory of magnetism, viz. that the latter is due to revolving electrons,** has been developed by Langevin and others, and certainly the idea accords well with many important and even complex phenomena in magnetism. This will receive treatment in Chapter XXV.

9. Preliminary Ideas on Permeability and Susceptibility. Magnetic Screens.

Exp. Note the distribution of the lines of force in the field between the unlike poles of two bar magnets placed in line (Fig. 12). Repeat this experiment with a piece of soft iron between the poles. It will be noted that the lines (Fig. 43) crowd into the iron as if *they found it easier to go through the iron than through the air.* The iron is, of course, magnetised by induction, a south pole appearing on the left where the lines enter, and a north pole on the right where they leave.

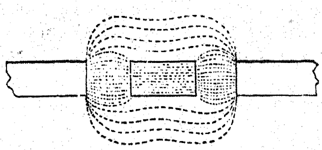


Fig. 43.

For practical purposes we regard every substance as possessing a certain power of conducting lines of force, and of offering a certain resistance to the passage of the lines; thus from the above experiment we deduce that iron is a better conductor of lines than air, or, as we now say, is more "permeable" or has greater "**permeability**" than air. Putting the same idea another way, we say that the magnetic resistance or "*reluctance*" of air is greater than that of iron. **The permeability of a medium may therefore be defined as its conducting power for lines**

of force as compared with air; thus in Fig. 43, if there were five unit tubes per square centimetre in the air space before the iron was placed in position, and 4,000 unit tubes per square centimetre in the iron when placed as indicated, the permeability of the iron would be said to be 800 (neglecting the demagnetising effect of the induced poles).

The above facts are usually expressed in a general form as follows:—If H denotes the number of unit tubes of force per square centimetre in the magnetic field (air) before the material is placed therein, then H measures the **magnetic** or **magnetising force.** If B denotes the number of unit tubes of induction per square centimetre in the material when placed in the field, then B measures the **magnetic induction** or **flux density. The ratio of B to H measures the permeability (μ) of the material.** Thus

$$\mu = \text{Permeability} = \frac{\text{Flux density}}{\text{Magnetising force}} = \frac{B}{H},$$

$$\therefore B = \mu H.$$

The magnetic field producing magnetisation may be due either to permanent or electro-magnets, or to a current in a coil of wire, but, to be exact, the value of the magnetising force used in the above expression should be the value *in the material*, and not the value in the field before the material is inserted, and it is clear that, owing to the demagnetising effect of the poles of the material, the former is less than the latter; in other words, a value of the magnetising force should be used in the expression for μ less than the value H given above.

The reduction in H owing to the demagnetising effect of the poles in the case of a bar of the material depends on the magnetisation of the bar and on the "dimension ratio" (Art. 7), but theory shows that if the bar has a length from 500 to 1,000 times the diameter, there is very little error in assuming that the magnetising force in the bar is identical with the magnetising force before the bar is introduced into the field. Further, if the material be in the form of a ring, the field being produced by a current in a coil of wire wrapped round it as shown in Fig. 39, there are no free poles developed in the ring (Art. 8), and there is no demagnetising action, so that the magnetising force in the material is identical with that in the empty coil.

In cases other than the two just mentioned, if H_1 denote the

value of the magnetising force *in the material* (supposed ellipsoidal),

$$H_1 = H - NI,$$

where H has the meaning given above, I is the intensity of magnetisation of the material, and N is a factor depending on the dimension ratio of the material. If the length is 500 times the diameter, the value of N is only ·0003. An *exact* definition of I appears later.

From the preceding pages it is clear that the values of B and H for air (to be more exact, for a vacuum) are taken as being the same, which means that the permeability (μ) for air is taken as unity. In other substances, it is important to remember that the permeability is not constant, as will be seen later, but that it depends upon many things —the amount of magnetism present, the magnetising force, the temperature, the previous history of the material, etc.

It has been indicated that the stronger the magnetisation developed in a material by a given magnetising force, the greater is said to be the **susceptibility** of the material to magnetisation. The expression "intensity of magnetisation" is defined in Art. 20; for the present we will define it in a somewhat vague manner as the amount of magnetism per unit sectional area. If I denote the intensity of magnetisation of the material in Fig. 43 and H the magnetising force, then **the ratio of I to H measures the susceptibility (κ) of the material.** Thus

$$\kappa = \text{Susceptibility} = \frac{\text{Intensity of magnetisation}}{\text{Magnetising force}} = \frac{I}{H},$$

$$\therefore\ I = \kappa H.$$

The previous remarks on the reduction of H due to the demagnetising effect of the poles and the opposing field set up thereby also apply in this case.

We shall see later that in such a case as the one considered

$$B = H + 4\pi I,$$

which means that the total number of tubes per square centimetre (B) in the specimen is equal to the sum of two lots, viz. H tubes per square centimetre due to the field, and $4\pi I$ tubes per square centimetre due to the magneti-

sation, *i.e.* the effect of the field upon the specimen. Dividing this expression throughout by H we have

$$\frac{B}{H} = 1 + 4\pi\frac{I}{H},$$

$$i.e.\ \mu = 1 + 4\pi\kappa,$$

and since μ for air (or vacuous space) is taken as unity, κ for air (or vacuous space) is taken as zero.

The above is an introductory elementary treatment only, to acquaint the reader with the general meaning of the terms in use; fuller details are given in Chapter XIX.

Exps. Place a bar magnet on a level with, and east or west of, a compass needle: the latter is deflected. Interpose sheets of wood, paper, glass, brass, etc., between the compass and the magnet; the deflection is still the same, showing that magnetic action takes place equally well through these media as through air. Place a large *thick* sheet of iron (Fig. 44) between the magnet and compass: the latter returns towards its normal position. In this case the lines from the magnet pass into the iron, but, owing to the greater permeability of iron, they do not pass right through into the space beyond, but prefer to continue

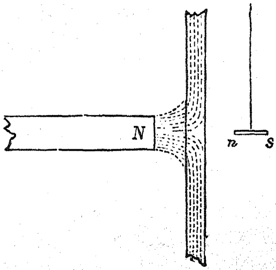


Fig. 44.

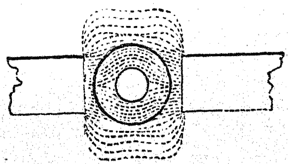


Fig. 45.

through the sheet towards the edge, and thence back to the other pole. Thus the needle is more or less *screened* from the action of the magnet, although in this case it may be affected by the induced pole in the centre of the iron sheet.

Place a soft iron ring between the unlike poles of two magnets as shown in Fig. 45, and sprinkle iron filings over the combination. It will be noted that the filings do not set themselves along definite lines *in the space within the ring*. The lines pass through the iron of the ring, preferring to keep in this rather than to cross the central

air space, so that this space is more or less effectively *screened* from the influence of the magnets. This is utilised in certain galvanometers which are placed inside a thick cylinder of soft iron, which practically protects the needle of the instrument from magnetic fields outside.

Theory shows that the screening effect of a cylinder of iron is given by the expression

$$H_i = \frac{H_o}{1 + \frac{1}{4}\left(\mu - 2\right)\left(1 - \frac{r^2}{R^2}\right)},$$

where H_i is the field inside, H_o the field outside, r the inner radius, R the outer radius, and μ the permeability of the iron. Thus the thicker the cylinder and the greater the permeability, the more effective will be the screening action. Since μ never equals infinity, the screening action can never be *theoretically* perfect.

10. Preliminary Ideas on Magnetisation Curves and Hysteresis.

Consider a long thin rod of any magnetisable material (the "dimension ratio" being such that the demagnetising effect of the poles may be neglected) to be placed in a solenoid through which a current of any desired strength may be passed. Let this current be gradually increased, and at various stages during the process let the value of the magnetising force (H), the flux density (B), and the intensity of magnetisation (I) be calculated *by methods to be given later*. We have thus a series of corresponding values of B, I, and H from which curves can be plotted showing the relationship between these quantities.

Two curves are frequently met with. In one, values of H are taken as abscissae and the corresponding values of B as ordinates, and the curve is known as a BH curve; the value of the permeability at any stage of the process is found from the corresponding point of the curve by the relation $\mu = B/H$. In the other, values of H are taken as abscissae and the corresponding values of I as ordinates, and the curve is known as an IH curve; the susceptibility at any stage is found from that point of the curve by the relation $\kappa = I/H$.

Fig. 46 gives the *BH* curves for various samples of iron and steel With a magnetising force of only 2·5 the soft iron (annealed) gives a value of *B* of about 8,000, the cast iron about 3,000, the hard iron about 1,000, and the glass-hardened steel about 400. When *H* has the value 10 the soft iron curve is becoming horizontal, *i.e.* the iron is approaching saturation, but this does not take place with the

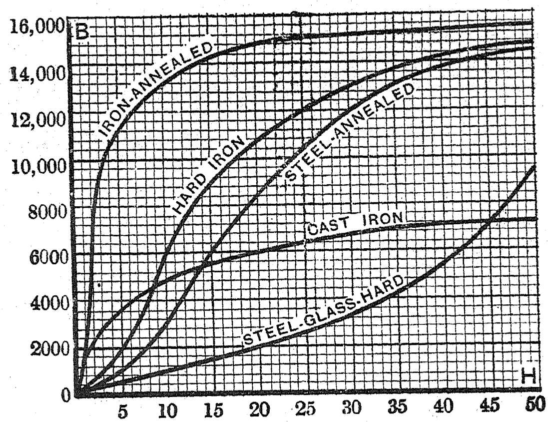


Fig. 46

steel and the hard iron until *H* is from 40 to 45. When *H* is 50 the soft iron, hard iron, and steel are all practically saturated, but the glass-hard steel is still rising, and its value of *B* is only equal to that in soft iron under a magnetising force of about 2·6. The *IH* curves would be somewhat similar.

There are three different stages indicated by a complete magnetisation curve drawn to a suitable scale :—

(*a*) A short initial stage when the magnetising force is small, in which a change in *H* produces only a very small change in *B*, and where the permeability is small. This part of the curve is better seen in Fig. 42 (*b*). For *very small values of H* the curve is a straight line inclined to the horizontal so that $\mu = B/H$ is constant.

(*b*) A stage where the curve is rising rapidly, *i.e.* where a small

change in H produces a large change in B, and where the permeability is increasing rapidly to a maximum value. Since $\mu = B/H = \tan\theta$ (Fig. 42*b*), the permeability has its maximum value at C in that figure.

(*c*) The saturation stage or nearly horizontal part of the curve. Here a large increase in H produces *comparatively* little effect on B, and the permeability drops to quite low values

In the figures the scale used for B is much smaller than that used for H since the changes in H are much smaller than the changes in B. If the same scale were used for both, the saturation part would be a straight line, not horizontal, but inclined at 45° to the axes, showing that the change in B is merely equal to the change in H. This is also clear from the relation $B = H + 4\pi I$, since I has become constant on saturation.

These three stages should be compared with the three parts of the "curve" of Fig. 42*a*.

Consider again a long thin unmagnetised rod in the solenoid. Let the current be gradually increased from zero to a certain maximum value, then gradually reduced to zero. Let it now be reversed in direction (thus tending to magnetise in the opposite direction) and gradually increased from zero to the same maximum value in this reverse direction, then gradually reduced to zero. Let it be again reversed into the original direction and gradually increased from zero to the same maximum value. Such an operation as the one indicated, say a variation from a maximum in one direction through zero to a maximum in the opposite direction and again back through zero to the first maximum, is termed a **cycle.**

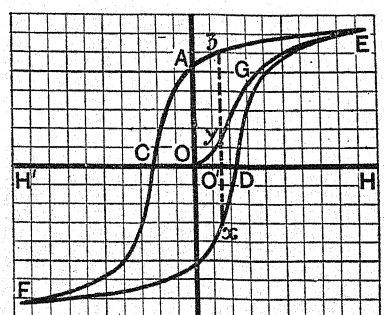


Fig. 47.

Fig. 47 shows the result of such an experiment. H

measured to the right is due to current flowing in the original direction, while H measured to the left is due to the reversed current B measured upwards is due to current in the original direction, while B measured downwards is due to the reversed current.

As the magnetising force H increases from zero to its maximum value represented by OH, the BH curve OGE is obtained, the maximum value of the flux density B being represented by HE. On reducing the magnetising force gradually to zero, the curve EA is obtained, so that OA represents the value of the flux density remaining when the magnetising force has been reduced to zero, *i.e.* **OA represents the residual magnetism** in the specimen to scale. On reversing the current and gradually increasing the magnetising force from zero to the maximum value represented by OH', the curve ACF is obtained; clearly OC represents the value of the reversed magnetising force necessary to bring the flux density to zero, in other words, to wipe out the residual magnetism, *i.e.* **OC represents what is called the "coercive force"** for the specimen to scale. On again decreasing the magnetising force to zero, reversing, and again increasing to the maximum value, the curve $FBDE$ is obtained. If the cycle be repeated the curve will be retraced, but the initial path OGE is never again traversed unless the specimen is first completely demagnetised.

It will be noted that the magnetic effects produced tend to persist, *i.e.* to lag behind the cause. Thus the value of B when H is diminishing is always greater than when H is increasing; when H is zero B still has a definite value, and it always requires a certain force in the opposite direction to bring B to zero. Prof. Ewing has given the name **hysteresis** (to lag behind) to this phenomenon, *i.e.* **hysteresis is the lagging of the magnetic flux or magnetic induction behind the magnetising force producing it.** The loop shown in Fig. 47 is termed the *hysteresis loop*, and *the area of the loop can be shown to be proportional to the energy wasted in the specimen due to the changing magnetic condition;* in fact in the BH case of Fig. 47 if the area is estimated in terms of B and H *and then divided by* 4π, the result is the waste of energy (in

ergs) per cubic centimetre per cycle, and if IH curves be plotted, the area, estimated in terms of I and H, is the waste of energy (in ergs) per cubic centimetre per cycle.

As in the case of Art. 9 *the above is an introductory elementary treatment only, for the purpose of acquainting the reader with the general meaning of the terms in use; fuller details are given in Chapter* XIX.

11. Ferromagnetics, Paramagnetics, and Diamagnetics.—It has already been indicated that *iron, steel, nickel, cobalt,* and *manganese* are the principal substances which can be made to exhibit magnetic properties, and of these iron and steel stand pronounced. In 1845 Faraday, by using powerful electromagnetics, demonstrated that all bodies are affected by a magnet, but that they are of two kinds, viz. *paramagnetic,* which are attracted and tend to move into the strongest part of the field, and *diamagnetic,* which are repelled and tend to move into the weakest part of the field.

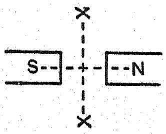


Fig. 48.

In experimenting with solids, small bars of various substances were suspended by a fine thread between the poles of a powerful electromagnet. If the substance was paramagnetic it set itself *axially, i.e.* along the line NS, if diamagnetic it set itself *equatorially, i.e.* along the line XX (Fig. 48).

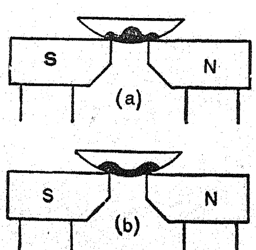


Fig. 49.

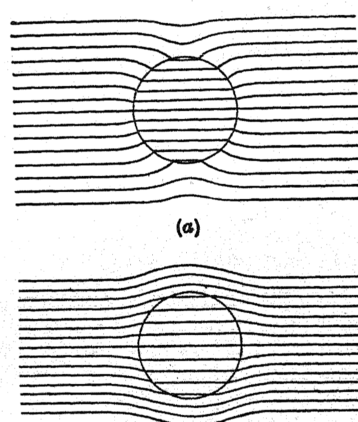
Liquids were enclosed in thin glass tubes and suspended between the poles, with the result that they set themselves either along NS or along XX and were classified accordingly. Plücker placed the liquids, in turn, in a watch-glass resting on the pole pieces as shown in Fig. 49. If the pole pieces are not more than about $\frac{1}{10}$ inch apart a paramagnetic liquid sets as at (a), *i.e.* it congregates where the field is strongest,

and if diamagnetic it sets as at (b), *i.e.* it moves away from this strongest part of the field. If the pole pieces are not close together the field is strongest *round about the poles* and the liquids set accordingly, so that the results are practically exactly opposite to those indicated above.

In dealing with gases the latter, rendered evident by traces of some other substance (*e.g.* ammonia and hydrochloric acid), were allowed to ascend between the poles, and it was noted whether they spread out between the poles or across them.

Experiments also indicated that the medium affected the results; thus a paramagnetic will act like a diamagnetic if it is surrounded by a medium more paramagnetic than itself.

The chief paramagnetics in descending order are iron, nickel, cobalt, manganese, crown glass, platinum, oxygen, titanium, palladium, osmium, and the chief diamagnetics are bismuth, phosphorus, antimony, flint glass, mercury, zinc, lead, tin, copper, water, alcohol. Diamagnetic phenomena are feeble compared with paramagnetic phenomena.



(*a*)

(*b*)

Fig 50.

Fig. 50 (a) shows the distribution of the lines when a paramagnetic body is placed in a magnetic field, and Fig. 50 (b) shows the same for a diamagnetic body. Since a diamagnetic is *repelled* by a magnet, it is evident that when acted on inductively it acquires polarity *opposite to that which would be acquired by a piece of iron.* Further, we have seen that for air (strictly a vacuum) B is taken as equal to H, therefore $\mu = B/H$ is unity and κ is zero (since $\mu = 1 + 4\pi\kappa$).

For a paramagnetic B is greater than H, therefore μ is greater than unity, and κ is positive and greater than zero. For a diamagnetic B is less than H, μ is less than unity, and κ is negative; κ in this case is *very small*, so that μ, although less than unity, never becomes negative. Strictly air is slightly diamagnetic.

Modern work on magnetism by Curie, Langevin, and others has led to a new classification in which iron, nickel, cobalt, and perhaps manganese are put into a class by themselves known as *ferromagnetics*. Briefly the following points of distinction may be mentioned:—

Ferromagnetics.—In these the magnetisation is *not* proportional to the magnetising force and κ is *not* constant, but depends on the magnetising force and the temperature; the variation in κ does *not* seem to follow any simple law. Ferromagnetics have the most pronounced magnetic properties and are attracted by a magnet.

Paramagnetics.—In these the magnetisation is proportional to the magnetising force, but κ varies with the temperature; over a wide range the variation in κ follows **Curie's Law,** viz. "with a given magnetising force κ is inversely proportional to the absolute temperature." Paramagnetics have only feeble magnetic properties and are attracted by a magnet.

Diamagnetics.—In these, the induced magnetisation is opposite to that of iron and they are repelled by a magnet. κ is practically constant, depending neither on the magnetising force nor on the temperature (bismuth is an exception).

The theories connected with this classification into ferromagnetics, paramagnetics, and diamagnetics are dealt with in subsequent chapters.

12. Magnetisation and Composition.—Reference has already been made to the magnetic difference between soft iron and hard steel, and to the fact that **tungsten steel** and steel containing a percentage of **molybdenum** are specially suitable for the manufacture of permanent magnets, the presence of the tungsten and molybdenum considerably increasing the coercive force of the material. Further, high grade carbon steels make better permanent magnets than low grade carbon steels for the same reason, *viz.* the coercive force increases with the percentage of carbon.

The presence of **manganese** causes a marked falling off

of the magnetic properties of iron. Thus in certain experiments of Barrett, Brown, and Hadfield the presence of 15·2 per cent. of manganese practically reduced the sample to a non-magnetisable body; with 12 per cent. of manganese the permeability was only about 1·2. Manganese itself is a very weak paramagnetic, but when fused in an electric furnace it passes into the ferromagnetic state and has marked coercive force although it does not attain strong magnetisation.

The presence of **nickel** in iron increases the coercive force until the percentage reaches 20, after which the coercive force decreases; thus in the experiments mentioned above, when the percentage of nickel was 19·64 the coercive force was 20, but when the percentage was raised to 31·4 the coercive force dropped to ·5. **Silicon** in iron has the effect of producing a lower permeability but greater coercive force if the amount present is very small, but if the percentage reaches about 2·5 the permeability is increased though the coercive force is reduced; a silicon iron containing 97·3 per cent. of iron, ·2 per cent. of carbon, and 2·5 per cent. of silicon, under a magnetising force up to about 10, has been stated to have a greater permeability even than best charcoal iron. The presence of **aluminium** in small percentages has been found to increase the permeability under magnetising forces up to about 60.

In addition to the non-magnetisable alloy *manganese steel* referred to above, others have been noted; thus an alloy consisting of 80·16 per cent. of iron, ·8 per cent. of carbon, 5·01 per cent. of manganese, and 14·05 per cent. of nickel is practically non-magnetisable even in a field of 320.

Hensler, in 1905, discovered that certain alloys of non-magnetisable substances *and manganese* became ferromagnetics almost equivalent to cast iron; thus an alloy 26·5 per cent. manganese, 14·6 per cent. aluminium, and 58·9 per cent. copper has a permeability of 37 under a magnetising force of about 150. In the **Hensler alloys** it seems that manganese must be present, and we have seen that this substance is normally a weak paramagnetic. It has been suggested that manganese is really a ferromagnetic having a critical temperature below the ordinary

temperature of the air, and that the effect of combining it with certain suitable substances is to raise its critical temperature so that the combination becomes magnetic at ordinary temperatures (Art. 14).

Recent experiments by Hadfield and Hopkinson (1911) seem to indicate that **in strong magnetic fields** no alloy has a greater intensity of magnetisation than pure iron.

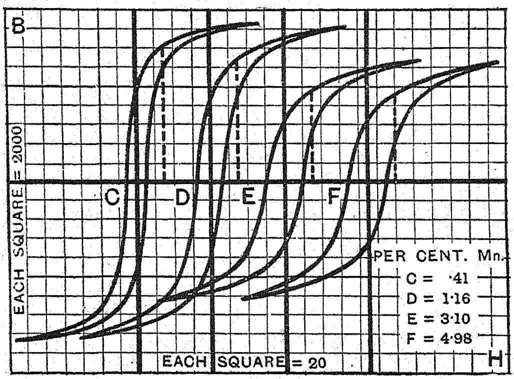


Fig. 51.

Fig. 51 shows the results of some recent experiments by E. Griffiths on *manganese steels*, and indicates the general falling off of the induction B with increasing percentage of manganese. The figure also shows the variation in energy loss by hysteresis—the greater the area of the loop the greater the loss—with varying percentages of manganese.

13. Magnetisation and Dimensions.—Continuing some observations of Joule, Bidwell in 1885 showed that a bar of iron subjected to a gradually increasing magnetising force (1) increased in length, reaching its maximum elongation with a magnetising force of from 60 to 120, (2) decreased in length, reaching its normal value with a magnetising force of from 200 to 400, (3) became actually shorter than originally with still higher magnetising forces, and (4) ceased to contract with a magnetising force of from

1,000 to 1,100. The maximum elongation is about two-millionths and the maximum contraction about six-millionths of the original length. Fig. 52 depicts the essentials of Bidwell's experiment showing the lamp, mirror, and scale method of magnifying the changes in length.

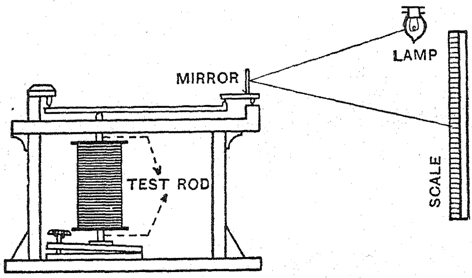


Fig. 52.

In the case of cobalt no change takes place until the magnetising force reaches about 50, and then the rod (1) decreases, reaching its minimum length with a magnetising force of about 400, (2) increases, reaching its normal value with a magnetising force of about 750, and (3) becomes actually longer than originally with still higher

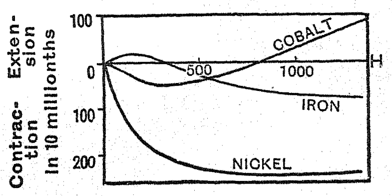


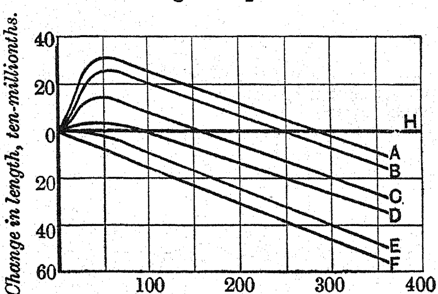
Fig. 53.

magnetising forces. Nickel decreases in length throughout, the decrease being much more pronounced than in iron, viz. about 1/40,000 of the original length with a field of about 900. Fig. 53 represents these results graphically.

To ascertain the change in volume on magnetisation, Joule placed the specimens in water contained in tubes

fitted with stoppers provided with capillary tubes, but he could not detect any variation. More recent experiments by Knott and others, however, show that iron slightly increases in volume, whilst nickel and cobalt slightly decrease.

Experiments have also been made with the object of finding the effect on the elongations and contractions of submitting the materials to tension during magnetisation. The effect of loading an iron wire with various weights at the time of magnetisation is shown in Fig. 54: the greater the load the less the elongation produced by magnetisation



Load in kilos:—A = 351; B = 585; C = 819;
D = 1170; E = 1600; F = 1950.

Fig. 54.

and the more marked the contraction, and with the maximum load used in the experiments (lowest curve) there is no elongation. With cobalt tension seems to produce no alteration in the contraction or elongation due to magnetisation. Tension diminishes the contraction shown by nickel if the magnetising force is small, but with greater magnetising forces a small tension makes the contraction due to magnetisation greater, and a large tension makes the contraction due to magnetisation smaller.

14. Magnetisation and Temperature.—It has been mentioned that a piece of iron raised to a bright red heat cannot be attracted by a magnet, and that a magnet

similarly heated loses the whole of its magnetisation. It appears, therefore, that magnetic bodies are rendered non-magnetisable if raised above a certain temperature, which is different for different substances. This **critical temperature**, as it is called, is between 700° C. and 900° C. for iron and steel, differing in different specimens.

The magnetic critical temperature of iron coincides with the temperature at which various rapid changes in the material take place, *e.g.* changes in density, specific heat, electrical conductivity, etc.; it is also the temperature of *recalescence*, *i.e.* the temperature at which a cooling mass of iron suddenly liberates heat and reglows. Evidently there is some marked molecular change at the critical temperature which settles the passage of the iron from a magnetisable to a non-magnetisable condition—to be more exact, to a condition where κ is practically zero and μ is practically unity, the same as for air.

Fig. 55 shows the relation between the flux density B and the magnetising force H for specimens of iron maintained at different temperatures all below the critical temperature. For weak fields below ·5 the magnetisation is greater the higher the temperature, but for strong fields it is less. Fig. 56 shows the effect on the permeability of gradually raising the temperature of the specimen when under a certain magnetising force. In curve A the magnetising force is ·3; as the temperature is raised the permeability increases slowly at first, then rapidly as the critical temperature is approached, and then drops suddenly to nearly unity at about 785° C., the critical temperature of the specimen. Curves B and C are for larger magnetising forces, 4 and 45 respectively; in these μ steadily falls, and then drops rather suddenly at about

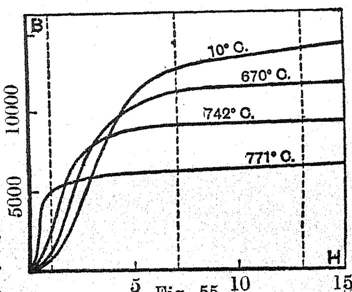


Fig. 55.

785° C., as before. Nickel behaves in much the same way, its critical temperature being about 310° C. Hysteresis loss diminishes with rise in temperature.

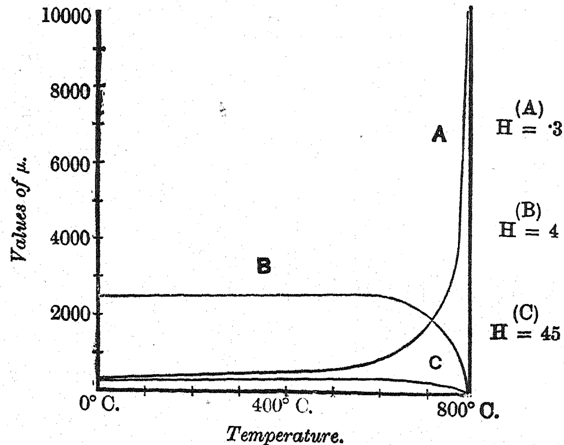


Fig. 56.

Recent experiments at very low temperatures down to that of liquid air (— 186° C.) seem to indicate that in the case of iron and nickel, if the magnetising force is small the permeability decreases as the low temperatures are approached, but if the magnetising force is large the permeability increases; with cobalt the permeability is always less at the very low temperatures.

15. Magnetisation and Magnetising Force.—The variation of magnetisation, etc., with the field producing the magnetisation has been frequently referred to in the preceding paragraphs, and Fig. 46 gives typical magnetisation curves. Lord Rayleigh in 1887 investigated the *BH* and *IH* curves for **very weak magnetic fields** (*i.e.* the part of the curves very near the origin *O* in Fig. 46) and came to the conclusion that they were practically straight lines inclined to the horizontal, so

that μ $(= B/H)$ and κ $(= I/H)$ were constant. More modern work shows that for fields up to about 2 the following may be applied in the case of tungsten steel:—

$$\kappa = 8{\cdot}9 + {\cdot}264\ H,$$
$$\mu = 112{\cdot}8 + 3{\cdot}31\ H.$$

For **medium fields** the variation of the permeability with the magnetising force is shown in Fig. 57; μ rises rapidly to a maximum in the case of soft iron (corresponding to the point C in Fig. 42 (b)) and then falls, as can also be seen from a consideration of the three stages of magnetisation, remembering that $\mu = B/H$.

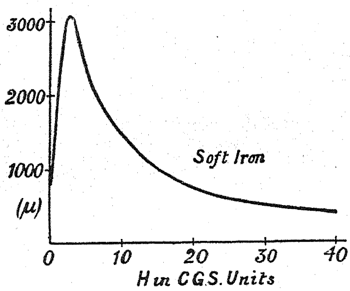


Fig. 57.

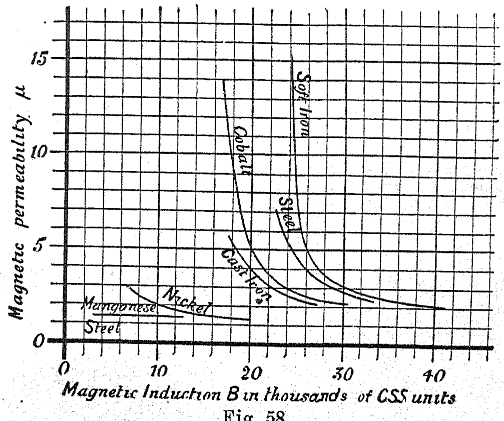


Fig. 58.

In **very strong magnetic fields** the magnetisation becomes very nearly constant (corresponding to the nearly

horizontal part of the curves of Fig. 46), hence both κ and μ decrease (Fig. 58), for even a large increase in H produces little change in B and practically no change in I. For manganese steel (the non-magnetisable alloy) the permeability is practically constant for all values of the induction, and just slightly above unity—the value for air.

It should be noted that the magnetisation produced by a given magnetising force depends on the previous history of the specimen and on the temperature. The latter has been referred to in Art. 14. The former can be readily seen from an examination of Fig. 47. Thus there are three values of B corresponding to the magnetising force OO', viz. $O'x$, $O'y$, $O'z$—$O'y$ being obtained on a first magnetisation, $O'z$ when the magnetisation has been pushed to a maximum and is being reduced to zero, and $O'x$ when the specimen has been magnetised in the opposite direction and this reversed magnetisation is being reduced to zero.

16. Magnetisation and Stress. Reciprocal Effects.— Villari discovered, in 1868, that if an iron bar or wire be feebly magnetised the effect of a pull on the bar or wire is to increase the magnetisation, whereas if it be strongly magnetised the effect of the pull is to diminish the magnetisation. This phenomenon is known as the **Villari Reversal**, and the value of the magnetisation at which, with a given stretching force, there is neither increase nor decrease in magnetisation is called the **Villari Critical Point** for that particular stretching force. The greater the stretching force applied, the smaller is the magnetisation of the Villari critical point. In the case of cobalt the effect is opposite to that in iron, *i.e.* if the magnetisation is weak the effect of a pull is to decrease the magnetisation, but if the magnetisation be stronger the effect of the pull is to increase it. With nickel, whatever be the value of the field, weak or strong, the effect of a pull is to diminish the magnetisation, so that the Villari reversal and critical point are absent.

Wiedemann discovered that if an iron wire is fixed at one end and free at the other, and magnetised so that the fixed end is a south pole and the free end a north pole, then on passing a current along the wire from the fixed end to the free end, the latter end will twist in a counter-clockwise direction, seen from the free end. If either the polarity or the direction of the current be altered the twist will alter in direction. Further, if the wire be unmagnetised and arranged as above, a current passing from the fixed to the free

end, then on giving the free end a twist in a counter-clockwise direction, viewed from that end, the wire will become magnetised, the free end being a north pole. The twist effect discovered by Wiedemann is known as the **Wiedemann Effect.**

The direction of the Wiedemann twist is reversed if the magnetising force exceeds a certain value. The twist in a nickel wire is opposite in direction to that mentioned above for iron. Finally, if a wire be magnetised, fixed as above, and the free end twisted, a momentary current is set up in the wire.

From the preceding and Art. 13 it will be noted that there are striking **reciprocal effects**, viz. :—

(*a*) Weak magnetisation lengthens a bar of iron, and strong magnetisation shortens it. Lengthening a weakly magnetised bar of iron increases the magnetisation, and lengthening a strongly magnetised bar decreases the magnetisation.

(*b*) Weak magnetisation shortens a bar of cobalt, and strong magnetisation lengthens it. Lengthening a weakly magnetised bar of cobalt decreases the magnetisation, and lengthening a strongly magnetised bar increases the magnetisation.

(*c*) Strong *or* weak magnetisation shortens a bar of nickel. Lengthening a strongly *or* weakly magnetised bar of nickel decreases the magnetisation.

(*d*) A wire magnetised in the direction of its length and carrying a current becomes twisted.

(*e*) A wire carrying a current becomes magnetised in the direction of its length when it is twisted.

(*f*) A wire magnetised in the direction of its length and then twisted has a momentary current set up in it.

It will be seen later that when a ferromagnetic wire carries a current it is *circularly magnetised.*

Exercises I.

Note.—*In most cases the Exercises at the ends of the chapters are classified into three sections. Section A consists of direct questions on the most important parts of the preceding chapter and are inserted so that the student may ascertain if he has thoroughly grasped these parts. Section B consists of questions (mainly calculations) taken from various examinations, including the Lower and Higher Examinations in Magnetism and Electricity of the Board of Education (B.E.). Section C consists of questions taken from the examinations of the Universities—Inter. B.Sc., B.Sc., etc.*

Section A.

(1) What do you understand by the following: Magnetic Poles, Magnetic Fields, Line and Tube of Force, Line and Tube of Induction, Laminated Magnet, Consequent Poles, Molecular Rigidity, Magnetising Force, Flux Density, Permeability, Susceptibility, Hysteresis, Residual Magnetism, Coercive Force, Critical Temperature, Villari Reversal, Villari Critical Point, Wiedemann Effect?

(2) What are the "rules for polarity" in the case of (*a*) a bar of iron being magnetised by "divided touch," (*b*) a bar of iron being magnetised by the current in a solenoid?

(3) Explain briefly the Molecular Theory of Magnetisation and mention any experimental facts which, in your opinion, support the theory.

(4) What is a "magnetisation curve"? Sketch (*a*) the magnetisation curves for soft iron and hard steel, (*b*) the curve obtained when a sample of iron is carried through a complete magnetic cycle. Show how residual magnetism, coercive force, and "loss of energy" per cubic centimetre per cycle are represented on the latter diagram.

(5) Distinguish between ferromagnetics, paramagnetics, and diamagnetics.

(6) Write a short essay on "Circumstances affecting Magnetisation," bringing out the *main* points relating to composition, dimensions, temperature, field, and stress.

Section B.

(1) A bar magnet is placed on a table. Describe how you would determine the direction of the lines of force in the plane of the table due to the magnet and to the earth's magnetism. Sketch the lines and explain the existence of the "null points." (B.E.)

(2) Two circular rings of iron are magnetised, the first by being placed between the poles of a strong horseshoe magnet so that the line joining the poles of the magnet is a diameter of the ring, the second by having one pole of a bar magnet drawn round it several times. Describe the magnetic state of each ring. (B.E.)

(3) A tall iron mast is just forward of the compass of a wooden ship. How will this affect the direction of the compass when the ship is sailing (*a*) to the east, (*b*) to the north, in the northern hemisphere? (B.E.)

What would be the effect if the ship were sailing (*a*) due east, (*b*) due west, in the southern hemisphere?

An iron ship is built with its length in the magnetic meridian, its bow being to the north, and it acquires permanent magnetism during construction. How would this permanent magnetism affect the compass when the ship is sailing (*a*) due east, (*b*) due west, (*c*) due north, (*d*) due south?

Section C.

(1) Describe the changes that are supposed to take place in the molecules of a steel bar when magnetised. How would you propose to determine whether this had any effect on the size of the bar? (Inter. B.Sc. Hons.)

(2) An elongated bar of bismuth is placed axially between the poles of a horseshoe magnet. What will be its condition with regard to magnetic polarity? How would you determine experimentally whether a body is paramagnetic or diamagnetic? (B.Sc.)

(3) Give a sketch of the molecular theory of magnetism as set forth by Weber and Ewing, adding any experimental proofs or criticisms you think desirable. (B.Sc.)

(4) What is meant by the magnetic permeability of a substance? *How can it be measured?* Describe how the permeability of a piece of soft iron varies with the magnetising force. (B.Sc.)

(5) What do you know respecting the Wiedemann Effect? If the torsional strain be right-handed which end of the wire will become a north pole? *How do you account for these observations of Wiedemann?* (D.Sc.)

NOTE.—In Questions C (4) and C (5) the parts in italics will be understood after reading Chapters XIX. and XXV.

CHAPTER II.

MAGNETISM.—FUNDAMENTAL THEORY.

17. Pole Strength. Laws of Magnetic Attraction and Repulsion. Unit Pole.—The force between two magnetic poles furnishes us with the best idea as to the strengths of the poles, and it has been agreed that *the strength of a pole be measured by the force it exerts on another given pole at a given distance*, the two being entirely free from the effect of other poles. Imagine then a point pole of strength m_1 at A (Fig. 59), and let F denote the force it exerts on another point pole at B; then, keeping B constant and varying the pole at A, we have

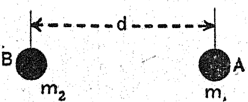


Fig. 59.

$$F \propto m_1.$$

Similarly, if m_2 be the strength of the point pole at B, and if A be kept constant and the pole at B be varied, we have

$$F \propto m_2.$$

Hence, since F is proportional to m_1 when B is constant, and proportional to m_2 when A is constant, it follows that

$$F \propto m_1 m_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

when both A and B vary. This constitutes the first law, viz.—*The force between two magnetic poles is directly proportional to the product of the pole strengths.*

Again, investigation shows that with two given point poles, if the distance between them be increased 2, 3, 4, . . . times the force F decreases to $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{16}$, . . . of its initial value, whereas if the distance be reduced to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, . . . the force is increased 4, 9, 16, . . . times. This constitutes a second law, viz.—*The force between two magnetic poles is inversely proportional to the square of their distance apart;* stated algebraically, if d denotes the distance

$$F \propto \frac{1}{d^2} \quad \text{.........................} \quad (2)$$

Combining (1) and (2) we obtain for the force F between two isolated point poles of strengths m_1 and m_2 and at distance d apart the expression

$$F \propto \frac{m_1 m_2}{d^2},$$

or, changing from a proportion to an equation,

$$F = \alpha \frac{m_1 m_2}{d^2}, \quad \text{.......................} \quad (3)$$

where α is a factor depending only on the medium and on the units we decide to adopt in our measurements.

Imagine our two point poles to be *in vacuo*, or *in air*, and let F be measured in dynes and d in centimetres. It will clearly be most convenient to so choose our unit pole that α in (3) becomes *unity* under these circumstances, and evidently this will be so if **unit pole** be defined as **that pole which when placed one centimetre in air from an equal pole exerts upon it a force of one dyne,** for if in (3) $m_1 = m_2 = 1$ (unit pole), $d = 1$ (cm.) and $F = 1$ (dyne), it follows that $\alpha = 1$. Hence, adopting the above unit pole, the force *in air* between two point poles of strengths m_1 and m_2 units and distance d cm. apart is given by

$$F = \frac{m_1 m_2}{d^2} \text{ dynes.} \quad \text{....................} \quad (4)$$

The more general formula is $F = \frac{1}{\mu}\frac{m_1 m_2}{d^2}$, where μ is the *permeability* of the medium (μ is taken as unity for air).

It should be noted that the laws established above apply to *isolated point poles.* The actual force between the like poles (say) of two ordinary bar magnets does not obey the laws, because (1) the poles are not concentrated at points and (2) the poles are not isolated —the second pole of each exerts an influence; by dealing with very long thin magnets, however, the results are approximated to very closely.

The unit pole defined above has no specific name; it is sometimes referred to as a "C.G.S. unit pole" and sometimes as a "unit of magnetism."

Examples. (1) *Two poles, one of which is twice as strong as the other, exert on each other a force equal to the weight of 500 milligrammes when placed 1 decimetre apart. Find the strength of each.*

The force must be in dynes and the distance in centimetres.

Let m = the weaker pole, $\therefore 2\,m$ = the stronger pole.

$$d = 1 \text{ dcm.} = 10 \text{ cm.}$$

Now 1 grm. weight = 981 dynes, $\therefore$ 1 milligramme $= \dfrac{981}{1000}$ dynes,

$$\therefore \text{ 500 milligrammes} = \frac{981}{1000} \times 500 = 490{\cdot}5 \text{ dynes,}$$

i.e. $\quad F = 490{\cdot}5$ dynes.

$$F = \frac{m \times 2m}{d^2},$$

$$490{\cdot}5 = \frac{2\,m^2}{10^2} = \frac{m^2}{50},$$

$$\therefore m^2 = 490{\cdot}5 \times 50 = 24525,$$

i.e. $\quad m = 156{\cdot}6.$

Thus the pole strengths are 156·6 and 313·2 C.G.S. units respectively.

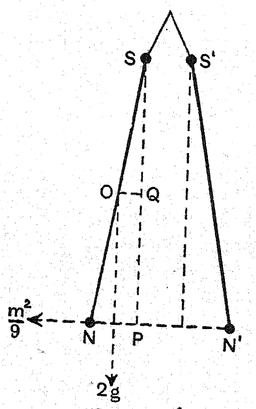


Fig. 60

(2) *Two equally magnetised magnets each 10 cm. long and weighing 2 grammes are hung by threads from their south poles as shown in Fig. 60. The distance between their south poles is found to be 1 cm. and the distance between their north poles 3 cm. Find the pole strengths.*

The forces on NS are (a) the repulsion $\dfrac{m^2}{9}$ dynes on the north pole, (b) the weight $2g$ dynes acting downwards from the mid point, (c) the repulsion on the south pole, and (d) the tension of the thread.

To eliminate the last two, take moments (see footnote, page 68) about S and we get

$$\frac{m^2}{9} \times SP = 2g \times OQ,$$

i.e.
$$m^2 = 18g \times \frac{OQ}{SP} = 18 \times 981 \times \frac{\frac{1}{2}}{\sqrt{10^2 - 1^2}}$$

$$= \frac{18 \times 981}{20} \text{ (approx.)} = 883 \text{ (approx.)},$$

$$\therefore\ m = 29{\cdot}7 \text{ C.G.S. units.}$$

Note.—In an actual case other forces would be acting (say) on the pole N, *viz.* the earth and the pole S'. These are neglected in the problem.

18. Field Strength. Unit Field.—It has been agreed that *the strength or intensity of a magnetic field at any point be defined as given in magnitude, direction, and sense by the force (in dynes) on a unit north pole placed at that particular point*, it being assumed that the unit pole itself does not affect the magnetic distribution in the district where it is placed. "Field" is a vector quantity and is subject to the law of vectors, *i.e.* can be resolved into components and follows the parallelogram law, and *the direction of a field is the direction in which a free north pole would be urged by the field.*

Unit magnetic field may thus be defined as **that field which exerts a force of one dyne on a unit north point pole placed in it,** and a field of strength H units is one which exerts a force of H dynes on a unit pole; if a pole of strength m units be placed in a field of strength H units, the force F on the pole will be given by the expression

$$F = Hm \text{ dynes.}$$

From the above it follows that, in order to determine the field strength at a point due to a number of magnetic poles, it is only necessary to imagine a unit *north* pole at the point in question, to calculate the force on this unit pole due to each of the others, and then to find the resultant by the parallelogram of forces; this resultant will give in magnitude and direction the intensity of the

field at the given point. Thus *the intensity of the field at distance d cm. from a north pole of strength* m *units is* $\frac{m \times 1}{d^2} = \frac{m}{d^2}$ *units in the direction of d,* i.e. *away from the given pole.*

Again, let NS (Fig. 61) represent a long thin magnet whose poles m and $-m$ may be regarded as point poles, and let it be required to find the intensity of the field at P. The force on a unit north pole at P due to N is given by $\frac{m \times 1}{NP^2}$ and acts in the direction NP; let it be represented by PR. Similarly, the force at P due to the pole S is given by $\frac{-m \times 1}{SP^2}$ and acts in the direction PS; let it be represented by PQ. Then, by the parallelogram of forces, the *resultant* force at P due to the magnet NS is represented by PT. The magnitude and direction of this resultant force determine the intensity of the magnetic field due to the magnet NS at the point P.

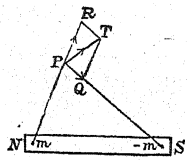


Fig. 61.

For the *magnitude* of the field we have the well-known expression

$$(PT)^2 = (PR)^2 + (PQ)^2 + 2PR \times PQ \times \cos RPQ,$$

i.e. $$F^2 = \left(\frac{m}{NP^2}\right)^2 + \left(\frac{m}{SP^2}\right)^2 + 2\left(\frac{m}{NP^2} \times \frac{m}{SP^2} \times \cos RPQ\right),$$

$$\therefore F = m\sqrt{\frac{1}{d^4} + \frac{1}{d_1^4} + \frac{2\cos\theta}{d^2 d_1^2}} \text{ C.G.S. units,}$$

where $NP = d$, $SP = d_1$, angle $RPQ = \theta$, and $F =$ the field. A neater expression for the case of a small magnet is given in Art. 31.

Exp. *Graphic determination of the direction of the field of a bar magnet at a given point.*

Assuming the pole strength m unknown, the *direction only* of the field at P may be graphically determined as follows. From the above,

$$PR : PQ = \frac{m}{NP^2} : \frac{m}{SP^2} = \frac{1}{NP^2} : \frac{1}{SP^2} = SP^2 : NP^2.$$

Now SP and NP are both known distances; hence mark off along

PR a distance representing on a suitable scale SP^2, and along PQ a distance representing NP^2 on the same scale. Complete the parallelogram and draw the diagonal PT; this is the direction of the field, and *is a tangent to the line of force at P.*

From the definition given above, the reader must *not* conclude that "intensity of field" and "force" are identical in nature. What is meant is that, if the force on unit pole is H dynes, the intensity of the field is H units of intensity. This will be better understood after reading Chapter XXI.

The conception of lines and tubes of force referred to in Chapter I. leads to another method of defining field intensity. Imagine a magnetic field traversed by lines of force, and further, imagine the lines to be grouped into tubes of force (Art. 5) which touch each other laterally and fill the entire space. When conceived on a certain definite plan, so that a certain definite number of tubes of force (to be given later) are imagined to emanate from a unit pole, the tubes are known as **unit tubes of force.** One property of these tubes has already been mentioned (Art. 5)—they tend to contract longitudinally and to expand laterally, a fact which explains the attraction and repulsion between magnetic poles.

Another property is that the tubes are narrower in the strong parts of the field than in the weak parts of the field; in fact, if Fig. 62 represents such a tube, and if the cross-section of the tube at B be twice the cross-section of the tube at A, the field at A will be twice the strength of the field at B. Further, there will evidently be more tubes passing through a square centimetre in the strong parts of the field than in the weak parts of the field, *e.g.* in the case just considered there will be twice as many tubes per square centimetre in the neighbourhood of A as in the neighbourhood of B. *Hence the strength of the field is inversely proportional to the cross-section of the unit tube, and directly proportional to the number of unit tubes per square centimetre taken perpendicular to their direction.*

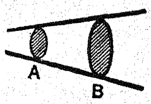


Fig. 62.

The actual numerical relation is established as follows. Imagine a unit north pole at the centre of a sphere of one centimetre radius; the force on another unit north pole anywhere on the surface of this sphere will be (Art. 17) 1 dyne, so that the field at the surface of the sphere will be a unit field. *It is agreed to conceive the unit pole sending out one tube of force per square centimetre of the surface of the sphere (unit field)*, and these are our unit tubes. As the surface of the sphere is 4π square centimetres, this leads to the results (a) 4π unit tubes of force radiate from unit pole, and (b) $4\pi m$ unit tubes of force radiate from a pole of strength m. The 4π, arbitrarily introduced in this way, is found as a consequence in a large number of electrical and magnetic formulae.

Unit magnetic field may thus be defined **as that field which has one unit tube of force per square centimetre,** and **the intensity of a magnetic field at any point is measured by the number of unit tubes per square centimetre at that point,** the area in each case being taken perpendicular to the direction of the tubes.

Imagine a pole of strength m at the centre of a sphere of radius r cm. The field intensity at any point on the sphere is m/r^2, and since the $4\pi m$ unit tubes occupy the area $4\pi r^2$, the cross section of each tube at the sphere is $4\pi r^2/4\pi m$, *i.e.* r^2/m. Thus the field intensity multiplied by the cross section of a tube is $\frac{m}{r^2} \times \frac{r^2}{m}$, *i.e.* unity.

In electrical engineering it is customary to say that "unit field has one line of force per square centimetre," that "a field is measured by the number of lines of force per unit area," that "4π lines of force emanate from unit pole," and, generally, to speak of the "number of lines of force" on a given area. These expressions are not scientifically accurate, although sanctioned by usage, *e.g.* every point in a field has its line of force, and therefore the number of lines on a given area is infinite. "Lines of force" here strictly means "unit tubes of force," but there is, perhaps, no harm in adopting the practical wording if the real meaning be understood.

The number of unit tubes passing through a given area is sometimes called the *magnetic flux* across that area; thus the strength of a field is measured by the *magnetic flux per unit area.*

The unit field is sometimes called a **Gauss**; thus a magnetic field of 15 C.G.S. units would exert a force of

15 dynes on a unit pole, would have 15 unit tubes of force per square centimetre taken perpendicular to their direction, and would be spoken of as a field of 15 gausses.

Example. *A north pole whose strength is* 10 *C.G.S. units experiences a force of* 20 *dynes when placed at a given point in a certain magnetic field: find the intensity of this field at the given point.*

Here a north pole of 10 units of strength experiences a force of 20 dynes. Therefore a north pole of *unit strength* would, at the same point in the field, experience a force of $\frac{20}{10} = 2$ dynes; that is, the intensity of the field at the given point is 2 C.G.S. units.

19. Magnetic Potential. Unit Potential. Field and Potential Gradient.—There is yet another way of defining a field, depending on the conception of *potential.* Let N (Fig. 63) be an isolated north pole, and imagine n to be a unit north pole at an infinite distance. To bring n from infinity to A, work must evidently be done against the repulsion of N, and to bring the unit pole to B clearly more work must be done. With a given pole N, the work in these cases depends only on the positions of the points in question, and such work is taken as a measure of what is termed the **magnetic potential** at these points. Thus, if W_1 ergs of work be done in moving the unit north pole from infinity to A, W_1 C.G.S. units is the potential at this point A; if W_2 ergs of work be expended in moving the unit north pole to B, W_2 C.G.S. units is the potential at the point B, and the difference in magnetic potential between A and B is $W_2 - W_1$ C.G.S. units.

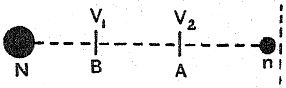


Fig. 63.

If N be replaced by a south pole the same reasoning holds, but in this case the unit north pole would move up under the attractive force; in other words, work is done *against* the magnetic force in the first case, and the unit north pole is moving from points of lower to points of higher potential, whilst work is done *by* the magnetic force in the second case, and the unit pole is moving from

points of higher to points of lower potential. Further, it is evident that at an infinite distance from N the force is zero; all points at infinity are, therefore, at the same potential, and in the above this is taken as the zero of potential.

Hence *the magnetic potential at a point is defined as measured by the work done, in ergs, in moving a unit north pole from infinity to that point, and the difference in magnetic potential between two points is defined as measured by the work done, in ergs, in moving a unit north pole from one point to the other.* Thus **the magnetic potential at a point in a field is 1 C.G.S. unit if one erg of work be done in moving a unit north pole from infinity to that point, and the magnetic potential difference between two points is 1 C.G.S. unit if one erg of work be done in moving a unit north pole from one point to the other.** The unit magnetic potential has no specific name.

In Fig. 63 let $V_1 =$ the potential of B and $V_2 =$ the potential of A; the work required to move a unit north pole from A to B is, therefore, $V_1 - V_2$ ergs. Suppose now that A and B be very near together, and let $H =$ the intensity of the field in the direction BA; the work required to move the unit north pole from A to B is, therefore, $H \times AB$. Hence

$$H \times AB = V_1 - V_2,$$

$$H = \frac{V_1 - V_2}{AB}.$$

The right-hand expression gives the **potential gradient** between B and A, hence—(*a*) **the intensity at a point in a magnetic field is numerically equal to the magnetic potential gradient at that point;** (*b*) the intensity of a magnetic field is greatest in those regions where the potential gradient is steepest, *i.e.* where the rate of variation of potential is greatest; this is evident, for given, say, a fixed value for V_1 and V_2, H is greater the smaller the distance AB.

Assuming A and B indefinitely close together, writing

dv for $V_1 - V_2$ and dx for the small length BA, we get the usual expression for the above, viz.

$$H = -\frac{dv}{dx},$$

the insertion of the negative sign being conventional, indicating *e.g.* that the potential *diminishes* as the distance x *from* N (Fig. 63) *increases.*

It should be noted that the work done in carrying the unit pole from one point to another is independent of the path described. Consider the two points A, B (Fig. 64) and the two paths ACB and ADB. Imagine the unit pole to be carried round the closed path $ACBDA$; the total work is zero, for everything is now in the initial condition again. The work for the path ACB must therefore be equal and opposite to that for the path BDA, and therefore equal to that for the path ADB. Thus the work is the same between A and B whether the path ACB or the path ADB be taken.

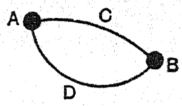


Fig. 64.

From the definition of magnetic potential the reader must **not** conclude that "magnetic potential" and "work" are identical in nature. What is meant is that if the work done in moving unit pole from one point to another is W ergs, the magnetic potential difference between the two points is W C.G.S. units of potential, *not* W *ergs*. This will be better understood after reading Chapter XXI.

Potential is a scalar quantity; thus the potential at a point due to a number of magnetic poles is simply the algebraic sum of the potentials due to each (see Arts. 24-26).

Example. *Find the work done "by the field" in moving* (1) *a north pole of strength* 2 *units from a point where the magnetic potential is* 50 *to a point where the potential is* 10 ; (2) *a north pole of strength* 5 *units from a point where the magnetic potential is* 4 *to a point where the potential is* 12.

(1) Magnetic potential difference = 50 − 10 = 40 C.G.S. units.

∴ Work done *by the field* in moving a unit north pole as indicated = 40 ergs.

Hence work done *by the field* in moving a north pole of 2 units as indicated = **80 ergs.**

(2) Magnetic potential difference = $12 - 4 = 8$ C.G.S. units.

∴ Work done *against the field* in moving a unit north pole as indicated = 8 ergs.

∴ Work done *by the field* in moving a north pole of 5 units as indicated = **− 40 ergs.**

20. Action of a Magnet in a Uniform Field. Magnetic Moment. Intensity of Magnetisation.

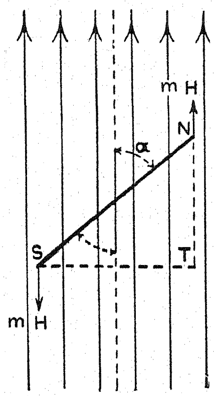


Fig. 65.

If the intensity is the same in magnitude and direction at every point of a magnetic field the latter is said to be a uniform field. In such a field the lines of force will be parallel straight lines, and the tubes will be everywhere uniform, straight, and equal in cross-section. The earth's magnetic field over fairly extended areas is uniform, whilst the field in the vicinity of a bar magnet is non-uniform.

Let m be the pole strength of the magnet NS (Fig. 65), H the intensity of the uniform field, and a the angle the magnet makes with the direction of the field. The force on each pole is mH; these constitute a couple the moment of which is obtained by the product of one of the forces and the perpendicular distance ST between them; thus

$$\text{Couple on } NS = mH \times ST = mH \times SN \sin a$$
$$= m \times SN \times H \sin a.$$

The product $(m \times SN)$ of the pole strength and the distance between the poles is called the **moment** of the magnet and is denoted by M; hence

$$\textbf{Couple on } NS = MH \sin a.$$

The effect of this couple is to rotate the magnet until it lies parallel to the direction of the field, in which position

the couple vanishes since $\alpha = 0°$, and $\sin \alpha$ is therefore zero. Clearly the couple has its maximum value when the magnet is at right angles to the field, in which case $\alpha = 90°$, $\sin \alpha = 1$, and the value of the couple is MH.

Since the magnet poles are of equal strength and the field is uniform, the forces acting on the two poles are equal, opposite, and parallel; *hence there is no resultant translatory force acting on a magnet in a uniform field,* i.e. *no force tending to make it move bodily, but only a directive couple setting it parallel to the uniform field.* Thus if a magnetic needle be placed on a cork in a vessel of water, the cork and needle will rotate until the latter lies in the magnetic meridian, but there will be no tendency to move over the surface of the water. It may be mentioned that the absence of a translatory force in such cases is one proof that the poles of a magnet are of equal strength.

In the preceding, if the magnet be at right angles to the field the couple acting on it is MH, and if the field be of unit strength the couple is M. Hence **the moment of a magnet is measured by the product of the pole strength and the distance between the poles, and is numerically equal to the moment of the couple acting on the magnet when at right angles to a uniform field of unit strength.**

If the magnetic state of a magnet is such that the quantities of free magnetism appearing on the opposite surfaces of a cross-section taken at right angles to the axis at any point are exactly equal but of opposite sign and uniformly distributed over the surfaces of the section, then the magnet is said to be *uniformly magnetised.* The intensity of magnetisation of such a magnet is the ratio of the magnetic moment to the volume; thus if M be the magnetic moment, m the pole strength, V the volume in cubic centimetres, l the length in centimetres, and a the cross-section in square centimetres, we have for the intensity of magnetisation I—

$$I = \frac{M}{V} = \frac{ml}{al} = \frac{m}{a};$$

hence **the intensity of magnetisation** may be defined

as **the magnetic moment per unit volume** or as **the pole strength per unit area of cross-section.**

From what has been said in Art. 2 it will be understood that uniform magnetisation, as defined above, is not found in the case of actual magnets. In the case of a uniformly magnetised magnet the free magnetism would appear at the ends only, and the intensity of magnetisation would be the same throughout its substance. Actual magnets, on the other hand, exhibit a distribution of free magnetism along their length, and the intensity of magnetisation varies from point to point in their volume. *The intensity of magnetisation at any point in a magnet may be defined as the magnetic moment per unit volume at the given point.*

Examples. (1) *A magnet* 10 *cm. long, with poles of unit strength, is freely suspended in a horizontal uniform magnetic field of intensity* 0·18 *unit: find the moment of the couple tending to restore the magnet to its position of rest when it is deflected in a horizontal plane through* 30° *from that position.*

The moment of the couple tending to restore the magnet to its position of rest is given by $\mathrm{MH} \sin \alpha$.

Here $\mathrm{M} = ml = 1 \times 10 = 10$ C.G.S. units,

$\mathrm{H} = 0{\cdot}18$ unit,

$\alpha = 30°$,

$\therefore$ moment of restoring couple $= 10 \times 0{\cdot}18 \times \sin 30°$

$= 1{\cdot}8 \times \frac{1}{2} = 0{\cdot}9$ (dyne cm. or C.G.S. unit).

(2) *A magnet suspended by a wire hangs in the magnetic meridian when the wire is untwisted. If on turning the upper end of the wire half round the magnet is deflected* 30° *from the meridian, show how much the upper end must be twisted to deflect the magnet* 45° *from the meridian.*

When the magnet is deflected through any angle α it is in equilibrium under the action of two couples, viz. (1) a restoring couple whose moment is $MH \sin \alpha$ tending to bring it back into the meridian, (2) a deflecting couple due to the torsion on the wire (whose moment is, within limits, proportional to the angle of torsion) tending to turn it out of the meridian; hence—

Case 1. Angle of torsion $\propto MH \sin 30°$,

$$\therefore 180° - 30° \propto MH \sin 30°.$$

Case 2. Let θ° = the required angle,

$$\therefore \theta^\circ - 45^\circ \propto MH \sin 45^\circ$$

$$\therefore \frac{\theta^\circ - 45^\circ}{150^\circ} = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \frac{\sqrt{2}}{1}$$

$$\therefore \theta^\circ - 45 = 150\sqrt{2}$$

$$\theta^\circ = (150\sqrt{2} + 45) \text{ degrees.}$$

(3) *Two magnets whose moments are M_1 and M_2 are rigidly fixed together at their centres so that their axes make an angle θ with each other, and the combination is floated on a cork in water. Find the position of equilibrium with respect to the magnetic meridian.*

Let Fig. 66 represent the position of rest, the line *mm* denoting the magnetic meridian.

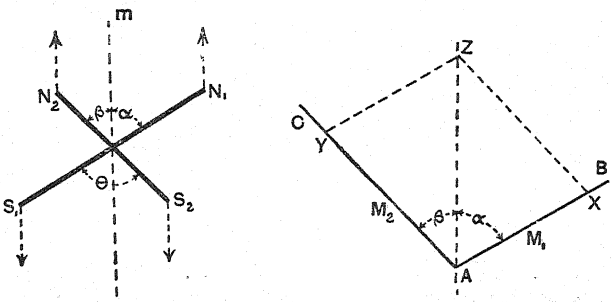


Fig. 66.

The couple on N_1S_1 is $M_1H \sin \alpha$ and tends to rotate the combination *counter-clockwise* into the meridian. The couple on N_2S_2 is $M_2H \sin \beta$ and tends to rotate the combination *clockwise* into the meridian. Since there is equilibrium

$$M_1H \sin \alpha = M_2H \sin \beta,$$

$$\therefore \frac{\sin \alpha}{\sin \beta} = \frac{M_2}{M_1}.$$

Thus the combination comes to rest in such a position that the sines of the angles which the magnets make with the meridian are inversely as their magnetic moments. If $M_1 = M_2$, $\alpha = \beta$.

To get the right position graphically, draw the two lines AB and AC enclosing an angle θ. On AB mark off a distance AX representing the moment M_1 on a suitable scale, and on AC mark off AY

representing M_2 on the same scale. The diagonal AZ of the parallelogram $AXZY$ will represent the meridian, and the angles α and β will be as indicated. From the figure—

$$\frac{\sin \alpha}{\sin \beta} = \frac{AY}{YZ} = \frac{AY}{AX} = \frac{M_2}{M_1},$$

which agrees with the condition found above.

21. Action of a Magnet in a Non-uniform Field.—Consider the action of the magnet ns in the non-uniform field of the larger magnet NS (Fig. 67). NS is supposed to be fixed, and ns to be pivoted at O so as to move freely in a horizontal plane. Let m and m_1 denote the respective strengths of the poles of the magnets. Then, considering the action of the poles of NS on the pole s of ns, we have that the pole N attracts it with a force represented by sa and equal to $\frac{mm_1}{(Ns)^2}$, and the pole S repels it with a force represented by sb and equal to $\frac{mm_1}{(Ss)^2}$.

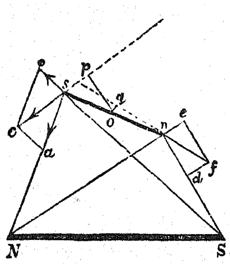


Fig. 67.

The resultant action of the magnet NS on the pole s is therefore represented by sc, and by an exactly similar construction the resultant action on the pole n may be represented by nf. Under the action of these forces the magnet ns will take up a position of equilibrium in which the moments of the forces represented by sc and nf round O are equal, that is, $sc \times Op = nf \times Oq$.*

If the magnet ns is not pivoted at O, but perfectly free to move in the plane of the paper, then it is evident that the action of the forces sc and nf must be to draw it up to NS, that is, the magnet ns experiences a force causing motion of translation, and not merely a couple setting it in

* The moment of a force about a point is the product of the force and the perpendicular distance of the point from the line of action of the force. Compare this with the definition of the "moment of a couple."

a particular position, as is the case with a magnet in a uniform field. When pivoted at *O* the resistance of the pivot prevents the translatory motion, but allows the magnet to set in the position of equilibrium indicated above.

The fact that a magnet in a non-uniform field has, in general, a resultant translatory force acting on it and is not merely subject to a directive influence should be carefully noted; as will be seen later, in this case the forces on the poles can be resolved into a couple and a single force, the combined effect of which is to tend to make the magnet not only take up a definite position, but also to move bodily (Art. 35).

22. The Period of Vibration of a Magnet.—We have seen that if a freely suspended magnet be deflected from its position of rest in a magnetic field, it at once experiences a couple urging it back into that position, and as a result it oscillates backwards and forwards about its equilibrium position, until finally it becomes stationary again. In such cases a double swing, *i.e.* a to *and* fro movement, is called a *vibration*, the number of complete vibrations in one second is called the *frequency*, the time taken to execute one vibration is called the *period*, and the extent of the motion from the central position to an extreme position is called the *amplitude*; a single swing is often referred to as an *oscillation.*

Experiment shows that with oscillating magnets, although the amplitude of the vibrations gradually diminishes as the magnet gradually comes to rest, the time of performing each vibration is (for small swings) the same, and further experiments show that the time of a vibration depends upon the size, shape, mass, and magnetic moment of the magnet, and upon the strength of the field in which it vibrates.

Exp. Suspend a magnetised knitting-needle of medium strength and start it oscillating by bringing a magnet towards it for a moment. Determine the number of vibrations (or oscillations) in a given time, say one minute. Now hang a piece of lead on each half of the needle, thus increasing the mass, etc., and determine the

number of vibrations (or oscillations) in the same time. The number will be found to be less, i.e. *the period of vibration is greater the greater the mass.*

Remove the lead and strengthen the needle by further magnetisation, thus increasing its magnetic moment, and again find the number of vibrations (or oscillations) in the same time. The number will be found to be greater than in the first case above, i.e. *the period of vibration is less the greater the magnetic moment.*

Strengthen the field by placing the north pole of a long magnet a short distance to the south of the needle and repeat. The number will be found to be greater in the same time, i.e. *the period of vibration is less the stronger the field.*

It has been shown in Art. 20 that the moment of the couple acting on the magnet when deflected through an angle a is $MH \sin a$, and if a be small this may be written MHa. Hence if K be the "moment of inertia" of the magnet about the axis of suspension, the angular acceleration is $\frac{MHa}{K}$, *i.e.* the angular acceleration is proportional to the angular displacement, and the motion of the magnet is *simple harmonic motion.* Thus, by the theory of simple harmonic motion, if t be the period—

$$t = \frac{2\pi}{\sqrt{\frac{MH}{K}}} = 2\pi \sqrt{\frac{K}{MH}} \quad \ldots\ldots\ldots\ldots \quad (1)$$

The moment of inertia (K) depends on the mass, size, and shape. For a rectangular bar magnet length a cm., breadth b cm., and mass m grammes vibrating about an axis through its centre, and perpendicular to the surface bounded by a and b, $K = m\frac{a^2 + b^2}{12}$, and for a cylindrical magnet length l cm., radius r cm., and mass m grammes vibrating about an axis through its centre and perpendicular to the axis of the cylinder, $K = m\left(\frac{l^2}{12} + \frac{r^2}{4}\right)$. From the formulae, the greater the mass the greater the value of K, and therefore the greater the period of vibration t, whilst the greater the value of M or H the less the value of t; this agrees with the experiments above.

If n be the "frequency" $n = 1/t$ and we get

$$n = \frac{1}{2\pi}\sqrt{\frac{MH}{K}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Squaring (1) and (2) we get

$$t^2 = \frac{4\pi^2 K}{MH} \quad \text{and} \quad n^2 = \frac{MH}{4\pi^2 K},$$

from which it follows that—

(*a*) If the same magnet be caused to vibrate in different fields, the square of the period is *inversely* proportional to the field strengths, and the square of the frequency is *directly* proportional to the field strengths (see Chapter III.) if we neglect any change in the moment of the magnet due to the inductive action of the field.

(*b*) If two magnets have equal moments of inertia (K), and if they be caused to vibrate in the same field, the square of the periods is inversely proportional to the magnetic moments, and the square of the frequencies is directly proportional to the magnetic moments (see Chapter III.).

The expression for t in (1) is only true for vibrations of very small amplitude, and in using this expression for accurate work it is necessary to make a correction for the average amplitude during observation; thus if t_1 be the observed period, and a the amplitude in circular measure,

$$t = t_1 \left(1 - \frac{a^2}{16}\right).$$

Example. *Two magnets are arranged parallel one above the other. When like poles are together the combination makes* 20 *vibrations per minute, and when unlike poles are together* 10 *vibrations per minute. Compare their magnetic moments.*

Clearly K is the same in both cases.

In Case 1 the magnets are in the same direction, so that if M_1 and M_2 denote their moments, t_1 the period, and n_1 the number of vibrations per minute,

$$t_1 = 2\pi \sqrt{\frac{K}{(M_1 + M_2) H}}.$$

If t_2 be the period in the second case, and n_2 the number of vibrations per minute,

$$t_2 = 2\pi \sqrt{\frac{K}{(M_1 - M_2) H}}.$$

$$\therefore \frac{t_1^2}{t_2^2} = \frac{M_1 - M_2}{M_1 + M_2}, \quad i.e. \ \frac{n_2^2}{n_1^2} = \frac{M_1 - M_2}{M_1 + M_2}$$

$$\therefore \frac{M_1 + M_2 + M_1 - M_2}{M_1 + M_2 - M_1 + M_2} = \frac{n_1^2 + n_2^2}{n_1^2 - n_2^2}$$

$$i.e. \ \frac{M_1}{M_2} = \frac{n_1^2 + n_2^2}{n_1^2 - n_2^2} = \frac{20^2 + 10^2}{20^2 - 10^2} = \frac{5}{3}.$$

23. The Product of "M" and "H." Bifilar Suspension of a Magnet.—The product of the moment of a magnet and the strength of a field in which it is placed—generally the earth's horizontal field H in practice—is often required.

One method is to suspend the magnet of moment M by a few fibres of unspun silk in the field of strength H, and to determine the time t seconds of one complete vibration. From the preceding section—

$$t = 2\pi\sqrt{\frac{K}{MH}}, \quad \therefore \ MH = \frac{4\pi^2 K}{t^2},$$

from which, knowing K and t, the value of MH is determined.

A second method is to suspend the magnet by a wire of phosphor, bronze, silver, or other suitable material, so that the magnet lies along the direction of the field when the wire is without twist, and then to rotate the upper end of the wire through an angle θ so as to deflect the magnet through an angle α from the direction of the field. The restoring couple due to the field is $MH \sin \alpha$, and the deflecting couple due to the twist on the wire is proportional to the twist $(\theta - \alpha)$ and therefore equal to $C(\theta - \alpha)$, where C is a constant for the wire ; hence, since these balance,

$$MH \sin \alpha = C(\theta - \alpha), \quad \therefore \ MH = \frac{C(\theta - \alpha)}{\sin \alpha}.$$

A third method is by means of a bifilar suspension of the magnet. Imagine the magnet (Fig. 68) suspended from the two points P and Q by two equal threads PR and QS, each of length l. Let θ be the angle between the horizontal support PQ and the magnet (also horizontal), *i.e.* the angle between $P'Q'$ and RS in the figure, where P' and Q' are vertically below P and Q. Let $PQ = 2x$, $RS = 2y$, $\gamma =$ the angle $P'PR$, *i.e.* the inclination of the threads to the vertical, and $W =$ mass of the magnet in grammes.

The tension T in PR can be resolved into a vertical component $T \cos \gamma$ and a horizontal component $T \sin \gamma$, and the same applies to the tension in QS. The two vertical components balance the weight (in dynes), hence—

$$2T \cos \gamma = Wg, \quad \therefore \ T = \frac{Wg}{2 \cos \gamma},$$

i.e.

$$T = \frac{Wg \, . \, l}{2\,PP'}.$$

The horizontal component $T \sin \gamma$ of the tension in PR has a moment about O equal to $T \sin \gamma \times p$, where p is the perpendicular from O on $P'R$, and similarly for the other horizontal component. Since these two constitute a couple we have

$$\text{Total turning moment} = 2\,T \sin \gamma \times p$$

$$= \frac{2\,T}{l}\,P'R \times p$$

$$= \frac{2\,T}{l}\,(\text{twice area of triangle } P'OR)$$

$$= \frac{2\,T\,xy \sin \theta}{l}.$$

Substituting the value of T found above,

Couple due to bifilar suspension

$$= \frac{Wgxy}{h} \sin \theta,$$

where h denotes the vertical height PP'.

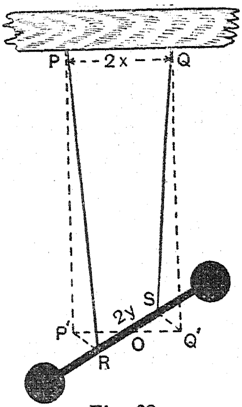


Fig. 68.

Imagine now that arrangements are such that PQ and the magnet are both in, say, the meridian, and let PQ be turned so as to set the magnet at right angles to the meridian. The restoring couple due to the earth is MH, and the deflecting couple due to the suspension is given by the above, θ *being the angle between* PQ *and the magnet*; hence—

$$MH = \frac{Wgxy}{h} \sin \theta,$$

$$\text{i.e. } MH = k \sin \theta, \quad \text{where } k = \frac{Wgxy}{h}.$$

(See worked example, p. 158.)

24. Potential at a Point due to a Magnetic Pole.

The point P at which the magnetic potential is required is d cm. from the north pole N of strength m units; P Q, R, S, T are points very near together (Fig. 69).

$$\text{Intensity of the field at } P = \frac{m}{NP^2} \text{ units,}$$

$$\text{,, \quad ,, \quad ,, \quad } Q = \frac{m}{NQ^2} \text{ ,,}$$

$\therefore$ Average intensity between P and $Q = \dfrac{m}{NP \,.\, NQ}$ units,

i.e. Force on unit pole between P and $Q = \dfrac{m}{NP \,.\, NQ}$ dynes.

$\therefore$ Work done in moving unit north pole from Q to P

$$= \text{Force} \times \text{distance}$$

$$= \frac{m}{NP \,.\, NQ} \times PQ = \frac{m}{NP \,.\, NQ}(NQ - NP)$$

$$= \left(\frac{m}{NP} - \frac{m}{NQ}\right) \text{ ergs.}$$

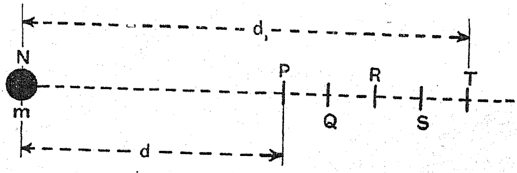


Fig. 69.

Similarly—

Work done in moving unit north pole from R to Q

$$= \left(\frac{m}{NQ} - \frac{m}{NR}\right) \text{ ergs.}$$

Work done in moving unit north pole from S to R

$$= \left(\frac{m}{NR} - \frac{m}{NS}\right) \text{ ergs.}$$

Work done in moving unit north pole from T to S

$$= \left(\frac{m}{NS} - \frac{m}{NT}\right) \text{ ergs.}$$

Hence, total work in moving unit north pole from T to P

$$= \left(\frac{m}{NP} - \frac{m}{NQ} + \frac{m}{NQ} - \frac{m}{NR} + \frac{m}{NR} - \frac{m}{NS} + \frac{m}{NS} - \frac{m}{NT}\right) \text{ ergs}$$

$$= \left(\frac{m}{NP} - \frac{m}{NT}\right) \text{ ergs}$$

$$= \left(\frac{m}{d} - \frac{m}{d_1}\right) \text{ ergs.}$$

But the work done in ergs in moving the unit pole from one point to another measures the magnetic potential difference between the two points; thus *the magnetic potential difference between two points at distances* d *and* d_1 *cm. from a pole of strength* m *is*

$$\text{Potential Difference} = \frac{m}{d} - \frac{m}{d_1} \text{ C.G.S. units.}$$

If T be at infinity $\frac{m}{d_1}$ is zero and the work done in moving unit north pole from infinity to P is therefore $\frac{m}{d}$ ergs; hence, by the definition, **the potential at P distant d cm. from the north pole N of strength m is given by**

$$\textbf{Potential at } P = \frac{m}{d} \textbf{ C.G.S. units.}$$

If N be replaced by a south pole of strength $-m$,

$$\text{Potential at P} = -\frac{m}{d} \text{ C.G.S. units.}$$

The reader acquainted with the Calculus will appreciate the neater proof. Since

$$F = -\frac{dV}{dx}, \quad dV = -Fdx.$$

$$\therefore \text{Potential at } P = V_d - V_\infty = -\int_\infty^d Fdx$$

$$= -m\int_\infty^d \frac{1}{x^2}dx = -m\left[-\frac{1}{x}\right]_\infty^d = \frac{m}{d}.$$

25. Potential at a Point (1) on the Axial Line, (2) on an Equatorial Line of a Bar Magnet.

The axial line is the prolongation of the axis of the magnet in either direction, *e.g.* P_1P in Fig. 70. If $2l$ be the length of the magnet, then, with the usual notation—

$$\text{Potential at } P = \frac{m}{d-l} - \frac{m}{d+l} = \frac{m \cdot 2l}{d^2 - l^2}$$

$$= \frac{M}{d^2 - l^2} \text{ C.G.S. units.}$$

$$\text{Potential at } P_1 = -\frac{m}{d-l} + \frac{m}{d+l} = -\frac{m\,.\,2l}{d^2-l^2}$$
$$= -\frac{M}{d^2-l^2} \text{ C.G.S. units.}$$

If the magnet be very small, so that l^2 may be neglected in comparison with d^2, these expressions become M/d^2 and $-M/d^2$ respectively.

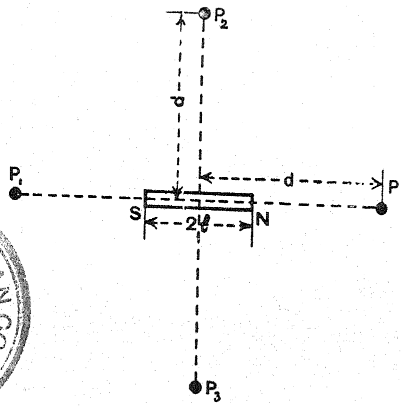


Fig. 70.

An equatorial line is a line bisecting the magnetic axis of the magnet at right angles, *e.g.* P_2P_3 in Fig. 70.

$$\text{Potential at } P_2 = \frac{m}{\sqrt{d^2+l^2}} - \frac{m}{\sqrt{d^2+l^2}} = 0.$$

It is evident that any point on P_2P_3 is at zero potential; in fact the *surface* bisecting the axis of a magnet at right angles is a surface of zero potential, for any point on it is equidistant from the north and south poles.

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26. Potential at any Point due to a Small Magnet.—The potential at any point due to a very small magnet can readily be deduced from the preceding.

Let NS (Fig. 71) represent the short magnet. If the strength of its poles be m and $-m$, then the potential at P, due to the north and south poles N and S, is $\frac{m}{NP} - \frac{m}{SP}$. Since NS is supposed to be very small compared with the distances NP and SP, this may be written as $\frac{m}{nP} - \frac{m}{sP}$. That is, if O be the middle point of NS, we have the potential at P given by

$$\frac{m}{(OP - On)} - \frac{m}{(OP + On)}$$

or
$$\frac{2m \,.\, On}{OP^2 - On^2},$$

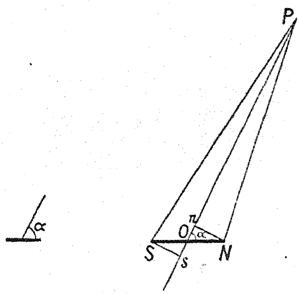


Fig. 71.

and, as On^2 is negligibly small compared with OP^2, this result reduces approximately to $\frac{2m \,.\, On}{OP^2}$ or $\frac{m \,.\, ns}{OP^2}$.

But if the angle between OP and ON be denoted by α, $ns = NS \cos \alpha$, and we get $\frac{m \,.\, ns}{OP^2} = \frac{m \,.\, NS \cos \alpha}{OP^2}$.

The quantity $m \,.\, NS$ is evidently the magnetic moment of the magnet, and if this be denoted by M, we get the result that the potential at P, due to the small magnet NS, is given by $M \cos \alpha / OP^2$, or if OP be denoted by d,

$$\textbf{Potential at } P = \frac{M \cos \alpha}{d^2} \textbf{ C.G.S. units.}$$

It should be noted that α is the angle between OP and the positive direction of the axis of NS, *i.e.* the direction SN.

If P be on the axial line, $\alpha = 0°$ or $180°$, $\cos \alpha = 1$ or -1, and the potential at P is M/d^2 or $-M/d^2$, as indicated in Art. 25.

If P be on an equatorial line, $\alpha = 90°$ or $270°$, $\cos \alpha$ is zero, and the potential is zero, as shown in Art. 25.

Let us assume that the magnet NS of moment M *can* be resolved into two components, viz. ns of moment $M \cos \alpha$ along OP, and n_1s_1 of moment $M \sin \alpha$ at right angles to OP. The potential at P due to the former is $M \cos \alpha/d^2$ (Art. 25) and that due to the latter is zero, the total being therefore $M \cos \alpha/d^2$. The agreement of this with the above may be taken as proof that our assumption as to the resolution of magnetic moments is correct (see Art. 28).

27. Potential Energy of a Magnet in a Uniform Field. Work done in deflecting a Magnet.—In Art. 26 it is shown that the potential at P (Fig. 71) due to a small magnet NS is $M \cos \alpha/d^2$. It is also the potential energy of the small magnet in the magnetic field at O due to a unit north pole at P, for it expresses both the work done in bringing a unit north pole from infinity up to P in the field of the magnet, and the work done in bringing the magnet from infinity up to the position NS at O in the field due to a unit north pole at P. That is $M \cos \alpha/d^2$ expresses the potential energy of either member of a system made up of a unit north pole at P and a very small bar magnet in the position NS at O. The field at O due to a unit north pole at P is $1/d^2$ in the direction Os, and the magnet being very small we may assume that it is in a uniform field of this intensity. The potential energy of the magnet in this field is $M \cos \alpha \,.\, 1/d^2$, where α is the *supplement* of the angle between the axis of the magnet and the direction of the field. That is, the potential energy of a magnet of moment M in a field of strength H, whose direction makes an angle β with the axis of the magnet, is given by the expression

$$\textbf{Potential energy} = -MH \cos \beta.$$

Consider, then, a magnet in its position of rest in the earth's field H. Here $\beta = 0°$ and the potential energy of the magnet has its minimum value $-MH$. If the magnet be rotated through 90° its potential energy becomes zero; hence the work done in rotating the magnet through 90° is evidently MH (the change in potential energy). If the magnet be rotated through 180° its potential energy has its maximum value MH (since $\cos 180 = -1$); hence the work done in rotating the magnet through 180° is evidently

$2MH$, for the energy has changed from $-MH$ to MH, *i.e.* by an amount $2MH$.

The work done in deflecting the magnet may also be found thus:—When the deflection is β (Fig. 72) the pole N has moved a distance AB along the direction of the field, and the work done on the pole N is measured by the product of the force and the distance, *i.e.* by $mH \times AB$; the work on the pole S is equal to this. Hence if $2l$ be the length of the magnet,

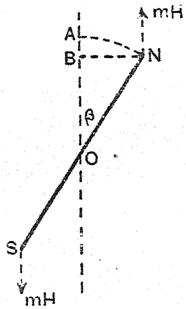


Fig. 72.

Total work

$$= 2mH \times AB = 2mH\,(AO - BO)$$
$$= 2mH\,(l - l\cos\beta)$$
$$= 2mlH\,(1 - \cos\beta),$$

i.e. **Total work** $= \boldsymbol{MH\,(1 - \cos\beta)}$.

When $\beta = 90°$ work $= MH$ and when $\beta = 180°$ work $= 2MH$.

28. Resolution of Magnetic Moment.—The moment of a magnet may be resolved in accordance with the rules that apply to the resolution of a force into components. Just as a force F may be resolved at any point into components $F_1, F_2, F_3, \ldots$ making angles $\alpha_1, \alpha_2, \alpha_3, \ldots$ with its direction, so the moment of a magnet, assumed to have the direction of the axis of the magnet, may be resolved, at the mid point of the axis, into components $M_1, M_2, M_3, \ldots$ associated with axes making angles $\alpha_1, \alpha_2, \alpha_3, \ldots$ with the axis of M. The truth of this may be proved as follows. If we resolve F and $F_1, F_2, F_3, \ldots$ along a fixed direction with which their directions make angles $\beta, \beta_1, \beta_2, \beta_3, \ldots$, then, by the principles of mechanics, we have

$$F\cos\beta = F_1\cos\beta_1 + F_2\cos\beta_2 + F_3\cos B_3 + \ldots$$

or

$$\mathbf{F}\cos\beta = \Sigma F\cos\beta.$$

Similarly, if the law of resolution be assumed to apply to

magnetic moments, and if $M, M_1, M_2, M_3, \ldots$ be supposed to be resolved along a fixed direction, making angles $\beta, \beta_1, \beta_2, \beta_3, \ldots$ with their axes, then we have

$$M \cos \beta = M_1 \cos \beta_1 + M_2 \cos \beta_2 + M_3 \cos \beta_3 + \ldots$$

or

$$\mathbf{M} \cos \beta = \Sigma M \cos \beta.$$

If P be a point at a distance d from the common mid point of intersection of these axes along the given fixed direction, then the potential at P due to magnets of moments $M, M_1, M_2, M_3, \ldots$ is given by

$$\frac{M \cos \beta}{d^2}, \; \frac{M_1 \cos \beta_1}{d^2}, \; \frac{M_2 \cos \beta_2}{d^2}, \; \frac{M_3 \cos \beta_3}{d^2}, \ldots$$

and since, as shown above,

$$M \cos \beta = M_1 \cos \beta_1 + M_2 \cos \beta_2 + M_3 \cos B_3 + \ldots$$

we have

$$\frac{M \cos \beta}{d^2} = \frac{M_1 \cos \beta_1}{d^2} + \frac{M_2 \cos \beta_2}{d^2} + \frac{M_3 \cos \beta_3}{d^2} + \ldots$$

that is,

$$\frac{\mathbf{M} \cos \beta}{d^2} = \Sigma \frac{M \cos \beta}{d^2}.$$

This means that the potential at P, due to the magnet of moment, M, is the algebraic sum of the potentials due to the components $M_1, M_2, M_3, \ldots$ into which it has been resolved. This result is in accord with the principles of magnetic potential. Hence the assumption that the rules for the resolution and composition of forces may be applied to magnetic moments leads to results in accord with magnetic theory and may therefore be accepted.

29. Intensity of the Magnetic Field at any Point on the Axial Line of a Bar Magnet.—This is obtained

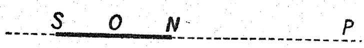


Fig. 73.

by estimating the resultant force on a unit north pole at the given point. Let NS, Fig. 73, represent the magnet;

then, to determine F, the intensity of the field at any point, P, on the axial line, we have

$$F = \frac{m}{NP^2} - \frac{m}{SP^2},$$

where m is the strength of the magnet poles. If O be the middle point of NS, and OP be denoted by d, and ON by l, then we have

$$F = \frac{m}{(d-l)^2} - \frac{m}{(d+l)^2}$$

or

$$F = \frac{4mdl}{(d^2 - l^2)^2},$$

and as M, the magnetic moment of the magnet, is equal to $2ml$, this may be written

$$F = M \frac{2d}{(d^2 - l^2)^2} \text{ C.G.S. units.}$$

If l is very small compared with d, this reduces to

$$F = \frac{2M}{d^3}.$$

The direction of F is that of OP produced, *i.e.* along the axial line. In this case NS is said to be "end on" to the point P.

30. Intensity of the Magnetic Field at any Point on an Equatorial Line of a Bar Magnet.—To determine the intensity F of the field at P in Fig. 74 we have

Intensity at P due to $N = \frac{m}{r^2}$ in the direction NP,

Intensity at P due to $S = -\frac{m}{r^2}$ in the direction PS.

Denoting these two by the lengths PT and PV, completing the parallelogram and drawing the diagonal PR, this latter represents in magnitude and direction the

resultant intensity F at the point P. From the figure it is clear that NSP and PRT are similar triangles, hence

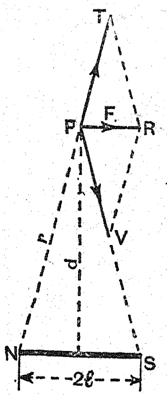


Fig. 74.

$$\frac{PR}{PT} = \frac{NS}{NP}, \text{ i.e. } \frac{F}{\frac{m}{r^2}} = \frac{2l}{r},$$

$$\therefore F = \frac{2ml}{r^3} = \frac{M}{r^3},$$

i.e. $$F = \frac{M}{(d^2 + l^2)^{\frac{3}{2}}} \text{ C.G.S. units.}$$

If l is very small compared with d so that l^2 may be neglected,

$$F = \frac{M}{d^3}.$$

The direction of F is parallel to the axial line of NS. In this case NS is said to be "broadside on" to the point P. From the two expressions for small magnets it is clear that the field due to such a magnet "end on" is twice the field due to the same magnet "broadside on" at the same distance.

Examples. (1) *A uniformly magnetised bar magnet 10 cm. long and of moment 200 C.G.S. units is placed in a horizontal position with its axis in the magnetic meridian and its north pole towards the north. A small compass 10 cm. due east of the centre of the magnet is in neutral equilibrium. Find the horizontal intensity of the earth's magnetic field.*

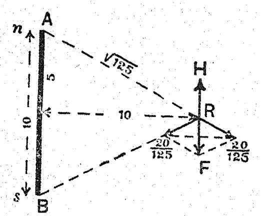


Fig. 75.

Let R (Fig. 75) be the position of the compass. Since the latter is in equilibrium the field F at R due to the magnet must be equal and opposite to the earth's horizontal field H, *i.e.* since the magnet is "broadside on" to R

$$H = \frac{M}{(d^2 + l^2)^{\frac{3}{2}}} = \frac{200}{(10^2 + 5^2)^{\frac{3}{2}}},$$

$$\therefore H = \cdot 14 \text{ C.G.S. unit.}$$

(2) *Two short bar magnets of moments* 108 *and* 192 *units are placed along two lines drawn on the table at right angles to each other. Find the intensity of the field at the point of intersection of the lines, the centres of the magnets being respectively* 30 *and* 40 *cm. from this point.*

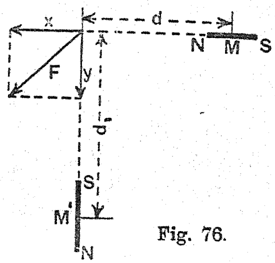


Fig. 76.

The conditions are shown in Fig. 76. The intensity x due to NS is $\frac{2M}{d^3}$ in the direction indicated, *i.e.*

$$x = \tfrac{216}{27000} = \cdot 008 \text{ unit.}$$

Similarly, the intensity y is $\frac{2M_1}{d_1{}^3}$ in the direction indicated,

i.e. $y = \tfrac{384}{64000} = \cdot 006$ unit.

Hence, for the total intensity F in the direction shown we have

$$F^2 = x^2 + y^2,$$

i.e. $$F = \sqrt{(\cdot 008)^2 + (\cdot 006)^2},$$

$$\therefore F = \cdot 01 \text{ C.G.S. unit.}$$

31. Intensity of the Field due to a small Bar Magnet at any Point.—Let NS (Fig. 77) represent the small magnet. Its moment M may be resolved into two components, $M \cos \alpha$ along OP, and $M \sin \alpha$ at right angles to OP, the angle NOn being denoted by α.

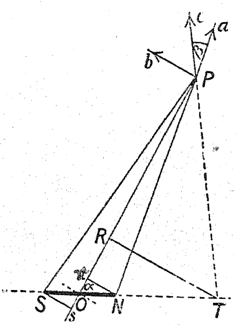


Fig. 77.

The field at P due to $M \cos \alpha$ is $\frac{2M \cos \alpha}{OP^3}$ along Pa, and that due to $M \sin \alpha$ at the same point is $\frac{M \sin \alpha}{OP^3}$ along Pb.

If the resultant of these fields acts along Pc and the angle aPc be denoted by β, then

$$\tan \beta = \frac{M \sin \alpha}{OP^3} \Big/ \frac{2M \cos \alpha}{OP^3} = \tfrac{1}{2} \tan \alpha,$$

that is $\tan \alpha = 2 \tan \beta$.

This result determines the *direction* of the resultant field, and a simple geometrical construction for finding this direction is readily deduced. From the figure $\tan\alpha = TR/OR$ and $\tan\beta = TR/RP$, and therefore $\frac{TR}{OR} = 2\frac{TR}{RP}$ or $RP = 2OR$, that is OR is $\frac{1}{3}OP$.

Hence, to find the direction Pc the construction is as follows. On OP take a point R such that OR is one third of OP. Through R draw RT at right angles to OP and cutting SN produced at T. Join TP and produce TP to c, thus obtaining the required direction Pc.

The *magnitude* of the resultant field is readily obtained: thus, if F denote the resultant, x and y the fields due to the component magnets, and d the distance OP,

$$F^2 = x^2 + y^2 = \left(\frac{2M\cos\alpha}{d^3}\right)^2 + \left(\frac{M\sin\alpha}{d^3}\right)^2,$$

$$\therefore\ F = \frac{M}{d^3}\sqrt{4\cos^2\alpha + \sin^2\alpha},$$

i.e.

$$F = \frac{M}{d^3}\sqrt{1 + 3\cos^2\alpha} \quad \text{C.G.S. units.}$$

If P be on the axial line $\alpha = 0$ or π, $\cos^2\alpha = 1$ and $F = 2M/d^3$, as in Art. 29. If P be on an equatorial line $\alpha = \pi/2$ or $3\pi/2$, $\cos^2\alpha = 0$ and $F = M/d^3$, as in Art. 30.

32. Potential and Field due to a Uniformly Magnetised Sphere.—Let the sphere be magnetised in the direction of the arrow (Fig. 78), so that the surface of the hemisphere on the right exhibits north magnetism and that on the left south magnetism. The sphere may be imagined to be made up of a large number of *very small* simple magnets (called "**doublets**"), each of pole strength m and length l, all lying parallel to the arrow, their north poles to the right and their south poles to the left. If there are x of these per unit volume, then, since the moment of each is ml, the moment per unit volume is xml, *i.e.* if I be the intensity of magnetisation of the sphere,

$$I = mxl.$$

Again, since each north pole is at a constant *very small* distance l to the right of its companion south pole, the north poles may be looked upon as distributed uniformly throughout a sphere whose centre is at n and the south poles may be looked upon, for a like

reason, as distributed uniformly throughout a sphere whose centre is at s, where $ns = l$. Since there are x north poles per unit volume in the north sphere and each is of strength m, the amount of north magnetism per unit volume is xm, and the total north magnetism is $\frac{4}{3}\pi r^3 xm$, where r is the radius of the sphere. Similarly, the total south magnetism in the south sphere is $-\frac{4}{3}\pi r^3 xm$. Hence, since a sphere of north *or* of south magnetism acts at external points as if the whole of the magnetism were concentrated at the centre (as will be shown later), the effect of the magnetised sphere is just the same as that of two poles, a north pole of strength $\frac{4}{3}\pi r^3 xm$ at n, and an equal south pole at s.

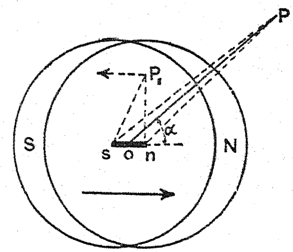


Fig. 78.

The *potential* at the point P due to the uniformly magnetised sphere is, therefore, obtained as in Art. 27, viz.

$$\text{Potential at } P = \tfrac{4}{3}\pi r^3 xm\left(\frac{1}{nP} - \frac{1}{sP}\right)$$

$$= \tfrac{4}{3}\pi r^3 xm\left(\frac{sP - nP}{nP \times sP}\right)$$

$$= \tfrac{4}{3}\pi r^3 xm\,\frac{l\cos a}{OP^2},$$

and, since $xml = I$ and $OP = d$, say

$$\text{Potential at } P = \tfrac{4}{3}\pi r^3 I\,\frac{\cos a}{d^2}.$$

This is identical with the result of Art. 27, since $\frac{4}{3}\pi r^3 I$ is the moment M of the sphere; *hence a uniformly magnetised sphere produces the same potential at a point outside as would be produced by a small magnet at its centre lying in the direction of magnetisation and having a moment numerically equal to the moment of the sphere.*

In the same way the *field* at P due to the uniformly magnetised sphere is given by the formula of Art. 31, M having the value $\frac{4}{3}\pi r^3 I$.

The *field* at a point P_1 *inside* may be calculated if we assume (what is shown later) that the force at P_1 is due entirely to the concentric sphere passing through P_1, the portion of the main sphere outside P_1 having no effect. The force at P_1 due to a north pole $\frac{4}{3}\pi(nP_1)^3 xm$ at n is $\frac{4}{3}\pi xm\,\frac{(nP_1)^3}{(nP_1)^2}$, *i.e.* $\frac{4}{3}\pi xm\ (nP_1)$ along nP_1, and the force at P_1 due to the south pole $\frac{4}{3}\pi(sP_1)^3 xm$ at s is

$\frac{4}{3}\pi xm\frac{(sP_1)^3}{(sP_1)^2}$, *i.e.* $\frac{4}{3}\pi xm(sP_1)$ along P_1s. By the triangle of forces the resultant of these two is $\frac{4}{3}\pi xm(ns)$ parallel to ns. But $ns = l$ and $xml = I$, where I is the intensity of magnetisation of the sphere; thus the *field* at an internal point P_1 is $\frac{4}{3}\pi \mathbf{I}$, and is parallel to ns.

33. Action of a Magnet in two Magnetic Fields at Right Angles. The "A" and "B" Positions of Gauss.—The most important example in practice is that of the earth's horizontal field (H) combined with the field due to a distant magnet lying east and west, and of this two cases arise.

Case 1. *Action of the Magnet NS under the combined influence of the Earth and an "End on" Magnet N′S′ (Fig. 79) lying east or west of NS.*

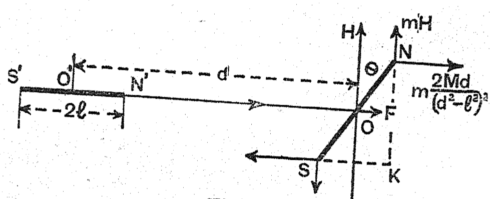


Fig. 79.

Let the small magnet NS be deflected through an angle θ from the direction of the earth's horizontal field H. If m be the pole strength of NS the force on each pole due to the earth will be mH as shown, and for the "couple" tending to rotate NS back into the meridian we have

Restoring couple due to the earth $= mH \times SK$.

Again, *if NS be small, the field at N and S due to N′S′ may be taken as equal to that at O*, viz. $\frac{2Md}{(d^2-l^2)^2}$ units, where M is the moment of $N'S'$, $2l$ its length, and d the distance $O'O$. The force on each pole due to the magnet $N'S'$ is therefore $m\frac{2Md}{(d^2-l^2)^2}$, and these constitute a couple

tending to set NS at right angles to the meridian; hence, as above, we have

$$\text{Deflecting couple due to } N'S' = m\,\frac{2Md}{(d^2-l^2)^2} \times NK.$$

Since these two couples balance each other

$$m\,\frac{2Md}{(d^2-l^2)^2} \times NK = mH \times SK,$$

i.e.
$$\frac{2Md}{(d^2-l^2)^2} = H \times \frac{SK}{NK} = H \tan\theta.$$

Hence the position of equilibrium of NS is such that

$$\tan\theta = \frac{2Md}{H(d^2-l^2)^2} \qquad\text{.................. (1)}$$

or, writing in another form more convenient in practice,

$$\frac{M}{H} = \frac{(d^2-l^2)^2}{2d}\tan\theta \qquad\text{.................. (2)}$$

If l be very small compared with d so that l^2 may be neglected, this becomes

$$\frac{M}{H} = \frac{d^3\tan\theta}{2} \qquad\text{...................... (3)}$$

This is known as the "tangent A position of Gauss."

Case 2. *Action of the Magnet NS under the combined influence of the Earth and a "Broadside on" Magnet N'S' (Fig. 80) lying north or south of NS.*

As before, we have

Restoring couple due to the earth $= mH \times SK$.

Again, the force on each pole of NS due to $N'S'$ is in this case

$$m\,\frac{M}{(d^2+l^2)^{\frac{3}{2}}} \text{ (Art. 30) and}$$

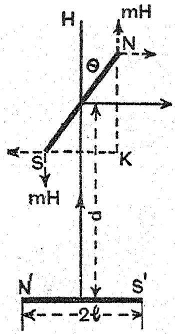


Fig. 80.

$$\text{Deflecting couple due to } N'S' = m\,\frac{M}{(d^2+l^2)^{\frac{3}{2}}} \times NK.$$

Hence, as before,

$$m\frac{M}{(d^2+l^2)^{\frac{3}{2}}} \times NK = mH \times SK,$$

i.e.

$$\frac{M}{(d^2+l^2)^{\frac{3}{2}}} = H \times \frac{SK}{NK} = H \tan\theta,$$

$$\therefore \tan\theta = \frac{M}{H(d^2+l^2)^{\frac{3}{2}}}, \quad \text{.................(4)}$$

i.e.

$$\frac{M}{H} = (d^2+l^2)^{\frac{3}{2}} \tan\theta. \quad \text{...........(5)}$$

If l be very small compared with d, this becomes

$$\frac{M}{H} = d^3 \tan\theta. \quad \text{....................(6)}$$

This is known as the "tangent B position of Gauss." The results of this section are largely used in magnetic measurements (see Chapter III.).

If F denote the field due to $N'S'$ (which is at right angles to the field H), a more "general" expression for (1) and (4) is

$$\tan\theta = \frac{F}{H}, \text{ i.e. } F = H\tan\theta.$$

34. Couples between Small Magnets.—In this and in Art. 35 the action of the earth on the magnets is not considered. Imagine two magnets NS and $N'S'$ laid under constraint in the position shown in Fig. 81, α being the angle between the field of $N'S'$ and the axis of NS. Let M and m

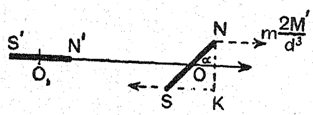


Fig. 81.

be the moment and pole strength of NS, M' the moment of $N'S'$, and d the distance O_1O; then, since the magnets are very small, we have (Art. 29)

$$\text{Couple on } NS \text{ due to } N'S' = m\frac{2M'}{d^3} \times NK$$

$$= m\frac{2M'}{d^3} \times SN \sin\alpha = \frac{2MM' \sin\alpha}{d^3}.$$

If $\alpha = 90°$ *so that the two magnets are at right angles, N'S' being "end on,"*

$$\text{Couple on } NS \text{ due to } N'S' = \frac{2MM'}{d^3},$$

and if $\alpha = 0°$ *so that the two magnets are in the same straight line,* the couple on NS vanishes.

Again, let the magnets be placed as in Fig. 82, α being again the angle between the field of $N'S'$ and the axis of NS; then we have

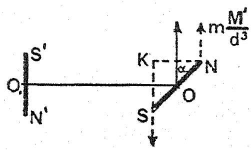


Fig. 82.

$$\text{Couple on } NS \text{ due to } N'S' = m\frac{M'}{d^3} \times NK$$

$$= m\frac{M'}{d^3} \times SN \sin\alpha$$

$$= \frac{MM' \sin\alpha}{d^3}.$$

If $\alpha = 90°$ *so that the two magnets are at right angles, N'S' being "broadside on,"*

$$\text{Couple on } NS \text{ due to } N'S' = \frac{MM'}{d^3},$$

and if $\alpha = 0°$ *so that the two magnets are parallel,* the couple on NS vanishes.

Consider now two magnets placed as in Fig. 83, where $N'S'$ is "end on" to NS, and NS "broadside on" to $N'S'$. The couple on NS due to $N'S'$ is $2MM'/d^3$ (clockwise) and the couple on $N'S'$ due to NS is MM'/d^3 (clockwise), so that regarding the two magnets as a single system the resultant couple will be $3MM'/d^3$ (clockwise). From this alone it would appear that such an arrangement, placed on some horizontal platform and suitably suspended, would undergo continuous rotation in a clockwise direction; this is contrary to experience, and the explanation is found in

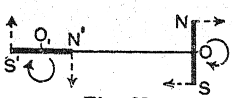


Fig. 83.

the existence of an equal and opposite balancing couple explained in Art. 35.

In the above, special cases of the couples between the two small magnets have been considered for simplicity, but a slight extension of the method enables the general case shown in Fig. 84 to be solved. With the usual notation we proceed as follows:—

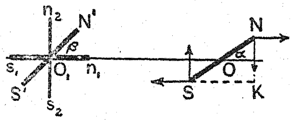


Fig. 84.

(1) *Couple on NS due to N'S'.*

Resolve $N'S'$ of moment M' into two components, viz. n_1s_1 of moment $M' \cos \beta$ along O_1O and n_2s_2 of moment $M' \sin \beta$ at right angles to O_1O. The force on each pole of NS due to n_1s_1 is (assuming it to be the same as at O) $m \dfrac{2M' \cos \beta}{d^3}$ parallel to O_1O as indicated, and the couple due to n_1s_1 is therefore

$$m \frac{2M' \cos \beta}{d^3} \times NK = m \frac{2M' \cos \beta}{d^3} \times SN \sin \alpha,$$

i.e. $$\textit{Couple due to } n_1s_1 = \frac{2MM'}{d^3} \cos \beta \sin \alpha.$$

Again, the force on each pole of NS due to n_2s_2 is $m \dfrac{M' \sin \beta}{d^3}$ at right angles to O_1O as indicated, and the couple due to n_2s_2 is therefore $m \dfrac{M' \sin \beta}{d^3} \times SK = m \dfrac{M' \sin \beta}{d^3} \times SN \cos \alpha$,

i.e. $$\textit{Couple due to } n_2s_2 = \frac{MM'}{d^3} \sin \beta \cos \alpha.$$

Since these are both clockwise we have

$$\textbf{Couple on } NS \textbf{ due to } N'S' = \frac{MM'}{d^3}(2 \cos \beta \sin \alpha + \sin \beta \cos \alpha).$$

(2) *Couple on N'S' due to NS.*

In the same way it can be proved that

$$\textbf{Couple on } N'S' \textbf{ due to } NS = \frac{MM'}{d^3}(2 \cos \alpha \sin \beta + \sin \alpha \cos \beta).$$

If $\beta = 0$ and $\alpha = 90°$ the couple on NS becomes $2MM'/d^3$ and that on $N'S'$ becomes MM/d^3, the "end on" and "broadside on" cases previously considered. If $\beta = 0°$ and $\alpha = 0°$ (magnets in the same straight line) both couples vanish, or if $\beta = 90°$ and $\alpha = 90°$ (magnets parallel) both couples also vanish, as previously indicated.

Each of the above couples is clockwise in direction, so that the resultant couple on the system as a whole is $\frac{3MM'}{d^3}(\cos\beta \sin\alpha + \sin\beta\cos\alpha)$ in a clockwise direction. Here again, however, as in the special case mentioned above, the apparent violation of mechanical principles is explained by the existence of an equal and opposite couple, dealt with in Art. 35.

933 **35. Forces between Small Magnets.**—First let the magnets be placed in line as shown in Fig. 85; then for the force on NS due to $N'S'$ we have

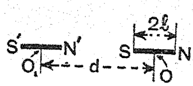


Fig. 85.

Force on pole N due to $N'S'$

$$= \frac{2M'}{(d+l)^3}m \text{ (repulsion)},$$

Force on pole S due to $N'S'$

$$= \frac{2M'}{(d-l)^3}m \text{ (attraction)}.$$

$\therefore$ Force on NS due to $N'S'$

$$= \frac{2M'}{(d-l)^3}m - \frac{2M'}{(d+l)^3}m$$

$$= 2M'm\left(\frac{(d+l)^3-(d-l)^3}{(d^2-l^2)^3}\right)$$

$$= 2M'm\left(\frac{6d^2l+2l^3}{(d^2-l^2)^3}\right),$$

and, neglecting l^2 and l^3,

$$\textbf{Force (translatory) on } NS = \frac{2M'm\,6l}{d^4} = \frac{6MM'}{d^4},$$

and it acts in the direction OO'. The force on $N'S'$ is equal to this, *but in the opposite direction $O'O$*. It will be remembered that the couples (Art. 34) in this case vanished.

Next let the magnets be placed as in Fig. 86, where $N'S'$ is end on to NS and NS broadside on to $N'S'$. So far, in discussing this arrangement, we have always assumed (Arts. 33, 34) the fields at N and S due to $N'S'$ equal to the

field at O and parallel to $O'O$, but it would be more exact to use the result of Art. 31 in stating the forces on N and S due to $N'S'$, and *the more exact treatment is necessary here.* For simplicity, however, we proceed as follows:—

To estimate the force on N, resolve $N'S'$ of moment M' into two components, viz. n_1s_1 of moment $M'\cos\theta$ along $O'N$, and n_2s_2 of moment $M'\sin\theta$ at right angles to $O'N$. The force on N due to n_1s_1 is $m\dfrac{2M'\cos\theta}{r^3}$ along NP, and the component of this perpendicular to $O'O$ (viz. in the direction NQ) is $m\dfrac{2M'\cos\theta}{r^3}\sin\theta$. We will neglect the other component in the direction NR for the present (there is really an equal and opposite component on the other pole S).

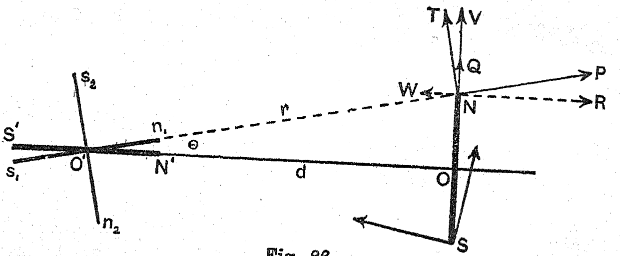


Fig. 86.

The force on N due to n_2s_2 is $m\dfrac{M'\sin\theta}{r^3}$ along NT and the component of this perpendicular to $O'O$ (viz. in the direction NV) is $m\dfrac{M'\sin\theta}{r^3}\cos\theta$. The other component in the direction NW we will neglect for the present as above (there is an equal and opposite one on S).

The total force on the pole N (due to $N'S'$) acting *upwards* in the figure, and perpendicular to OO', is therefore $\dfrac{3M'm}{r^3}\sin\theta\cos\theta$.

In the same way, by resolving $N'S'$ into two component

moments along and at right angles to $O'S$, neglecting the horizontal components of the forces on S (which are equal and opposite to those neglected above on N), it will be found that the vertical components on S due to $N'S'$ act *upwards* in the figure, perpendicular to OO', and are equal to the ones above; hence

$$\left.\begin{array}{c}\text{Translatory force on } NS \\ \text{(upwards)}\end{array}\right\} = \frac{6M'm}{r^3} \sin\theta\cos\theta,$$

and since the magnets are supposed small compared with their distance apart, we may put $r = d$, $\cos\theta = 1$, and $\sin\theta = \frac{l}{d}$, where $2l =$ length of NS; thus

$$\textbf{Translatory force on } NS = \frac{3MM'}{d^4} \textbf{ (upwards).}$$

If the horizontal components on N and S (which we have so far neglected) be now considered, it will be found that the resultant horizontal component on each pole of NS is $m\frac{M^1}{r^3}(3\cos^2\theta - 1)$ and that these form a couple on NS whose arm is $2l$ and moment therefore $\frac{MM'}{r^3}(3\cos^2\theta - 1)$. Putting $r = d$ and $\cos\theta = 1$ this becomes $\frac{2MM'}{d^3}$, the couple on NS due to $N'S'$ found in Art. 34. Thus the total effect of $N'S'$ on NS really resolves itself into a couple and a single translatory force (see Art. 21).

In the same manner as above the translatory force on $N'S'$ due to NS can be proved equal to $\frac{3MM'}{d^4}$, but acting *downwards* in the figure. Regarding then the two magnets as a single system, these two translatory forces perpendicular to OO' constitute a couple on the system, acting *counter-clockwise*, of moment $\frac{3MM'}{d^4} \times d = \frac{3MM'}{d^3}$; this balances the resultant *clockwise* couple $\frac{3MM'}{d^3}$ referred to in Art. 34.

A neater estimation of the forces in Fig. 86 involves the Calculus. To illustrate the method, we will complete the above solution by

finding the force on $N'S'$. Let the distance $OO' = x$, $2l'$ = length of $N'S'$, and m' = pole strength of $N'S'$, then

$$\text{Force on the pole } N' = m' \frac{M}{x^3},$$

Rate of change of field of NS in direction x

$$= \frac{d}{dx}\left(\frac{M}{x^3}\right) = -\frac{3M}{x^4},$$

$$\therefore \text{ Field at } S' = \frac{M}{x^3} - \frac{3M}{x^4} 2l',$$

i.e.

$$\text{Force on the pole } S' = m' \left(\frac{M}{x^3} - \frac{3M}{x^4} 2l'\right),$$

$$\therefore \text{ Resultant force on } N'S' = m' \frac{M}{x^3} - m' \left(\frac{M}{x^3} - \frac{3M}{x^4} 2l'\right)$$

$$= \frac{3Mm'2l'}{x^4} = \frac{3MM'}{x^4} = \frac{3MM'}{d^4}$$

and its direction is perpendicular to OO'.

If the magnets be parallel, the force on NS can be proved to be $\frac{3MM'}{d^4}$ along the line joining their centres, whilst the force on $N'S'$ is equal and opposite to this. It will be remembered that the couples in this case vanished.

If the general case shown in Fig. 84 be taken, it will be found that the force on NS has a component $\frac{3MM'}{d^4}(\cos\beta \sin\alpha + \sin\beta \cos\alpha)$ perpendicular to OO', and that the same applies to $N'S'$. Regarding the two magnets as a single system, it will be found that these two perpendicular components form a counter-clockwise couple on the system with arm d. This balances the clockwise couple referred to at the end of Art. 34.

36. Magnetic Shells.—Practically, a magnetic shell is a thin sheet of metal magnetised so that all one face exhibits north magnetism and all the other face south magnetism, but the precise definition is "A magnetic shell is a thin sheet of magnetic material, magnetised at every point in a direction normal to the shell at the point considered." *The strength of a shell at any point is defined as measured by the product of the intensity of magnetisation and thickness of the shell at that point;* thus if I be the intensity of magnetisation, t the thickness, and ϕ the strength,

$$\phi = It.$$

If the strength be the same at all points of the shell, and if M be the moment of the shell and A the face area,

$$\phi = It = \frac{M}{V}t = \frac{M}{At}t = \frac{M}{A},$$

so that for such a shell *the strength may also be defined as the moment per unit face area.*

The **potential at any point due to a magnetic shell** may be determined as follows. Consider (Fig. 87) a small element of the shell at O. The length of the element taken parallel to the direction of magnetisation, that is normal to the shell, is equal to t, the thickness of the shell at O. Let the area of the faces of the element be denoted by a, where a is very small. Then the magnetic moment of the element is equal to Iat, where I denotes the intensity of magnetisation. By

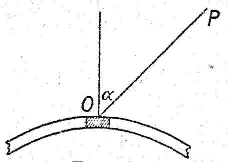


Fig. 87.

Art. 20 the potential at P due to this element is $\frac{Iat \cos \alpha}{OP^2}$, where α is the angle between OP and the normal at O in the direction of the magnetic axis of the element. But It is the strength of the shell, ϕ, and $\boxed{\frac{a \cos \alpha}{OP^2}}$ is the measure of the solid angle* subtended by the small area a at P. If this small solid angle be denoted by $\delta\omega$, then the potential at P due to the element of the shell at O is given by $\phi\,(\delta\omega)$. Hence, if ϕ be the same at all points of the shell, we have

$$\text{Potential at } P = \phi\omega,$$

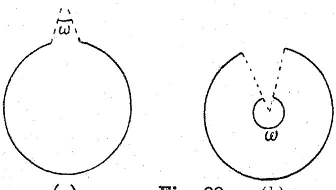
where ϕ denotes the strength of the shell and ω the solid angle subtended **by the shell** at the point P.

The theory of magnetic shells derives its importance from the fact that a closed circuit carrying a current gives rise to the same magnetic field, and is subject to the same action when placed in a magnetic field, as a shell of the same contour, and of strength numerically equal to the

* See Appendix on the "Measurement of Solid Angles."

strength of the current in the circuit; hence the following important points may be noted with advantage at this stage:—

(1) *Potential Inside and Outside a Closed Shell.*—The solid angle subtended by a shell at a point depends only on its boundary. Fig. 88 (a) shows that for a nearly closed shell the solid angle subtended at an *external* point is very small, and indicates that for a closed shell the value of ω for any point outside is zero. The potential due to a closed shell at any point outside it is therefore zero. Similarly Fig. 88 (b) shows for a nearly closed shell the solid angle subtended at an *internal* point and indicates that for a closed shell this angle is 4π. The potential at any point inside a closed shell is therefore $4\pi\phi$. (As there is no difference in potential there is no magnetic force.)



(a) Fig. 88. (b)

(2) *Difference in Potential between two Points on the same Normal, one close to the Positive Surface, the other close to the Negative Surface.*—Let A be on the positive side and A' on the negative side. If the shell be closed and A' outside, the potential at A is $4\pi\phi$, and at A' it is zero; hence the difference in potential is $4\pi\phi$. If the shell be not closed (Fig. 89) the potential at A is $(4\pi - \omega)\,\phi$, and at A' the potential is $-\omega\phi$; hence the difference in potential is $4\pi\phi$ as before.

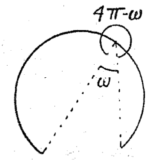


Fig. 89.

(3) *Potentials at a Point due to an Infinite Plane Shell and at the Centre of a Hemispherical Shell.*—In each of these cases the solid angle subtended at the point P is evidently 2π (Fig. 90), so that the potential at P is, in both cases, $2\pi\phi$.

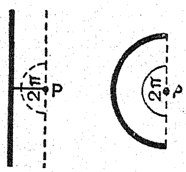


Fig. 90.

(4) *Potential and Field due to a Plane Circular Shell.*—In Fig. 91 AB is the shell, and P the point at which the potential and field are required. The solid angle subtended by the shell at P is

$$2\pi\,(1 - \cos\theta).$$

Hence $$\text{Potential at } P = 2\pi\phi\left(1 - \frac{x}{(x^2 + r^2)^{\frac{1}{2}}}\right)$$

For the *field* at P we have (Art. 19)

$$H = -\frac{dv}{dx},$$

i.e.

$$H = -2\pi\phi \frac{d}{dx}\left(1 - \frac{x}{(x^2 + r^2)^{\frac{1}{2}}}\right)$$

$$= 2\pi\phi \left\{\frac{(x^2 + r^2)^{\frac{1}{2}} - \frac{1}{2}(x^2 + r^2)^{-\frac{1}{2}} 2x . x}{x^2 + r^2}\right\}$$

$$= 2\pi\phi \frac{(x^2 + r^2)^{-\frac{1}{2}}(x^2 + r^2 - x^2)}{x^2 + r^2},$$

$$\therefore \text{ Field at } P = \frac{2\pi r^2 \phi}{(x^2 + r^2)^{\frac{3}{2}}}.$$

If P is indefinitely near the shell x is zero and the potential is $+ 2\pi\phi$ at one side and $- 2\pi\phi$ at the other, the potential difference between two such points being again $4\pi\phi$.

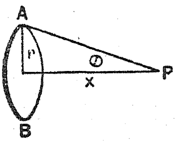


Fig. 91.

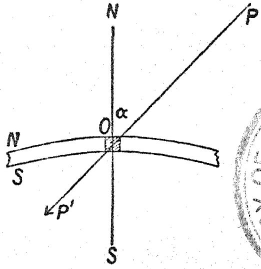


Fig. 92.

(5) *Potential Energy of a Shell in a Field.*—The potential at P (Fig. 92), due to the element of a shell at O, has been shown to be $\frac{Iat \cos a}{OP^2}$. That is, the potential energy of the element of the shell in the field due to a unit north pole at O is given by $\frac{Iat \cos a}{OP^2}$.

But the field at O, due to a unit north pole at P, is $\frac{1}{OP^2}$ along OP', and the flow of force across the area a at O is given by $\frac{1}{OP^2} . \cos a . a$, or $\frac{a \cos a}{OP^2}$ in the direction OS, that is by $- \frac{a \cos a}{OP^2}$ in the direction of ON.

Now the potential energy of the element may be written as $It \cdot \frac{a \cos \alpha}{OP^2}$ and $It = \phi$, the strength of the shell, and $-\frac{a \cos \alpha}{OP^2} = f$, or $\frac{a \cos \alpha}{OP^2} = -f$, where f is the flow of force across the surface area of the element in the direction of the magnetic axis of the element. The potential energy of the element is therefore equal to $-\phi f$.

This argument applies to every element of the shell, and, therefore, the potential energy of the shell as a whole is given by $-\Sigma\phi f$ or $-\phi\Sigma f$. But Σf is the total flow of force across the shell, that is, the total flow of force through the boundary of the shell in the direction of magnetisation, that is from the south face to the north face. If this flow of force be denoted by F, then the potential of a shell in a magnetic field is given by $-\phi F$, where ϕ is the strength of the shell and F the total flow of force through the boundary of the shell from the negative to the positive side.

(6) *Mutual Energy of Two Magnetic Shells.*—If we consider a magnetic field due to two magnetic shells in the field, the mutual energy of the shells is readily deduced. Let ϕ and ϕ' be the strengths of the shells. The flow of force F, as defined above, through the shell of strength ϕ, is proportional to ϕ' and may be taken as equal to $M\phi'$, where M is a constant. The potential energy of this shell in the field of the other is $-\phi \cdot M\phi'$. Similarly, the flow of force through the shell of strength ϕ', due to the other shell, is proportional to ϕ and may be taken as equal to $M'\phi$, where M' is a constant. The potential energy of this shell is therefore given by $-\phi' \cdot M'\phi$.

But the potential energy of either shell in presence of the other is the same, and, therefore, $\phi M\phi' = \phi' M'\phi$ or $M = M'$, and the mutual energy of the system is $-M\phi\phi'$, where ϕ and ϕ' are the strengths of the shells.

The constant M is called the *coefficient of mutual induction* for the two shells, and the fact that $M = M'$ means that the flow of force from one shell through the contour of the other *per unit strength of shell* is the same *for* both shells.

37. Equipotential Surfaces and Lines of Force.—*A line or surface of which all points are at the same potential is called an equipotential line or surface.* Thus imagine an isolated north pole of strength m units and let a sphere be described round it of radius r, the pole being at the centre. The potential at every point on the sphere is m/r (Art. 24), so that the sphere is an equipotential surface; the intersection of the sphere by a plane passing through its

centre would be a circle of radius r, every point on which would have the potential m/r, so the circle would be an equipotential line. Since no work is done in carrying a pole along an equipotential line or surface, it follows that *lines of force cut equipotential lines and surfaces at right angles*, for if not, the force would have a component along the line or surface, and work would therefore be done in moving a pole along it; this is contrary to the conception of equipotentials.

Since the potential at a point P due to a magnet NS is $m/NP - m/SP$, and this has the same value for all points of the equipotential through P, it is evident the equation of an equipotential line of a simple magnet is

$$\frac{1}{d} - \frac{1}{d_1} = \text{a constant} = c.$$

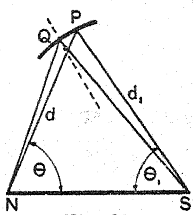


Fig. 93.

The equation to a line of force of a simple magnet is readily found. Let PQ be two points on the line indefinitely near together (Fig. 93), and let the distance $PQ = ds$. Let $NP = d$, $SP = d_1$, the angle $PNS = \theta$, $PSN = \theta_1$, $QNS = \theta + d\theta$, and $QSN = \theta_1 - d\theta_1$.

Now

$$\sin SQP = -\frac{d_1\, d\theta_1}{ds} = \cos SQR,$$

where QR is the normal

$$\cos NQR = \frac{d.d\theta}{ds}.$$

The forces at Q are (say) m/d^2 along NQ, and $m/d_1{}^2$ along QS. Since there can be no component normal to a line of force, resolving along QR we have

$$\frac{m}{d^2}\, d\frac{d\theta}{ds} + \frac{m}{d_1{}^2}\, d_1\frac{d\theta_1}{ds} = 0,$$

$$\frac{1}{d}\,\frac{d\theta}{ds} + \frac{1}{d_1}\,\frac{d\theta_1}{ds} = 0,$$

and since $d : d_1 = \sin\theta_1 : \sin\theta$

$$\sin\theta\, d\theta + \sin\theta_1\, d\theta_1 = 0.$$

Integrating,

$$\cos\theta + \cos\theta_1 = \text{a constant} = C,$$

which is the equation required.

37a. Magnetic Induction. Tubes of Force and Induction.—Before proceeding with this section the student should again read Arts. 5 and 9 of Chapter I.

Up to this point in the present chapter, the poles and magnets have been assumed to be *permanent* poles and magnets *situated in air* (strictly in vacuo) and therefore the necessity for any differentiation between magnetic force and magnetic induction, tubes of force and tubes of induction, etc., referred to in Chapter I. has not arisen. In fact in the study of fundamental magnetic theory we are mainly concerned with permanent magnets in air and even the substitution of another medium *unless it be iron or one of a very few substances* would not alter the results to any marked extent; thus the force between two magnetic poles for example is practically the same whether the poles be in air, water, oil, wood, vulcanite, etc. (Art. 9). Should the medium consist of iron or other more or less important "magnetic" material, however, complications arise and further considerations are necessary; these are of the utmost importance in electromagnetic theory and in the study of applied magnetism.

Imaginary Case of an Isolated Pole in a Medium of Indefinite extent and of Permeability μ.—Consider two poles of strengths m and m_1 situated d cm. apart in a medium of permeability μ; the force between them is $mm_1/\mu d^2$, and if m_1 be a unit pole the force $m/\mu d^2$ on it measures the strength of the field in C.G.S. units at distance d cm. from the pole m. Thus the **field intensity** (H) depends on the strength of the pole, on the distance, and *on the medium.*

Now it is convenient to have a quantity which depends only on the pole and on the distance, *i.e.* which is the same at distance d from the pole m *whatever the medium*, and this quantity is called the **magnetic induction** (B). Clearly from the preceding it must be taken as μ times the field intensity (*i.e.* $B = \mu H$) for then the induction at distance d cm. from the pole m becomes $\mu \times m/\mu d^2$ or m/d^2, *i.e.* it depends upon m and d, *but not upon the medium.* To summarise:—for a point at distance d cm. from a pole of strength m in a medium of permeability μ:—

$$H = \text{Field Intensity} = \frac{m}{\mu d^2},$$

$$B = \text{Magnetic Induction} = \mu H = \mu \frac{m}{\mu d^2} = \frac{m}{d^2},$$

and if the medium be air ($\mu = 1$)—strictly a vacuum—both H and B are measured by the same expression, viz. m/d^2.

Imagine now that the lines of induction (page 17) emanating from the pole m are grouped into tubes of induction so that at any point the number of tubes of induction per unit area (perpendicular to their direction) measures the induction at that point. Picture a sphere of radius d cm. drawn in the medium the pole m being at the centre. The induction at any point of the sphere is m/d^2 so that m/d^2 tubes of induction pass through unit area. But the area of the sphere is $4\pi d^2$ so that the total number of tubes passing through the surface is $m/d^2 \times 4\pi d^2$, *i.e.* $4\pi m$. **Thus the number of tubes of induction emanating from the pole m is $4\pi m$ and this is so whatever the medium.** The number of tubes of force is $1/\mu$ of the number of tubes of induction. In air ($\mu = 1$), the tubes of induction and tubes of force *become identical*, so that **in air the number of tubes of force emanating from the pole m is $4\pi m$** as stated in Art. 18.

The cross section of a tube of induction at distance d from the pole m is evidently $4\pi d^2/4\pi m$, viz. d^2/m and the induction is m/d^2; thus the induction multiplied by the cross section of the tube of induction is unity. Further, as has been stated in Art. 18, "the intensity at *a* point multiplied by the cross-section of the tube of force at the point is unity."

Case of a Permanent Magnet in Air.—As already indicated (Art. 5) tubes of force pass from the north pole through the field to the south pole, their number per unit area (perpendicular to their direction) at any point measuring the intensity of the field at that point. These tubes of force are continuous with other tubes inside the magnet, but these latter are not tubes of force, i.e. *they do not by their number per unit area measure the intensity of the field inside.* They are in fact tubes of induction and their number per unit area (perpendicular to their direction) at any point measures

the induction at that point. If m be the strength of the poles it may be assumed that $4\pi m$ tubes of induction pass through the substance of the magnet from the south pole to the north pole, and if a be the cross sectional area the number of tubes of induction per unit area may be taken as $4\pi m/a$, *i.e.* $4\pi I$ where I is the intensity of magnetisation: thus the magnetic induction $B = 4\pi I$. Of course when the $4\pi m$ tubes of induction pass from the north pole into the air they spread out and are both tubes of force and tubes of induction, *i.e.* their number per unit area now measures both the intensity of the field and the induction at any point.

To be more exact the demagnetising effect of the poles (Art. 4) should be taken into account in writing down the induction in the magnet. Thus the magnetisation effect $4\pi I$ is directed in the magnet from the south pole to the north pole whilst the demagnetising effect of the poles is directed from the north pole to the south pole. Calling this latter h we have for the induction $B = 4\pi I - h$.

Case of a Bar of Iron lying in a Magnetic Field, its length parallel to the Field.—In this case if the field be of intensity H and the bar be magnetised by induction to an intensity of magnetisation I then (neglecting the demagnetising effect of the poles) there are per sq. cm. inside the iron H tubes of force due to the field and $4\pi I$ tubes due to the magnetisation (sometimes called tubes of magnetisation). These are both directed from the south pole to the north pole inside the iron so that the total number of tubes of induction per sq. cm. is the sum of these two sets, viz. $H + 4\pi I$: this measures the magnetic induction B so that $B = H + 4\pi I$. It follows from this that since $B = \mu H$ and the susceptibility $k = I/H$, $\mu = 1 + 4\pi k$. *More exact* proofs are given in Art. 207.

In the case of a permanent magnet in air the magnetising force H has been removed so that there is no magnetising field but only residual magnetism, and $B = 4\pi I$ as previously indicated (neglecting the effect of the poles).

The subject of magnetic induction in iron, etc., is more exhaustively dealt with in Chapter XVI.

37b. Gauss's Theorem.—Imagine *any* closed surface surrounding a point pole m in a medium of permeability μ, and consider a small area a containing a given point (Fig. 93a). Let H denote the field intensity at this point and let α be the angle between the direction of H and the *outward* drawn normal to the surface at the given point. The component of H along this normal is $H \cos \alpha$, and since magnetic induction $= \mu \times$ field intensity, the induction in this direction is $\mu H \cos \alpha$. The product $\mu H \cos \alpha \times a$ is the flow of induction across the small area a. The

total flow of induction or the total normal induction over the whole closed surface is obtained by supposing the whole surface divided up into a very large number of small

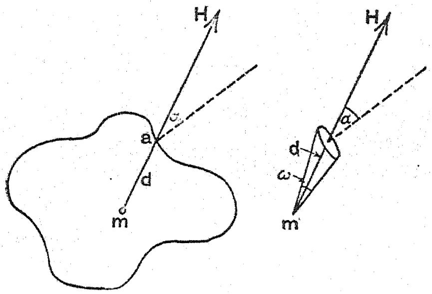


Fig. 93*a*.

areas such as a and summing up the values of $\mu H \cos \alpha \,.\, a$ for all these areas. Writing **F** for this total normal induction we have

$$\mathbf{F} = \mu \Sigma H \cos \alpha \,.\, a.$$

Now in Fig. 93*a* if d be the distance of the pole m from the point in the area a we have:—

$$\text{Normal induction over } a = \mu H \cos \alpha \,.\, a,$$

$$= \mu \frac{m}{\mu d^2} \cos \alpha \,.\, a,$$

$$= m \frac{\cos \alpha \,.\, a}{d^2}.$$

But $\cos \alpha \,.\, a/d^2$ is the solid angle ω subtended at the point pole m by the area a; hence

$$\text{Normal induction over } a = m\omega,$$

$$\therefore \left.\begin{array}{c}\text{Total normal induction for}\\ \text{the whole closed surface}\end{array}\right\} = m\Sigma\omega,$$

$$\textit{i.e. } F = 4\pi m,$$

for $\Sigma\omega$ is the solid angle subtended at the pole by the whole closed surface and is equal to 4π. This is a simple

proof of Gauss's Theorem applied to magnetism which states that **"The total normal magnetic induction over a closed surface drawn in a magnetic field is 4π times the total 'magnetism' inside."** A more complete proof of the same theorem as applied to electrostatics is given in Art. 89.

If the pole be *outside* the closed surface, the total normal induction over the surface is zero (see Art. 89).

The theorem still holds if there is more than one pole included also if they are "actual" and not "point" poles, and if some are + and some − ; in this case the *sum* of the pole strengths is involved.

In the case of an air medium μ is taken as unity; hence *for this medium* induction and intensity coincide and therefore, *for this medium*, the reader will come across the statement **"the total normal magnetic intensity over a closed surface drawn in a magnetic field is 4π times the sum of the strengths of all the magnetic poles inside."**

37c. Applications.—Imagine a plane sheet of magnetism (N pole) of *infinite* extent, the amount of magnetism per unit area being denoted by ρ and **let it be required to find the intensity of the field at P due to the sheet** (Fig. 93*b*). Picture a unit area at P parallel to the sheet. From the boundary of this area draw lines perpendicular to the sheet thus forming a prism, the other end of the prism being a unit area at P_1. In this closed surface formed by the prism the total amount of magnetism is that on the unit area S, viz. ρ. By Gauss's theorem the total induction over the closed surface is $4\pi\rho$, and as the induction is everywhere perpendicular to the sheet, the induction over the sides of the prism is zero whilst that over the two ends together is $4\pi\rho$. Half of this is at P and half at P_1 so that the induction at P is $2\pi\rho$. As the medium is air this also measures the intensity at P; hence

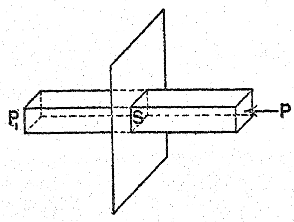


Fig. 93*b*.

$$\textbf{Intensity at } P = 2\pi\rho,$$

and is independent of the distance of P from the sheet.

The intensity of the field at the point P (Fig. 93*c*) **between two poles** (assumed close together) may also be found from the above. If I be the intensity of magnetisation, *i.e.* the amount of pole per unit area, then if the poles are very close together the intensity at P due to N is $2\pi I$ and the intensity due to S is also $2\pi I$. These are in the same direction, viz. right to left so that the total intensity at P is given by the expression

Fig. 93*c*.

Intensity at $P = 4\pi I$.

The force between two such poles in contact may also be deduced from the above. The field due to N say is $2\pi I$ and as S is in this field and has an amount of pole I per unit area the force on this area is field × pole, *i.e.* $2\pi I \times I$ or $2\pi I^2$; thus:—

Force on each per unit area $= 2\pi I^2$.

This also holds in the case of a magnet pole and a piece of *soft* iron. If the induced intensity of magnetisation of the iron is not I but I_1, the force per unit area is $2\pi I I_1$. Again, since $B = 4\pi I$ (approx.) $I = B/4\pi$ and $2\pi I^2 = B^2/8\pi$ as stated in Art. 7. (See also Art. 276).

Exercises II.

Section A.

(1) Define unit pole, unit potential, unit field, and establish expressions for the potential and field at any point due to a very small bar magnet.

(2) Find the potential at a point due to (a) a magnetic shell, (b) a uniformly magnetised sphere.

(3) Prove that the field due to an "end on" magnet is twice the field due to the same magnet "broadside on" at the same distance, and establish the formulae for the A and B positions of Gauss.

(4) Two short bar magnets are arranged so that the axis of one produced bisects the axis of the other at right angles. Find expressions for the "couple" and the "translatory force" on each due to the other.

(5) Show that the work done in deflecting a magnet through 180° from the meridian is $2MH$, and that the minimum potential energy of the magnet in the field is $-MH$.

Section B.

(1) Two very long thin straight electromagnets are placed in a line with their north poles facing each other, and at such a distance that they repel with a force equal to the weight of 10 pounds. What will be the force if the distance apart of the poles be doubled, whilst at the same time the currents are increased in such a way that one magnet is three times and the other five times as strong as at first? Express the result in pounds and in dynes.

(2) The force between two poles P and Q which are 2 cm. apart is equal to the weight of ·5 gramme, and the force between P and R when 3 cm. apart is equal to the weight of ·35 gramme. Find the strength of the poles Q and R if that of P is 100 C.G.S. units. At what distance must Q and R be placed apart so that the force between them may be 1 dyne?

(3) Two magnets A and B are in turn suspended horizontally by a vertical wire so as to hang in the magnetic meridian. To rotate A through 45° the upper end of the wire has to be turned once round. To deflect B through the same angle the upper end of the wire has to be turned one and a half times round. Compare the moments of A and B. (B.E.)

(4) Two exactly equal magnets are attached at their mid points so that their axes are at right angles, and the combination is pivoted so that the axes are horizontal, and they can turn freely about a vertical axis. How will the system set itself under the influence of the horizontal component of the earth's field? If the moment of each magnet is M and the moment of inertia about the axis round which it can turn K, what will be the period of vibration of the system? (B.E.)

(5) Prove that the magnetic force exerted by a short bar magnet at a point A on the line passing through its centre and perpendicular to its axis is the same as the force exerted at a point on the axis the distance of which from the centre of the magnet is $\sqrt[3]{2}$ times the distance of A from the centre. (B.E.)

(6) A magnet 10 cm. long is placed in the magnetic meridian, the north end of the magnet being to the south. The field due to this magnet just counterbalances the earth's field (·18 C.G.S. unit) at a place 35 centimetres from the centre of the magnet (along the axis produced). Find the strength of each pole of the magnet. (B.E.)

(7) Find the force of attraction or repulsion between two short bar magnets of moments 10 and 20 C.G.S. units, with their centres 20 cm. apart and their axes pointing in the same direction along the same line. (B.E.)

Section C.

(1) A solid cylindrical bar magnet of steel 10 cm. long and 1 cm. diameter vibrates once in 10 seconds in the earth's horizontal field. If the density of steel is 8 grammes per c.cm. and $H = \cdot 18$ C.G.S. unit, calculate the value of the magnetic moment of the magnet. (Inter. B.Sc.)

(2) If two short magnets of equal moment are placed with the line joining their centres along the axis of one and perpendicular to the axis of the other, calculate the intensity (magnitude and direction) of the field at the middle point of the joining line. (Inter. B.Sc.)

(3) The moment of a magnet is 1,000. How much work is done in turning it through 90° from the meridian at a place where $H = \cdot 16$? (Inter. B.Sc.)

(4) Define magnetic potential, and find the magnetic potential at a point due to a very short magnet. Show that the magnetic moment of such a magnet may be treated as a vector. (B.Sc.)

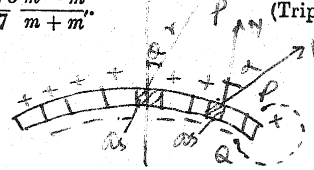
(5) Two magnets of the same length l are placed with their axes parallel, and their centres at a considerable distance R from a point P, one with its axis passing through P, the other with P on the line through its centre perpendicular to its axis. Find how the magnets must be oriented, and what must be the relation between the moments, in order that the magnetic field at P due to them may be independent of powers of l/r lower than the fourth. (B.Sc.)

(6) Two magnets of the same steel, dimensions $10 \times 3 \times \cdot 5$ and $20 \times 4 \times \cdot 7$ respectively, were found, when pivoted horizontally in the earth's field so as to oscillate about their shortest axes, to swing at the same rate. Compare the intensities of magnetisation of the two. (B.Sc.)

(7) Find an expression for the couple and the translatory force which a small magnet B exerts on a second small magnet A whose axis produced bisects that of B at right angles. Find also the couple and force exerted by A upon B, and reconcile your results with the axiom that "action and reaction are equal and opposite." B.Sc. Hons.)

(8) Show how mechanical principles are not violated by the fact that the couple exerted by one magnet on another is, in general, not the same as that of the second on the first. (B.Sc. Hons.)

(9) Two magnetic particles of moments m and m' are fixed at two corners of an equilateral triangle, with their axes bisecting the angles. A third magnetic particle is free to move at the other angular point. Show that its axis makes with the bisector of the third angle an angle $\tan^{-1}\dfrac{\sqrt{3}}{7}\dfrac{m-m'}{m+m'}$. (Tripos.)



The work done in carrying a unit +ve pole from the +ve to the −ve side of the shell along a path which goes round the edge is

$$W = V_P - V_Q.$$

where P & Q are the two points very near the shell and on the +ve and −ve side of the shell respectively. Then we have

$$W = \phi\omega - \{-(4\pi - \omega)\phi\}$$
$$= 4\pi\phi.$$

where ω is the solid angle subtended by the shell at P.

CHAPTER III.

MAGNETISM.—MAGNETIC MEASUREMENTS.

38. The Torsion Balance.—This instrument is mainly of historical interest and finds no place in the equipment of a modern laboratory. It is only introduced here because of its connection with the history of the subject and to illustrate certain principles: *practically* it is of little value. It consists (Figs. 94, 95) of a fine vertical wire of silver carrying at its lower end a suitable stirrup to hold the

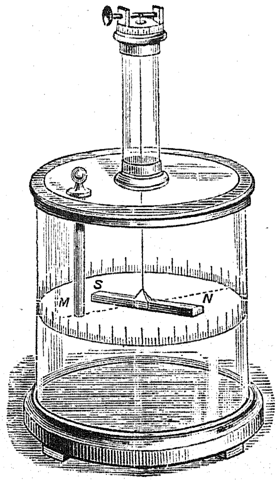


Fig. 94.

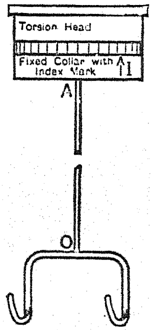


Fig. 95.

horizontal magnet *NS*, its upper end being clamped to a torsion head whereby it can be rotated. The torsion head is graduated in degrees and the collar round which it turns is provided with an index mark, so that in any experiment the angle through which the torsion head (and therefore top of the wire) is turned is known.

On that portion of the glass case which surrounds the magnet NS, and on a level with the latter, a scale of degrees is etched, so that the deflection of NS in any experiment is known. Through an aperture in the cover a vertical magnet M can be inserted so that its lower end just comes on a level with the magnet NS and opposite the zero mark on the scale.

Before commencing an actual measurement it is essential that the suspension wire be free from torsion when the magnet NS lies in the meridian, the vertical magnet M being absent; this is effected by suspending in the place of the magnet a *copper* bar of about the same size and weight, and then adjusting the apparatus until this bar lies in the meridian with its end pointing northwards opposite the zero on the scale. When the magnet is now replaced in its stirrup it lies in the meridian without putting any twist on the wire.

A working formula for the instrument may be developed as follows:—The vertical magnet M is inserted, north pole downwards, and the north pole of the suspended magnet is repelled. The torsion head is then turned in the opposite direction so as to lessen the deflection (Fig. 96). Let α = the deflection and β = the angle the torsion head is turned through. The twist on the wire is $(\alpha + \beta)$ and the couple, which is proportional to the angle, is $c\,(\alpha + \beta)$, where c is a constant for the wire.

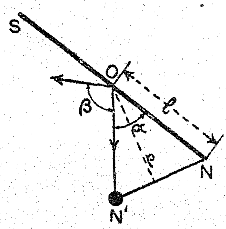


Fig. 96.

It is clear that this couple tends to bring NS towards its initial position. The couple due to the earth, also tending to move NS back again to its initial position, is $MH \sin \alpha$. Thus the total turning effect in this direction is

$$MH \sin \alpha + c\,(\alpha + \beta).$$

If F be the force between the poles, its moment about O is $F \times p$, *i.e.* $Fl \cos \frac{\alpha}{2}$, and this is in the opposite direction

to the above. Since there is equilibrium,

$$Fl \cos \frac{\alpha}{2} = MH \sin \alpha + c\,(\alpha + \beta);$$

and since $F = \dfrac{mm'}{(NN')^2} = \dfrac{mm'}{\left(2l \sin \dfrac{\alpha}{2}\right)^2} = \dfrac{mm'}{4l^2 \sin^2 \dfrac{\alpha}{2}}$,

we have
$$\frac{mm' \cos \dfrac{\alpha}{2}}{4l \sin^2 \dfrac{\alpha}{2}} = MH \sin \alpha + c\,(\alpha + \beta)$$

as the equation for the torsion balance. If α be small this may be written

$$P \frac{1}{\alpha^2} = Q\alpha + \beta,$$

where P and Q are constants.

39. Experiments with the Torsion Balance.

Exp. 1. *To verify the law of inverse squares by Coulomb's method.*—Set up the balance so that the magnet is in the meridian and the wire without twist. Turn the torsion head through an angle $a°$ so as to deflect the magnet through an angle $b°$. The torsion on the wire is $(a - b)°$ and this balances the earth's action when the deflection is $b°$. Hence the earth's action for 1° deflection may be assumed equivalent to $\left(\dfrac{a - b}{b}\right)$ degrees of torsion on the wire. Let $\dfrac{a - b}{b} = T°$. Note that in this case the twist on the wire is opposing the action of the earth.

Bring the torsion head and the magnet back to the zero positions, insert the vertical magnet and let α be the deflection. The torsion on the wire tending to bring the magnet back is $\alpha°$ and the action of the earth also tending to bring it back is measured by $\alpha T°$ of torsion. Thus the force of repulsion between the poles is balanced by an equivalent $(\alpha + \alpha T)$ degrees of torsion.

Turn the torsion head in the opposite direction so as to reduce the deflection to $\dfrac{\alpha°}{2}$. Let the torsion head be turned $\beta°$. The twist on the wire is $\left(\beta + \dfrac{\alpha}{2}\right)$ degrees, and the action of the earth

is now measured by $\frac{\alpha}{2}T^{\circ}$ of torsion. Thus the force of repulsion is balanced by $\left(\beta + \frac{\alpha}{2} + \frac{\alpha}{2}T\right)$ degrees of torsion.

If the law be true, at half the distance the force should be four times as great, so that, assuming—what is not strictly right—that the distance has been halved, $\left(\beta + \frac{\alpha}{2} + \frac{\alpha}{2}T\right)$ should equal $4(\alpha + \alpha T)$; but the method is not very accurate, for it involves various assumptions which are not strictly true.

In Coulomb's actual experiment the torsion head was turned through 360° to deflect the magnet 10°, so that T was equal to 350/10 = 35°. The deflection α was 24°, so that the equivalent torsion in the first case was $(24 + 24 \times 35)^{\circ}$, *i.e.* **864°**. The torsion head was turned eight times round to reduce the deflection to 12°, so that β° was 2880° and the equivalent torsion in the second case was $(2880 + 12 + 12 \times 35)$, *i.e.* **3312°** and $3312/864 = 3{\cdot}8$ = say 4 nearly, which roughly proves the law of inverse squares.

Exp. 2. *To verify the Torsion Balance formula and therefore the Law of Force between magnetic poles.*—Turn the torsion head through an angle β to deflect the magnet through an angle α; then in the formula $Q\alpha + \beta = \frac{P}{a^2}$ we have $Q\alpha - \beta = 0$ (the torsion is acting *against* the earth and the vertical magnet is absent). Hence $Q = \beta/\alpha$, so that Q is determined.

Now insert the vertical magnet, turn the torsion head so as to lessen the deflection, and note the value of β and of α. Repeat, obtaining a series of corresponding values of β and α.

From the formula $Q\alpha + \beta = \frac{P}{a^2}$ we have $a^2(Q\alpha + \beta)$ = a constant. Substitute the value of Q and the various values of β and α and verify that the expression is constant. Also $Q\alpha + \beta \propto \frac{1}{a^2}$, hence plot $Q\alpha + \beta$ against $\frac{1}{a^2}$ and verify.

Exp. 3. *To compare the moments of two magnets.*—Suspend the first magnet A so that it hangs in the meridian when the wire is without twist, and then turn the torsion head through an angle $\beta_1{}^{\circ}$ to deflect the magnet through an angle α°. Repeat with the second magnet B and let $\beta_2{}^{\circ}$ be the angle the torsion head must be turned through to deflect the magnet through the same angle α°.

The couple due to the torsion in the first case is $c(\beta_1 - \alpha)$ and this balances the couple due to the earth, viz. $M_a H \sin \alpha$; hence

$$M_a H \sin \alpha = c(\beta_1 - \alpha).$$

Similarly, in the second case

$$M_b H \sin \alpha = c(\beta_2 - \alpha),$$

$$\therefore \frac{M_a}{M_b} = \frac{\beta_1 - \alpha}{\beta_2 - \alpha}.$$

(See worked example page 66.)

Exercise. Solve the following (1) by Coulomb's method, (2) by means of the formula: "The torsion head of a torsion balance is turned through 50° and the suspended magnet is deflected through 10°. If the instrument be brought back to its initial position and the vertical magnet inserted, the suspended magnet is deflected 30°. How much must the torsion head be turned to reduce this deflection to 15°? (Inter. B.Sc.)."

40. The Oscillation Magnetometer.—Searle's form of this apparatus consists (Fig. 97) of a small cylindrical magnet (15 mm. in length) fixed in a brass bob whereby the moment of inertia of the system is increased and therefore the period of vibration increased for greater convenience in counting vibrations. Below the magnet is fixed an aluminium pointer to further facilitate the observation of the vibrations. The centre of the magnet lies vertically above the lowest point of the bob, so that the middle of the magnet is readily placed over (say) any desired point in a field. The whole is suspended by a silk fibre 15 cm. long from a torsion head fixed on a small stand. If desired the pointer may be omitted and the whole arranged in a narrow bottle or test tube fixed to a suitable base.

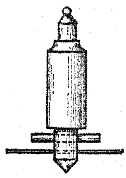


Fig. 97.

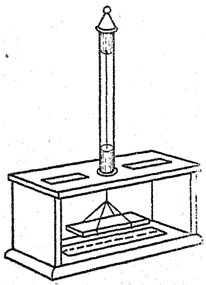


Fig. 98.

Fig. 98 shows a vibration box for the study of the vibration of different magnets. The magnet is held in the double loop of the silk support hung from the torsion head at the top of the tube. Before commencing an experiment, any torsion on the thread must be removed by the method indicated in Art. 38, and when

experimenting, the extent of the swings should not exceed 3° on either side of the neutral position.

Oscillation Experiments.—When great accuracy is required in vibration experiments corrections must be made for amplitude and torsion, and frequently for the effects of temperature and the inductive action of the fields on the magnetic moments. These are dealt with later and will be omitted in this section.

Exp. 1. *To compare the moments of two magnets.*—Suspend the first magnet, of moment M_1 and moment of inertia K_1, in the oscillation box, and find the time taken to perform (say) 30 or 40 complete vibrations: from this calculate the time t_1 of one complete vibration. Repeat with the second magnet, of moment M_2 and moment of inertia K_2, and let t_2 be the time of a vibration; then

$$t_1 = 2\pi\sqrt{\frac{K_1}{M_1H}} \quad \text{and} \quad t_2 = 2\pi\sqrt{\frac{K_2}{M_2H}},$$

$$\therefore t_1^2 \propto \frac{K_1}{M_1} \quad \text{and} \quad t_2^2 \propto \frac{K_2}{M_2},$$

$$\therefore \frac{M_1}{M_2} = \frac{K_1}{K_2}\left(\frac{t_2}{t_1}\right)^2.$$

If n_1 and n_2 be the number of vibrations performed in the same time (say one minute) by the two magnets, then, since $n = \frac{1}{t}$ (Art. 22), $n_1 \propto \frac{1}{t_1}$ and $n_2 \propto \frac{1}{t_2}$,

$$\therefore \frac{M_1}{M_2} = \frac{K_1}{K_2}\left(\frac{n_1}{n_2}\right)^2.$$

If the moments of inertia of the two magnets are equal,

$$\frac{M_1}{M_2} = \left(\frac{n_1}{n_2}\right)^2 = \left(\frac{t_2}{t_1}\right)^2.$$

This method has the disadvantage that it invariably necessitates the determination of K_1 and K_2. This may be avoided by causing the two magnets to oscillate together as one system, first with like poles, and then with unlike poles pointing in the same direction. The magnets are arranged with parallel axes in the same vertical plane, each magnet being symmetrical to the axis of oscillation, so that when one is reversed in direction the moment of inertia of the oscillating system is unchanged. If t_1 and t_2 are

the times of vibration and K the moment of inertia of the compound system we have

$$t_1 = 2\pi\sqrt{\frac{K}{(M_1 + M_2)H}} \quad \text{and} \quad t_2 = 2\pi\sqrt{\frac{K}{(M_1 - M_2)H}}.$$

This gives

$$\frac{M_1}{M_2} = \frac{t_1^2 + t_2^2}{t_2^2 - t_1^2} = \frac{n_1^2 + n_2^2}{n_1^2 - n_2^2}. \quad \text{(See example Art. 22.)}$$

Exp. 2. *To compare the earth's horizontal field at two places.*—Using Searle's magnetometer, find the time t_1 of a complete vibration or the number of vibrations (n_1) in a given time at the first place where the earth's field is H_1. Repeat with the same magnetometer at the second place, and let t_2, n_2, and H_2 be the corresponding values. Since K and M are the same in both cases (neglecting the inductive action of the earth),

$$\frac{H_1}{H_2} = \left(\frac{n_1}{n_2}\right)^2 = \left(\frac{t_2}{t_1}\right)^2.$$

Exp. 3. *To compare the field at a point P due to a magnet with the earth's horizontal field.*—Place a magnet on the table with its axis in the meridian and its north pole pointing northwards, and let the given point P be on the axial line and (say) north of the magnet. It is evident that at P the field (F) due to the magnet is in the *same* direction as the earth's horizontal field H. Hence place the magnetometer at the point P and find the values of t_1 or n_1 in the field $F + H$. Remove the magnet and find t_2 or n_2 in the earth's field H alone; then

$$\frac{F + H}{H} = \frac{n_1^2}{n_2^2}, \qquad \therefore \frac{F}{H} = \frac{n_1^2 - n_2^2}{n_2^2},$$

or

$$\frac{F + H}{H} = \frac{t_2^2}{t_1^2}, \qquad \therefore \frac{F}{H} = \frac{t_2^2 - t_1^2}{t_1^2}.$$

If the magnet had been placed with its south pole pointing northwards, the field of the magnet at P would have been *opposed* to the earth's field, *i.e.* the total field would have been $F - H$, assuming F the greater: in this case

$$\frac{F - H}{H} = \frac{n_1^2}{n_2^2}, \qquad \therefore \frac{F}{H} = \frac{n_1^2 + n_2^2}{n_2^2} = \frac{t_2^2 + t_1^2}{t_1^2}.$$

Exp. 4. *To compare any two magnetic fields.*—If the effect of the earth's field can be neglected, the method of Exp. 2 above may be used. If the earth's field cannot be neglected, then arrange the two fields in turn parallel with, and in the same direction as, the earth's

field. Let n_1, n_2, and n_0 be the number of vibrations in equal times in the fields $F_1 + H$, $F_2 + H$, and H alone; then

$$n_0^2 \propto H,$$
$$n_1^2 \propto F_1 + H, \quad \therefore n_1^2 - n_0^2 \propto F_1,$$
$$n_2^2 \propto F_2 + H, \quad \therefore n_2^2 - n_0^2 \propto F_2,$$
$$\therefore F_1 : F_2 = n_1^2 - n_0^2 : n_2^2 - n_0^2.$$

If the earth's field be opposed to the others

$$F_1 : F_2 = n_1^2 + n_0^2 : n_2^2 + n_0^2.$$

In practice the *time* for 30 or 40 vibrations is observed and then n found, or t may be calculated and the corresponding formulae involving t employed.

Exp. 5. *To verify the law of inverse squares for a magnetic pole to find the pole strength* (m) *of a long magnet, and to compare the pole strengths of two long magnets.*—Consider a **long** magnet—preferably a long Robison magnet—lying in the meridian with (say) its north pole northwards. The field *due to the magnet* at a point P on the axial line and near the north pole of the magnet may be regarded as *due to this pole only* and equal to m/d^2 (Art. 18), where d is the distance between the pole and the point P. Thus the field *varies inversely as the square of the distance*, and the law may be verified by the oscillation magnetometer.

Let n_1 = the number of vibrations in a given time under the influence of the earth (H) alone, and n_2 the number in the same time when the long magnet is placed as above, its north pole being (say) 2 inches from the centre of the oscillating needle; then $n_2^2 - n_1^2$ is proportional to the force due to the pole 2 inches distant. This is repeated with the bar magnet at various distances. Taking any pair of results it will be found that approximately

$$\frac{\text{Force due to pole at distance } d_1}{\text{Force due to pole at distance } d_2} = \frac{d_2^2}{d_1^2},$$

which is the law of inverse squares. Further, if $F \propto 1/d^p$, Fd^p = a constant; hence

$$\log F + p \log d = \text{a constant.}$$

Thus from the various numbers obtained proportional to F at various distances find the various values for $\log F$ and $\log d$, and plot $\log F$ against $\log d$; the tangent of the angle of slope of the curve will give p, and this will be found to be approximately 2.

The pole strength of the long magnet may be found if the earth's horizontal field H be known. Thus, if d = the distance between P and the pole, n_2 = the number of vibrations under the influence of

$F + H$, and n_1 = the number in the same time under H alone, we have $\frac{F}{H} = \frac{n_2^2 - n_1^2}{n_1^2}$ and $F = \frac{m}{d^2}$, hence

$$m = d^2 H \frac{n_2^2 - n_1^2}{n_1^2}.$$

Again, the pole strengths of two *long* magnets may be compared by placing them, in turn, in the position above and at *equal distances* from the magnetometer; if n_2 and n_3 be the vibrations in equal times when magnets A and B are in position respectively,

Pole strength of A : Pole strength of $B = n_2^2 - n_1^2 : n_3^2 - n_1^2$.

Exp. 6. *To verify the inverse cube law for a small magnet and to find the moment of the magnet.*—In Art. 29 it is shown that the field due to a small magnet at a point on its axial line is $2M/d^3$, *i.e.* is *inversely as the cube of the distance* **if the distance is great compared with the length of the magnet.** Hence, with these conditions satisfied, distances being measured from the centre of the small magnet to the magnetometer, the law is capable of verification by the method of Exp. 5. In this case p should be 3.

The moment of the small magnet may also be found. Thus, with the notation of Exp. 5, $F = \frac{2M}{d^3}$ and $\frac{F}{H} = \frac{n_2^2 - n_1^2}{n_1^2}$; hence

$$M = \tfrac{1}{2} d^3 H \frac{n_2^2 - n_1^2}{n_1^2}.$$

Exp. 7. *To investigate the distribution of "free" magnetism along an ordinary bar magnet.*—Some idea of this may be obtained by allowing a small magnet to oscillate opposite successive points in the length of the bar. The bar magnet is fixed vertically (Fig. 99) with its south pole upwards. In testing the upper half, the needle is fixed due magnetic south of the bar, and in testing the lower half, due magnetic north of it, so that in both cases F and H are in the same direction. The needle must be heavy to prevent it being attracted up to the magnet, *i.e.* it must be weighted. The table below gives the result of an experiment of this kind; but such is not of great value, for the moment of the needle is constantly varying owing to the varying inductive action of different parts of the magnet, and the magnet is itself affected by the needle. A much more satisfactory method of performing this test is given in Chapter XVII.

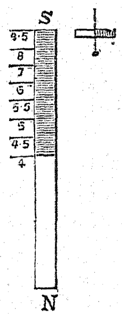


Fig. 99.

Needle makes four vibrations per minute when under the influence of the earth's field only.

Distance of needle from centre of magnet.	Number of vibrations of needle.	Relative intensities of fields due to magnet.	Distribution of magnetism from centre towards N. pole.
0 cm.	4	$4^2 - 4^2 =$	0
2 „	4·5	$4{\cdot}5^2 - 4^2 =$	4·25
4 „	5	$5^2 - 4^2 =$	9
6 „	5·5	$5{\cdot}5^2 - 4^2 =$	14·25
8 „	6	$6^2 - 4^2 =$	20
10 „	7	$7^2 - 4^2 =$	33
12 „	8	$8^2 - 4^2 =$	48
13 „	10	$10^2 - 4^2 =$	84
14 „	9·5	$9{\cdot}5^2 - 4^2 =$	74·25

Exercise. A magnetic needle makes a complete vibration in a horizontal plane in 2·5 seconds under the influence of the earth's magnetism only, and when the pole of a long bar magnet is placed in the magnetic meridian in which the needle lies and 20 centimetres from its centre, a complete vibration is made in 1·5 seconds. Assuming $H = {\cdot}18$ C.G.S. units, and neglecting the torsion of the fibre by which the needle is suspended, determine the strength of the pole of the long magnet.

41. The Deflection Magnetometer.—This consists essentially of a small magnetic needle pivoted or suspended so as to move freely in a horizontal plane. When the needle is pivoted, as in the form shown in Fig. 100, the

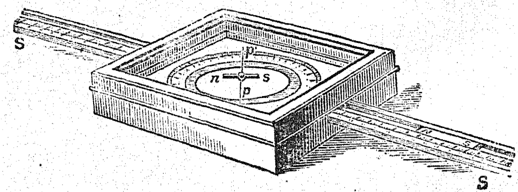


Fig. 100.

deflection of the needle can be read off on a circular scale by means of a light pointer pp' attached to the needle ns, with its length at right angles to the axis of the needle. To eliminate error due to parallax in reading the position

of the needle the circular scale should be on mirror glass, or on a ring of paper or other substance lying on a sheet of mirror glass.

The deflecting magnet is placed on the graduated arms S, S, so that its distance from the needle is known. In this form of magnetometer, when the pointer is at zero the arms S, S are at right angles to the needle, so that the deflecting magnet is mainly placed in the "end on" position shown. Fig. 101 gives another type, with two pairs of graduated arms, so that when one pair is along the direction of the field the other pair is at right angles thereto, and thus the deflecting magnet can be placed either "end on" or "broadside on" quite readily. The arms are much longer in proportion than is indicated in the figure.

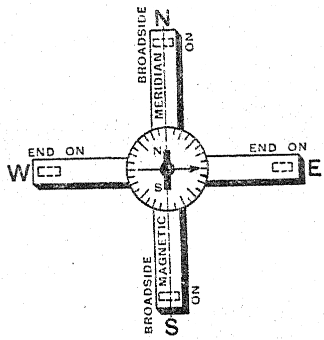


Fig. 101.

Fig. 102 shows a simple form of mirror magnetometer. In this form the needle is either fixed, with its axis horizontal, at the back of a light plane or concave mirror, or rigidly attached, as shown in the same figure, to a light frame carrying the mirror. The needle and mirror system is suspended by a single silk fibre, f, in the simple wooden stand shown in the figure. The deflection of the needle of a

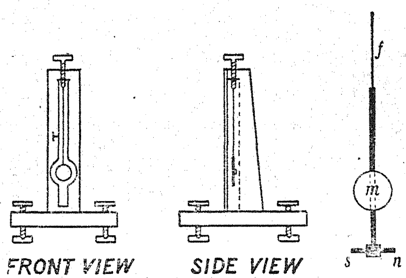


Fig. 102.

mirror magnetometer is obtained by the "lamp and scale" method.

Rays of light from a lamp pass through a slit, fall upon the magnetometer mirror, and are reflected on to a scale placed at right angles to the line joining the slit and mirror, thereby producing an image of the slit upon the scale, so that any rotation of the needle and mirror will be indicated by a motion of the image along the scale.

If the mirror be concave the slit is placed at a distance equal to its radius of curvature, and a real image is therefore formed on a scale placed at the same distance (Fig. 103). If the mirror be plane a lens L is placed in such a

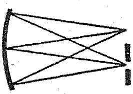


Fig. 103.

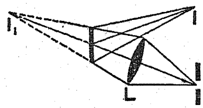


Fig. 104.

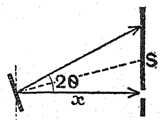


Fig. 105.

position that the image I_1 it would form of the slit would be as far behind the mirror as the scale is in front; the mirror reflects the rays and produces the image I on the scale (Fig. 104).

To obtain the actual deflection of the needle, imagine the apparatus so adjusted that, before deflection, the image on the scale is vertically above the slit; then if $\theta°$ be the deflection of the needle, the reflected rays move through an angle $2\,\theta°$ and (Fig. 105) $\tan 2\,\theta = \frac{s}{x}$, where s = displacement of image along the scale in centimetres and x = distance between slit and mirror in centimetres; thus $2\,\theta$ is obtained from tables and θ found. Approximately

$$\tan\,\theta = \frac{1}{2}\,\tan 2\,\theta = \frac{1}{2}\,\frac{s}{x}.$$

Fig. 106 shows a modern reflecting magnetometer (lamp and scale not shown). In yet another form, a telescope occupies the position of the lamp and lens, and this is used to view the image of the scale in the mirror (plane); the posi-

tion of the cross wires on the image of the scale enables the deflections to be obtained.

In deflection experiments the following errors may arise :—

(1) *The deflecting magnet may not be symmetrically magnetised.* This is eliminated by turning the magnet round so that its north pole occupies the position previously occupied by its south pole, and repeating the observations (Fig. 107).

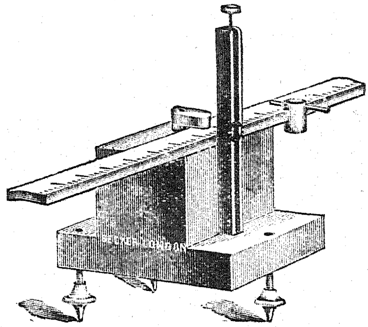


Fig. 106.

(2) *The needle may not be pivoted or suspended at the centre of the graduated arm.* This is eliminated by repeating the observations above with the magnet at the same distance on the other side of the needle (Fig. 107).

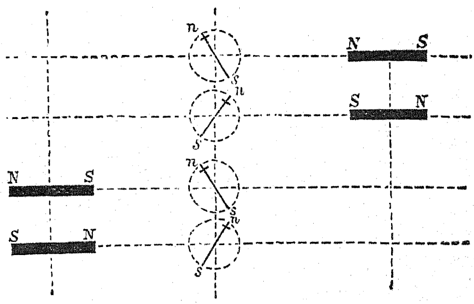


Fig. 107.

(3) *In the form shown in Figs.* 100, 101 *the needle may not be pivoted at the centre of the circular scale.* This is eliminated by reading both ends of the pointer each time.

The mean of the eight observations may be taken as the true deflection.

42. Experiments with the Deflection Magnetometer.—In this section it will be assumed that the magnets are so short compared with their distance from the magnetometer needle that the approximate formulae of Art. 33 may be employed.

Exp. 1. *To compare the moments of two magnets.*—Set up the magnetometer for (say) the A position of Gauss, *i.e.* with the graduated arms at right angles to the meridian.

Place the first magnet A "end on," its neutral line at a certain distance d (which must be great compared with half the length of the magnet), and read the deflection from both ends of the pointer.

Turn the magnet round so that its north and south poles change places and again read both ends of the pointer.

Place the magnet on the other side of the needle so that its neutral line is at the same distance d from the needle and repeat the above four observations. Let θ_1 be the mean of the eight readings.

Repeat with the second magnet B, its neutral line being at the same distance d from the needle, and let θ_2 be the mean of the eight readings. Clearly (Art. 33)—

$$\frac{M_1}{H} = \frac{d^3 \tan \theta_1}{2} \text{ and } \frac{M_2}{H} = \frac{d^3 \tan \theta_2}{2},$$

where M_1 and M_2 are the moments of the magnets and H the horizontal component of the earth's field. Hence

$$\frac{M_1}{M_2} = \frac{\tan \theta_1}{\tan \theta_2}.$$

If the magnetometer be of the mirror type four readings of the deflection of the image on the scale are taken for each magnet (*i.e.* for the four positions shown in Fig. 107). Let s_1 cm. = the mean of the four deflections produced by A, s_2 cm. = the mean of the four produced by B, and x cm. = the distance between the slit and the mirror; then

$$\frac{M_1}{M_2} = \frac{\tan \theta_1}{\tan \theta_2} = \frac{\frac{1}{2} \cdot \frac{s_1}{x}}{\frac{1}{2} \cdot \frac{s_2}{x}} = \frac{s_1}{s_2}.$$

Another method is to place the first magnet with its neutral line d_1 cm. from the needle and note the deflection. The second magnet then takes the place of the first and its distance is altered until it

produces the *same* deflection. If d_2 cm. be the distance of its neutral line from the needle when this condition holds, we have

$$\frac{M_1}{H} = \frac{d_1^3 \tan \theta}{2} \text{ and } \frac{M_2}{H} = \frac{d_2^3 \tan \theta}{2},$$

$$\therefore \frac{M_1}{M_2} = \frac{d_1^3}{d_2^3}.$$

Exp. 2. *To compare the earth's horizontal field at two places.*—At the first place let θ_1 be the mean of the eight readings given when the neutral line of the magnet is d cm. from the needle. The same magnet and magnetometer are taken to the second place and the experiment repeated with the magnet at the same distance d cm. from the needle. Let θ_2 be the mean of the eight readings, H_a the earth's field at the first place, and H_b the earth's field at the second place; then, if the "end on" position has been used,

$$\frac{M}{H_a} = \frac{d^3 \tan \theta_1}{2} \text{ and } \frac{M}{H_b} = \frac{d^3 \tan \theta_2}{2},$$

$$\therefore \frac{H_a}{H_b} = \frac{\tan \theta_2}{\tan \theta_1}.$$

Another method is to note the deflection θ and the distance d_1 at the first place, and then at the second place to alter the distance until the same deflection θ is obtained; if d_2 be the distance

$$\frac{H_a}{H_b} = \frac{d_2^3}{d_1^3}.$$

Exp. 3. *To determine the moment of a magnet* (M) *or the strength of the earth's field* (H).—If the earth's field H be known, the moment of the deflecting magnet may be found from the relation $M = \dfrac{Hd^3 \tan \theta}{2}$ or $M = Hd^3 \tan \theta$, according to whether the "end on" or "broadside on" position be employed. Similarly, if the moment of the deflecting magnet be known, H may be found from the relation $H = 2M/d^3 \tan \theta$ or $H = M/d^3 \tan \theta$.

Exp. 4. *To verify the inverse cube law for a small magnet.*—For this experiment a good mirror magnetometer should be employed and "distances" should be as great as possible consistent with readable deflections. The small magnet is placed (say) "end on" and the deflection s and distance d between the neutral line and needle noted. This is repeated at various distances. Taking any pair of results, it will be found that approximately

$$\frac{\text{Deflection } s_1 \text{ with magnet at distance } d_1}{\text{Deflection } s_2 \text{ with magnet at distance } d_2} = \frac{d_2^3}{d_1^3}.$$

Now $F = H \tan\theta$ (Art. 33), *i.e.* F is proportional to $\tan\theta$, and therefore proportional to s; hence

$$\frac{\text{Field due to magnet at distance } d_1}{\text{Field due to magnet at distance } d_2} = \frac{s_1}{s_2} = \frac{d_2^3}{d_1^3},$$

which verifies the inverse cube law. Again, if $F \propto \frac{1}{d^p}$, since $F \propto s$, we have sd^p = a constant and

$$\log s + p.\log d = \text{a constant}.$$

Thus from the various values of s and d plot $\log s$ against $\log d$; the tangent of the angle of slope of the curve will give p and this will be found to be approximately 3. The actual value of F is $2M/d^3$ (Art. 29).

Exp. 5. *To verify* (*indirectly*) *the law of inverse squares for a magnetic pole.*—In Art. 30 it is proved that the field F_1 due to a small "end on" magnet is twice the field F_2 due to the same magnet "broadside on" at the same distance, and *in the proof it was assumed that the law of inverse squares for a magnetic pole was true.* Now let a magnet be placed "end on" to the magnetometer needle and let θ_1 be the mean of the eight deflections. Let the same magnet be then placed "broadside on" at the same distance and let θ_2 be the mean of the eight deflections. Since $F = H \tan\theta$ we have

$$F_1 : F_2 = \tan\theta_1 : \tan\theta_2.$$

But *if the law of inverse squares be true* $F_1 = 2F_2$, and therefore *if the law be true* $\tan\theta_1$ should be twice $\tan\theta_2$; this will be found to be so within the range of experimental error (see Art. 47).

Exp. 6. *To determine the temperature coefficient of a magnet.*—In permanent magnets a rise in temperature produces a slight decrease in the moment of the magnet, so that if M_t be the moment at $t°$ C., M_o the moment at 0° C., and a the **temperature coefficient,** *i.e.* **the decrease in unit moment for unit rise in temperature,** we have

$$M_t = M_o(1 - at).$$

To determine a, the magnet is placed in a vessel of oil and "end on" to the magnetometer needle. The oil is heated and then allowed to cool and readings are taken, at intervals, of the deflection and the temperature. Considering any pair of observations, the deflections being θ_1 and θ_2 and the temperatures t_1 and t_2, we have

$$\frac{M_{t_1}}{M_{t_2}} = \frac{\tan\theta_1}{\tan\theta_2}, \quad i.e. \quad \frac{M_o(1 - at_1)}{M_o(1 - at_2)} = \frac{\tan\theta_1}{\tan\theta_2},$$

$$\therefore a = \frac{\tan\theta_1 - \tan\theta_2}{t_2 \tan\theta_1 - t_1 \tan\theta_2}.$$

Good consistent results can only be obtained if the magnet has previously been subjected to several increases and decreases in temperature.

Exercise. A short bar magnet is placed, at Gibraltar, perpendicular to the magnetic meridian and "end on" towards a compass needle, from which it is distant 100 centimetres. When the experiment is repeated at Portsmouth the magnet has to be placed at a distance of 110 centimetres from the compass to produce the same deflection of the needle. Compare the horizontal forces of the earth's magnetism at Gibraltar and Portsmouth.

43. Absolute Determination of the Horizontal Component (H) of the Earth's Field.—This important determination is effected by combining the results of a deflection and an oscillation experiment.

(*a*) *Deflection Experiment.*—A good mirror magnetometer is arranged (say) with the graduated arm exactly at right angles to the meridian, *i.e.* for the *A* position of Gauss. A small deflecting magnet is then placed "end on" with its neutral line at a convenient distance d cm. east of the needle, and the deflection of the image along the scale is observed. The magnet is then turned round so that its north and south poles change places and the deflection is again noted. Finally, two more deflections are taken with the magnet at the same distance d cm. west of the needle.

The mean of the four deflections is then determined; let it be s cm. and let x cm. be the distance between the magnetometer mirror and the scale. If $\theta°$ be the actual deflection of the needle $s/x = \tan 2\theta$; thus $2\theta°$ may be found from tables and therefore $\theta°$ is known. Now (Art. 33)

$$\frac{M}{H} = \frac{(d^2 - l^2)^2}{2d} \tan \theta,$$

where M is the moment of the deflecting magnet, l half the distance between its poles (say half its length approximately), and H the earth's horizontal field. Thus all the terms on the right hand side are known,

$$i.e. \ \frac{M}{H} = \text{some number} = a.$$

(*b*) *Oscillation Experiment.*—The deflecting magnet is suspended in the oscillation box (Fig. 98) and, with the precautions mentioned in Art. 40, the time taken to execute (say) 50 complete vibrations is observed and from this the time (t seconds) of one vibration is calculated. This is repeated once or twice and the mean value of t taken. The moment of inertia K of the magnet is next found by weighing and measuring and then applying one or other of the formulae given in Art. 22. Now (Art. 22)

$$MH = \frac{4\pi^2 K}{t^2},$$

and all the terms on the right hand side are known,

$$i.e. \quad MH = \text{some number} = b.$$

$$\text{Hence} \qquad MH \div M/H = H^2 = b \div a,$$

$$i.e. \quad H = \sqrt{\frac{b}{a}}.$$

The actual form of the expression is

$$H = \frac{2\pi}{t(d^2 - l^2)} \sqrt{\frac{2Kd}{\tan\theta}},$$

or if the magnetometer be used in the "broadside on" or B position, so that $M/H = (d^2 + l^2)^{\frac{3}{2}} \tan\theta$,

$$H = \frac{2\pi}{t}\sqrt{\frac{K}{(d^2 + l^2)^{\frac{3}{2}} \tan\theta}}.$$

Exp. The reader must carefully carry out the above experiment. To further indicate the method, the following data of an actual determination are given. The deflection experiment was carried out in the A position.

Distance between neutral line of magnet and needle $= 40$ cm. $= d$.
Mean of eight readings of the deflection $= 10{\cdot}5° = \theta$.
Length of magnet $= 15{\cdot}28$ cm. $= 2l$, $\therefore$ $7{\cdot}64$ cm. $= l$.

$$\frac{M}{H} = \frac{(d^2 - l^2)^2}{2d}\tan\theta = \frac{\{1600 - (7{\cdot}64)^2\}^2}{80}\tan 10{\cdot}5°,$$

$$i.e. \quad \frac{M}{H} = 6371{\cdot}4.$$

In the oscillation experiment, the mean of four observations was taken in finding the period.

Time of one complete vibration = 19·5 seconds = t.

Breadth of magnet = ·65 cm.

Weight of magnet = 95·34 grammes = w.

∴ Moment of inertia of magnet (rectangular)

$$= w\frac{(\text{Length})^2 + (\text{Breadth})^2}{12} = 95{\cdot}34\,\frac{(15{\cdot}28)^2 + ({\cdot}65)^2}{12}$$

$$= 1858 = K.$$

$$MH = \frac{4\pi^2 K}{t^2} = \frac{4\pi^2(1858)}{(19{\cdot}5)^2},$$

i.e.
$$\boldsymbol{MH = 192{\cdot}93.}$$

$$\therefore H^2 = MH \div \frac{M}{H} = 192{\cdot}93 \div 6371{\cdot}4 = {\cdot}030281,$$

$$\therefore \boldsymbol{H} = \sqrt{{\cdot}030281} = \boldsymbol{{\cdot}174.}$$

44. Errors, Corrections, and Precautions in the Determination of H.

(*a*) *Deflection Experiment.*—As mentioned in Art. 41, the deflecting magnet may not be symmetrically magnetised, and the magnetometer needle may not be suspended at the centre of the graduated arm; these errors are eliminated by taking the four readings of the deflections as explained in Art. 41.

The length l really denotes half the distance between the poles of the magnet, and is approximately equal to half the length of the magnet, only for a very thin magnet. Where l cannot be determined directly with sufficient accuracy it may be eliminated from the result by the following method. We have

$$F = 2\,M\,\frac{d}{(d^2 - l^2)^2}.$$

This may be expanded and written—

$$F = \frac{2M}{d^3}\left\{1 + 2\left(\frac{l}{d}\right)^2 + 3\left(\frac{l}{d}\right)^4 + \dots\right\}.$$

As $\frac{l}{d}$ is a small quantity, the higher powers of it may be neglected, and we get as a sufficiently accurate result

$$F = \frac{2M}{d^3}\left(1 + 2\,\frac{l^2}{d^2}\right).$$

Since l^2 is unknown, we may write x^2 for $2l^2$ and we get

$$F = \frac{2M}{d^3}\left(1 + \frac{x^2}{d^2}\right).$$

And from the relation $F = H \tan\theta$ we get

$$H \tan\theta = \frac{2M}{d^3}\left(1 + \frac{x^2}{d^2}\right),$$

$$\therefore \tan\theta = \frac{2M}{H}\left(\frac{1}{d^3} + \frac{x^2}{d^5}\right).$$

Now if two observations of θ are made for distances d_1 and d_2, and if θ_1 be the mean of the four deflections at distance d_1 and θ_2 the mean deflection at distance d_2,

$$\tan\theta_1 = \frac{2M}{H}\left(\frac{1}{d_1^3} + \frac{x^2}{d_1^5}\right),$$

$$\tan\theta_2 = \frac{2M}{H}\left(\frac{1}{d_2^3} + \frac{x^2}{d_2^5}\right),$$

and eliminating x from these results we get

$$\frac{M}{H} = \frac{d_1^5 \tan\theta_1 - d_2^5 \tan\theta_2}{2(d_1^2 - d_2^2)},$$

an expression not involving l.

Another method is to find l^2, *i.e.* (half the magnetic length)^2 for the deflecting magnet, and then to use this value in the formulae of the preceding article. Thus, with θ_1, θ_2, d_1, and d_2 having the same meaning as above,

$$\frac{M}{H} = \frac{(d_1^2 - l^2)^2}{2d_1}\tan\theta_1 \quad\text{and}\quad \frac{M}{H} = \frac{(d_2^2 - l^2)^2}{2d_2}\tan\theta_2,$$

$$\therefore \frac{(d_1^2 - l^2)^2}{2d_1}\tan\theta_1 = \frac{(d_2^2 - l^2)^2}{2d_2}\tan\theta_2.$$

Solving for l^2 and neglecting higher powers of l,

$$d_2 d_1^4 \tan\theta_1 - 2d_2 d_1^2 l^2 \tan\theta_1 = d_1 d_2^4 \tan\theta_2 - 2d_1 d_2^2 l^2 \tan\theta_2,$$

i.e. $$d_1^3 \tan\theta_1 - 2d_1 l^2 \tan\theta_1 = d_2^3 \tan\theta_2 - 2d_2 l^2 \tan\theta_2,$$

$$\therefore l^2 = \frac{d_1^3 \tan\theta_1 - d_2^3 \tan\theta_2}{2(d_1 \tan\theta_1 - d_2 \tan\theta_2)}.$$

We have seen that the moment of a magnet is affected by variations in temperature, decreasing as the temperature increases, so that if M be the moment at the temperature of the room ($t°$ C.) and M_0 the moment at 0° C., $M = M_0(1 - at)$, where a is the *temperature coefficient* of the magnet (Art. 42). The moment can therefore be reduced to 0° C. if necessary by the relation $M_0 = M/(1 - at) = M(1 + at)$, the coefficient a being determined by a separate experiment. As a is very small and the change in temperature during an experiment is also small, this reduction is rarely necessary.

When the needle is deflected there is, of course, a twist on the suspension making the deflection *less* than it would otherwise be, and this torsion error may be allowed for by increasing the observed deflection by an amount depending on a constant for the suspension known as its coefficient of torsion. In practice the error is not great and it is more usual either to neglect it or to eliminate it by using what is known as the "sine" method instead of the "tangent" method above.

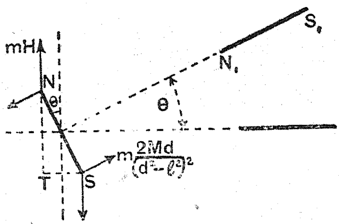


Fig. 108.

Imagine the magnetometer so constructed that the whole can be rotated, and let this be done until the magnet is always at right angles to the needle (Fig. 108), in which case the magnet and needle are in the same relative position as at the "starting" position and *there is no twist on the suspension.* Clearly,

$$m\frac{2Md}{(d^2 - l^2)^2} \times SN = mH \times ST,$$

$$i.e. \quad \frac{M}{H} = \frac{(d^2 - l^2)^2}{2d} \sin\theta.$$

Thus the tangent of the angle must be replaced by the

sine in the deflection formula. Similarly for the formula eliminating l we would have

$$\frac{M}{H} = \frac{d_1^5 \sin \theta_1 - d_2^5 \sin \theta_2}{2(d_1^2 - d_2^2)}.$$

(*b*) *Oscillation Experiment.*—When the magnet is oscillating, the twist on the suspension reduces the period of vibration, and thus for accurate work a correction must be made for torsion as follows: Let the top of the suspension be turned through β to deflect the magnet through α. The twist on the suspension is $\beta - \alpha$, and therefore $c(\beta - \alpha) = MH \sin \alpha = MH\alpha$, say, *i.e.*

$$c = \frac{MH\alpha}{\beta - \alpha}.$$

When the magnet is oscillating the restoring couple at any instant is $MH \sin \theta + c\theta = MH\theta + c\theta = (MH + c)\theta$, and the period is therefore $t = 2\pi\sqrt{\frac{K}{MH + c}}$,

$$i.e.\ MH + c = \frac{4\pi^2 K}{t^2}.$$

$$MH\left(1 + \frac{\alpha}{\beta - \alpha}\right) = \frac{4\pi^2 K}{t^2},$$

$$\therefore\ MH = \frac{4\pi^2 K}{t^2\left(1 + \frac{\alpha}{\beta - \alpha}\right)}.$$

The employment of this formula, instead of the one previously used for MH, will therefore correct for torsion; the error is not of much importance, however, in the case of the long fine suspensions employed in practice.

In the deflection experiment the magnet is *at right angles to the meridian*, whereas in the oscillation experiment it is more or less *in the meridian*. In the latter case it is, therefore, subject to the earth's inductive action and its magnetisation *slightly increased*, *i.e.* if M be the moment in the deflection experiment, zM will be the

moment in the oscillation experiment, where z is a factor greater than unity. It can be shown that

$$z = 1 + \frac{P}{Q},$$

where $Q = \frac{M}{H}$, as found in the deflection experiment, and P is a constant depending on the *volume* of the magnet and on the *material* of which it is made. In practice, however, z may be found approximately by experiment. The magnet is placed "end on" at distance d, and the deflection s_1 is noted; if $\theta_1°$ be the deflection of the needle

$$\frac{M}{H} = \frac{d^3 \tan \theta_1}{2} \text{ (approximately).}$$

A long solenoid is then placed "end on" with its centre at distance d from the magnetometer needle, and a current of electricity is passed which will produce at the centre of the solenoid a field approximately equal to the earth's field about to be determined. As will be shown later, the necessary current is obtained by the formula $I = \frac{10Hl}{4\pi n}$, where I = current in amperes (Art. 169), l = length of solenoid in centimetres, n = number of turns in the solenoid, and H = earth's horizontal field (say ·18). When a current passes the needle will be deflected, and so another coil is included in the circuit, and its position is adjusted with respect to the needle until it neutralises the effect of the solenoid.

The magnet is then placed inside the solenoid so that its centre coincides with the centre of the solenoid,* and the deflection s_2 is noted. Since the magnet is lying *along* a field of strength H, its moment is zM, and if $\theta_2°$ be the deflection of the needle

$$z\frac{M}{H} = \frac{d^3 \tan \theta_2}{2} \text{ (approximately),}$$

$$\therefore\ z = \frac{\tan \theta_2}{\tan \theta_1} = \frac{s_2}{s_1}.$$

* N. pole of magnet is towards N. end of solenoid.

Hence, since $zMH = 4\pi^2K/t^2\left(1 + \frac{a}{\beta - a}\right)$, we have

$$MH = \frac{4\pi^2Ks_1}{t^2 1 + \left(\frac{a}{\beta - a}\right)s_2},$$

correcting for torsion and the inductive action of the earth's field.

The amplitude of the vibrations must be small, otherwise a correction must be made for amplitude (Art. 22). This is, however, less than ·02 per cent. if the swing does not exceed 3° on either side, and for such swings may therefore be neglected.

If the effect of change of temperature on magnetic moment be taken into account in the deflection experiment and the moment reduced to 0° C., the same must be done in the oscillation experiment.

The Kew magnetometer for the determination of H and the Kew method are given in Chapter IV.

45. Absolute Determination of the Moment (M) of a Magnet.—It should be noted that the experiment for the determination of H (Art. 43) also serves for the determination of M, the magnetic moment of the deflecting magnet. We have

$$\frac{M}{H} = a \quad \text{and} \quad MH = b,$$

$$\therefore \frac{M}{H} \times MH = M^2 = a \times b,$$

$$i.e.\ M = \sqrt{ab},$$

where $a = \frac{(d^2 - l^2)^2}{2d} \tan\theta$ and $b = \frac{4\pi^2K}{t^2}$, the magnet being "end on" in the deflection experiment. The details of Art. 44 also apply in the determination of M.

Thus, referring to the experiment of Art. 43, we have

$$\frac{M}{\mathrm{H}} = 6371{\cdot}4 \text{ and } MH = 192{\cdot}93,$$

$$\therefore M = \sqrt{6371{\cdot}4 \times 192{\cdot}93} = 110{\cdot}87 \text{ units.}$$

46. Determination of H or M by tracing Lines of Force.—Knowing M, the value of H can be found, or, knowing H, the value of M can be found, from the diagrams plotted in Art. 5.

Exp. 1 (Fig. 23). The field at the neutral point X due to the magnet is equal and opposite to the earth's horizontal field H, *i.e.* $2Md/(d^2 - l^2)^2 = H$, where d is the distance from the neutral line of the magnet to X and l is half the magnetic length. Hence

$$H = \frac{2Md}{(d^2 - l^2)^2} \quad \text{and} \quad M = \frac{H(d^2 - l^2)^2}{2d}$$

will give either of these if the other be known. (See Exercises II. B. (6).)

Exp. 2 (Fig. 24). For the same reason we have in this case

$$H = \frac{M}{(d^2 + l^2)^{\frac{3}{2}}} \quad \text{and} \quad M = H(d^2 + l^2)^{\frac{3}{2}},$$

where d is the distance of the neutral point along the equatorial line of the magnet. (See worked Example 1, Art. 30.)

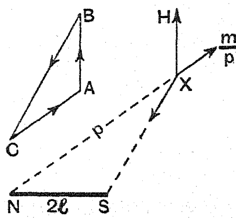


Fig. 109.

Exp. 3 (Fig. 25). This may similarly be solved by an application of the results of Art. 31, but the following method is simpler. The forces on unit pole at X are as indicated in Fig. 109. As the unit pole would be in equilibrium under the action of these three forces, if the triangle ABC be drawn with its sides parallel to the direction of the forces, the forces will be proportional to the sides to which they are parallel, *i.e.*

$$\frac{H}{\frac{m}{p^2}} = \frac{AB}{AC}, \qquad \therefore H = \frac{AB}{AC} \cdot \frac{m}{p^2} = \frac{AB\,2ml}{AC\,2p^2l},$$

$$\therefore H = \frac{AB}{AC} \cdot \frac{1}{2p^2l} \cdot M \quad \text{and} \quad M = \frac{AC}{AB} \cdot 2p^2l \cdot H.$$

47. Gauss's Proof of the Inverse Square Law for Magnetic Poles.—The principle of this has already been given in Exp. 5, Art. 42, which the reader should again note before proceeding with this section.

Now let us assume the more general case, that *the force between two poles varies inversely as the p^{th} power of the distance.* On this assumption we obtain the following results :—

(*a*) *Field due to an "end on" magnet* (Fig. 73).

If F_1 be the field at P (Fig. 73) then, as in Art. 29, we get :—

$$F_1 = \frac{m}{(d-l)^p} - \frac{m}{(d+l)^p} = m\{(d-l)^{-p} - (d+l)^{-p}\}$$
$$= m\{(d^{-p} + pd^{-(p+1)}l + \ldots) - (d^{-p} - pd^{-(p+1)}l + \ldots)\}$$
$$= 2mpd^{-(p+1)}l \text{ (neglecting higher powers of } l)$$
$$= \frac{p \,.\, 2ml}{d^{p+1}} = \frac{pM}{d^{p+1}}.$$

(*b*) *Field due to a "broadside on" magnet* (Fig. 74).

If F_2 be the field at P (Fig. 74) then, as in Art. 30, PR represents F_2 and PT represents m/r^p, and we get : —

$$\frac{F_2}{\frac{m}{r^p}} = \frac{2l}{r}, \quad \therefore F_2 = \frac{2l}{r} \,.\, \frac{m}{r^p} = \frac{M}{r^{p+1}}, \quad i.e.\ F_2 = \frac{M}{d^{p+1}}$$

if l is small compared with d. Thus the field F_1 due to such a magnet "end on" is p times the field F_2 due to the same magnet "broadside on" at the same distance, if the force between poles is inversely as the p^{th} power of the distance.

Now let the magnet be placed "end on" to a magnetometer needle at distance d, and let θ_1 be the deflection. Let it then be placed "broadside on" at the same distance, and let θ_2 be the deflection ; we have

$$F_1 = H \tan\theta_1 \quad \text{and} \quad F_2 = H \tan\theta_2,$$
$$i.e.\ F_1 : F_2 = \tan\theta_1 : \tan\theta_2.$$

But F_1 is equal to pF_2 ; hence

$$\frac{\tan\theta_1}{\tan\theta_2} = p.$$

Experiment shows that $\tan\theta / \tan\theta_2 = 2$; hence $p = 2$,

i.e. the force between magnetic poles is inversely as the square of the distance.

Gauss worked to a higher degree of accuracy than is indicated above, and showed that

$$\text{(end on)}\ldots\tan\theta_1 = Pd^{-(p+1)} + Qd^{-(p+3)} +$$

$$\text{(broadside on)}\ldots\tan\theta_2 = P_1d^{-(p+1)} + Q_1d^{-(p+3)} +$$

where P, P_1 are numerical coefficients such that $P/P_1 = p$.

From the results of his experiments he found that

$$\tan\theta_1 = \cdot086870d^{-3} - \cdot002185d^{-5}$$

$$\tan\theta_2 = \cdot043435d^{-3} + \cdot002449d^{-5}.$$

Thus $p = P/P_1 = \cdot086870/\cdot043435 = 2$; also $d^{-(p+1)} = d^{-3}$, $\therefore p = 2$; also $d^{-(p+3)} = d^{-5}$, $\therefore p = 2$.

48. Geometrical Construction for the Equipotential Lines of a Simple Magnet.—A geometrical construction for the equipotential lines of a simple magnet is as follows :—

Exp. Imagine the poles N and S (Fig. 110) to be $+20$ and -20, units respectively. Since the potential at distance r from a pole of

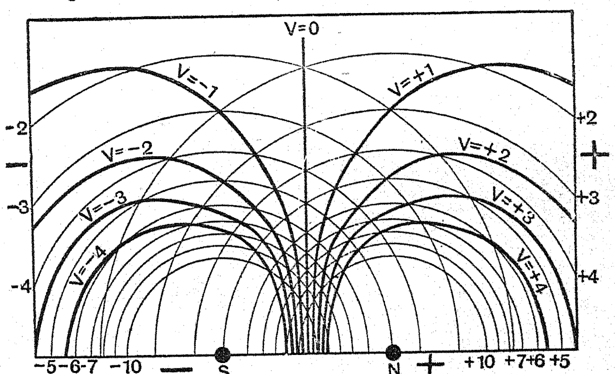


Fig. 110.

strength m is $\frac{m}{r} = v$ say, we have $r = \frac{m}{v}$. Hence with N as centre draw a series of circles representing the equipotentials $+1$, $+2$, $+3$, ... for N alone ; thus for the equipotential $+5$ we have

$r = \frac{20}{5} = 4$ cm., for $+ 15$ $r = \frac{20}{15} = 1\frac{1}{3}$ cm., and so on. With S as centre draw a series representing the equipotentials $- 1$, $- 2$, $- 3$, ... for S alone.

Since the potential at a point due to two poles is the algebraic sum of the potentials due to each, the resultant equipotential lines will be obtained from the intersections of the two sets of circles drawn. Thus the equipotential line $+ 10$ will be given by the intersection of $+ 11$, $- 1$; $+ 12$, $- 2$; $+ 13$, $- 3$; $+ 14$, $- 4$, etc.; the equipotential $- 3$ by the intersection of $- 4$, $+ 1$; $- 5$, $+ 2$; $- 6$, $+ 3$, etc.; the equipotential zero by the intersection of $+ 6$, $- 6$; $+ 4$, $- 4$; $+ 8$, $- 8$, etc. Fig. 110 gives a portion of the construction.

If a small compass be fitted with a short brass bar at its centre at right angles to its axis, the equipotential lines may be mapped by a modification of the method of Art. 5, for the needle will set along the lines of force and the bar along the equipotential lines.

49. Geometrical Construction for the Lines of Force of a Simple Magnet.

—A geometrical construction for the lines of force is as follows :—

Exp. Imagine the poles N and S to be $+ 12$ and $- 12$ units respectively (Fig. 111). Further imagine each pole to be at the

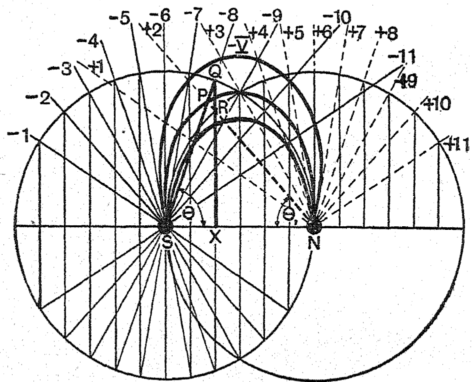


Fig. 111.

centre of a sphere as shown and imagine each sphere to have its surface divided into a number of equal areas; this is done by dividing the diameter into a number of equal parts and then erecting

planes through these points perpendicular to the diameter, in which case the sphere will have its surface divided into as many equal areas as the diameter has been divided into equal parts (12 only in the figure).

The number of lines of force through these equal areas due to the pole at the centre will be equal, *i.e.* the number of lines passing through the spaces contained by the radial lines is the same, and such may therefore be taken as representing the distribution due to the pole at the centre. The radial lines drawn from the pole N are marked *plus* and those from the pole S *minus*. The resultant lines are really the diagonals of a number of (approximate) parallelograms formed by the intersection of the radial lines; this is more apparent the greater the number of lines. In sketching the resultant lines, it should he noted that resultant $-$V passes through the intersection of $-8, +3$; $-7, +2$; $-6, +1$, etc., and so on.

If P be any point on a line of force, the angle PSN be θ, and the angle PNS be θ', it has been shown (Art. 37) that the line satisfies the condition

$$\cos\theta + \cos\theta' = \text{constant},$$

and this may be used to verify the construction above. Consider a line, say NPS, and let the circles round N and S be of such a radius r that the constant for this line is NS/r. Taking any ordinate XR cutting the circles in Q and R, we see that the intersection of NR and SQ gives us the point P on the line. Now

$$\cos\theta = SX/r \text{ and } \cos\theta' = NX/r,$$

$$\therefore \cos\theta + \cos\theta' = \frac{SX}{r} + \frac{NX}{r} = \frac{NS}{r},$$

which verifies that P is a point on the line required.

Examples.

1. *Two magnets, A and B, are caused to oscillate in the same magnetic field; A performs* 15 *vibrations per minute, and B* 10 *vibrations per minute. The magnet A is then caused to oscillate in one magnetic field and B in another; A now performs* 5 *vibrations per minute and B* 20 *vibrations per minute. Compare the intensities of the fields in which A and B now oscillate, and compare also the magnetic moments of these magnets.*

From the data of the question the second field in which the magnet A oscillates is to the first field in which both A and B oscillate in the ratio

$$\frac{5^2}{15^2} = \frac{1}{3^2} = \frac{1}{9}.$$

That is, the intensity of the second field is $\frac{1}{9}$th that of the first.

Similarly, the second field in which B oscillates is to the first as

$$\frac{20^2}{10^2} = \frac{2^2}{1} = \frac{4}{1}.$$

That is, the intensity of the second field is 4 times that of the first Therefore the ratio of the intensities of the second fields in which A and B respectively oscillate is given by

$$\frac{1}{9} : 4 \text{ or } 1:36.$$

That is, the intensity of the second field in which B oscillates is 36 times as great as that in which A oscillates a second time.

From the data given in the beginning of the question we have, assuming the magnets to have the same moment of inertia,

$$\frac{\text{The magnetic moment of } A}{\text{The magnetic moment of } B} = \frac{15^2}{10^2} = \frac{3^2}{2^2} = \frac{9}{4}.$$

2. *A suspended magnetic needle makes 20 vibrations per minute in the earth's field. When the north pole of a long bar magnet is 5 inches due magnetic south of it, the suspended needle makes 30 vibrations per minute. How many vibrations per minute will it make if the north pole of the long bar magnet be 3 inches due magnetic south of it?*

Let x = the number of vibrations per minute.

In both cases the earth's field and the field off the pole of the long magnet are in the same direction; hence

$30^2 - 20^2 \propto$ magnet's field in Case 1,

i.e. $30^2 - 20^2 \propto$ force due to the magnet pole at a distance of 5 inches,

and $x^2 - 20^2 \propto$ force due to the magnet pole at a distance of 3 inches,

$$\therefore \frac{\text{Force due to pole at } 3''}{\text{Force due to pole at } 5''} = \frac{x^2 - 20^2}{500}.$$

But by the law of inverse squares for a magnetic pole

$$\frac{\text{Force due to pole at } 3''}{\text{Force due to pole at } 5''} = \frac{5^2}{3^2} = \frac{25}{9},$$

$$\therefore \frac{x^2 - 400}{500} = \frac{25}{9},$$

$$9x^2 - 3600 = 12500,$$

$$\therefore x = \mathbf{42 \cdot 3} \text{ nearly.}$$

3. *The period of vibration of a uniformly magnetised magnetic needle is 3 seconds. The needle is then broken into exact halves. What is the period of vibration of each half?*

With the usual notation—

$$t_1 = 2\pi\sqrt{\frac{K_1}{MH}}, \quad t_2 = 2\pi\sqrt{\frac{K_2}{M_2 H}}, \quad \therefore \frac{t_1}{t_2} = \frac{\sqrt{K_1 \times M_2}}{\sqrt{K_2 \times M_1}}$$

Again—

$$K_1 = w_1\left(\frac{l_1^2}{12} + \frac{r_1^2}{4}\right) = \frac{w_1 l_1^2}{12} \text{ approx.}$$

and

$$K_2 = \frac{w_2 l_2^2}{12},$$

$$\therefore \frac{K_1}{K_2} = \frac{w_1 l_1^2}{w_2 l_2^2}.$$

Further—

$$M_1 = ml_1, \quad M_2 = ml_2, \quad \therefore M_2/M_1 = l_2/l_1.$$

Hence

$$\frac{t_1}{t_2} = \frac{\sqrt{w_1 l_1^2 \times l_2}}{\sqrt{w_2 l_2^2 \times l_1}} = \frac{\sqrt{w_1 \times l_1}}{\sqrt{w_2 \times l_2}}.$$

Now

$$w_1 = 2w_2 \quad \text{and} \quad l_1 = 2l_2,$$

$$\therefore \frac{3}{t_2} = \frac{\sqrt{2w_2 \times 2l}}{\sqrt{w_2 \times l_2}} = \frac{2}{1},$$

i.e.

$$2t_2 = 3, \quad \therefore t_2 = \mathbf{1\tfrac{1}{2}\ seconds.}$$

4. *A small compass needle makes 10 oscillations per minute under the influence of the earth's magnetism. When an iron rod 80 cm. long is placed vertically with its lower end on the same level with and 60 cm. from the needle and due (magnetic) south of it, the number of oscillations is 12 per minute. Calculate the strength of pole of the iron rod* (i) *neglecting,* (ii) *taking account of, the influence of the upper end* (B. E.)

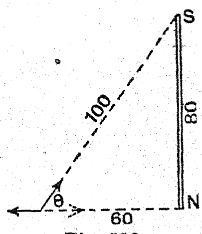


Fig. 112.

(*a*) *Neglecting upper pole.* The bar is magnetised, the lower end being a north pole, and the horizontal field due to this pole only is $F = \frac{m}{60^2}$. Now (Fig. 112)—

$$\frac{F+H}{H} = \frac{12^2}{10^2} = 1{\cdot}44, \quad \textit{i.e.} \ \frac{F}{H} = {\cdot}44, \quad \therefore F = {\cdot}44 \times H;$$

Hence

$$\frac{m}{60^2} = {\cdot}44 \times {\cdot}18,$$

i.e.

$$m = {\cdot}44 \times {\cdot}18 \times 60^2 = \mathbf{285{\cdot}12 \ units.}$$

(*b*) *Taking upper pole into account.* The field due to the upper pole is $\frac{m}{100^2}$ in the direction shown and the horizontal component of this is $\frac{m}{100^2}\cos\theta$, *i.e.* $\left(\frac{m}{100^2} \times \frac{60}{100}\right)$. The total horizontal field due to the magnet is therefore $\frac{m}{60^2} - \left(\frac{m}{100^2} \cdot \frac{60}{100}\right)$,

i.e.
$$F = m\left(\frac{1}{60^2} - \frac{60}{100^3}\right).$$

Now
$$F/H = \cdot 44, \quad i.e.\ F = \cdot 44 \times \cdot 18,$$

$$\therefore\ m\left(\frac{1}{60^2} - \frac{60}{100^3}\right) = \cdot 44 \times \cdot 18,$$

i.e.
$$m = \frac{\cdot 44 \times \cdot 18 \times 36000000}{7840}$$

$$\therefore\ m = \mathbf{363 \cdot 6} \text{ units.}$$

Exercises III.

Section A.

(1) Explain how you would compare the moments of two magnets (*a*) by the torsion balance, (*b*) by the method of oscillations, (*c*) by the method of deflections. How would you compare two magnetic fields?

(2) Describe fully Gauss's proof of the Law of Inverse Squares.

(3) Explain briefly three methods for the verification of the Law of Inverse Squares for a magnetic pole and two methods for the verification of the Inverse Cube Law for a small magnet.

(4) Describe and explain a method by which the magnetic moment of a steel magnet and the horizontal intensity of the earth's magnetic field may be determined in absolute measure.

(5) Summarise the principal errors, and give briefly the corrections, in connection with the experiment for the determination of the earth's horizontal magnetic field referred to in Question 4.

Section B.

(1) A bar magnet which can move only in a horizontal plane is caused to vibrate at three different stations, *A*, *B*, and *C*. At *A* it makes 20 vibrations in 1 minute 30 seconds; at *B*, 25 vibrations in

1 minute 40 seconds ; at C, 20 vibrations in 2 minutes. Find three numbers proportional to the forces which act upon the magnet at the three places. (B.E.)

(2) A mariner's compass is placed upon a table and a bar magnet is placed upon the floor below it, the centre of the bar magnet being straight underneath the centre of the compass needle. When the N. end of the bar magnet is northwards the compass needle, after being disturbed, makes 100 oscillations in 16 minutes. When the N. end is southwards the compass makes 100 oscillations in 12 minutes. When the bar magnet is removed so that the needle is under the influence of the earth alone, how long will it take to make 100 oscillations? (B.E.)

(3) A short bar magnet is placed on a table with its axis perpendicular to the magnetic meridian and passing through the centre of a compass needle. In London the compass needle is deflected through a certain angle when the centre of the magnet is 25 inches from the centre of the needle. If the experiment be repeated in Bombay, the magnet must be moved 5 inches nearer the needle to produce the same deflection. Use these data to compare the horizontal force in London and Bombay. (B.E.)

(4) A magnet placed due east (magnetic) of a compass needle deflects the needle through 60° from the meridian. If at another station where the horizontal force of the earth's magnetism is three times as great as at the first the same magnet be similarly placed with respect to the compass needle, what will be the deflection of the latter? (B.E.)

(5) A thin uniform magnet 1 metre long is suspended from the N. end so that it can turn freely about a horizontal axis which lies magnetic east and west. The magnet is found to be deflected from the perpendicular through an angle D ($\sin D = \cdot 1$, $\cos D = \cdot 995$). If the weight of the magnet is 10 grammes, the horizontal component of the earth's field is ·2 C.G.S. unit, and the vertical component ·4 C.G.S. unit, find the moment of the magnet. (B.E.)

(6) In an experiment to find H the following observations were taken:—Deflection experiment: End on position. Distance from centre to centre = 40 cm. Mean deflection = 8° 36′. Oscillation experiment: 6 complete vibrations took 140 seconds, mass of magnet 100·5 grammes, length 15·2 cm., breadth ·65 cm., depth 1·2 cm. Find the values of M and H.

(7) What is meant by a neutral point in a magnetic field? There is found to be a neutral point on the prolongation of the axis of a bar magnet at a distance of 10 centimetres from the nearest pole. If the length of the bar be 10 cm. and $H = 0{\cdot}18$ C.G.S. unit, find the pole strength of the magnet. (B.E.)

Section C.

(1) Describe the principle of measurement employed in the torsion balance. A magnet suspended by a fine vertical wire hangs in the magnetic meridian when the wire is untwisted. If on turning the upper end of the wire half round the magnet is deflected 30° from the meridian, show how much the upper end of the wire must be turned in order to deflect the magnet 45° and 60° respectively. (Inter. B.Sc.)

(2) Investigate the intensity due to a straight bar magnet (*a*) at any point on its axis, (*b*) at any point in the line through its centre perpendicular to its axis, the distance from the magnet being great compared with its length.

Describe how you would carry out an experiment to test the inverse square law of magnetic action, using the results of this investigation. (B.Sc.)

(3) Give an outline of the determination of H—the horizontal magnetic field of the earth—by employment of a bar magnet and a magnetic needle. How is the effect of the length of the bar magnet eliminated by observing deflections of the needle by the bar magnet at two different distances of the latter? (B.Sc.)

(4) Give an account of the method to be employed for an exact determination of the horizontal component of the earth's magnetic field. Discuss the corrections on account of (*a*) the torsion of the suspending fibre, (*b*) the amplitude of vibration, (*c*) the length of the deflecting magnet, (*d*) magnetic induction due to the earth. (D.Sc.)

CHAPTER IV.

MAGNETISM.—TERRESTRIAL MAGNETISM.

50. The Magnetic Field of the Earth.—We have already learnt that, when a magnetic needle is freely suspended at any point on the earth's surface, it invariably sets in a particular position, with its magnetic axis pointing approximately north and south. This shows that at all points on the earth's surface a magnetic field exists. This magnetic field is supposed to be due to the earth's magnetism, but whether the earth is a permanent magnet, or whether its magnetism is due to some external cause, or both, is, at present, an open question. The simplest explanation is to consider the earth as a permanent magnet, with its poles not far distant from the north and south poles of the earth. Assuming the distribution of magnetism to be somewhat irregular, the magnetic field that should result agrees approximately with observations of the actual field of the earth.

The quantities which determine the magnetic field of the earth are sometimes called the **magnetic elements** and these are (1) the **declination,** (2) the **inclination** or **dip,** and (3) the **horizontal component of the field.** The determination of (1) fixes the vertical plane in which the magnetic force acts, the determination of (2) fixes the direction of the force in that plane, and from (2) and (3) the **total intensity** of the field can be calculated.

The *magnetic meridian* at a place is the vertical plane containing the direction of the magnetic force at that place, *i.e.* the vertical plane containing the magnetic axis of a freely suspended magnetic needle. The *geographical meridian* at a place is the vertical plane through the

geographical north and south poles ~~and~~ at the given place. *Declination is the angle between the geographical and magnetic meridians.*

The *inclination or dip at a place is the angle which the resultant field (or the direction of the resultant force in the magnetic meridian) makes with the horizontal at that place.* It may also be defined as *the angle between the magnetic axis of a dip needle able to move in the magnetic meridian and the horizontal.*

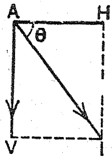


Fig. 113.

The total intensity of the earth's field is never determined directly. The horizontal component (H) and the dip θ (Fig. 113) are measured. If I be the total intensity we have

$$I = \frac{H}{\cos\theta};$$

thus I is determined. We have also the relations

$$V = H\tan\theta \quad \text{and} \quad I = \sqrt{H^2 + V^2},$$

where V is the vertical component of the field.

51. Simple Determination of Declination.—To determine the declination it is necessary to determine the geographical and magnetic meridians and to measure the angle between them. A simple method is as follows:—

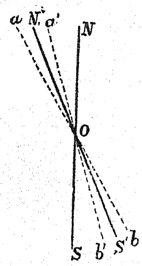


Fig. 114.

Exp. Suppose we wish to mark out the two meridians on the table in the laboratory. Let a straight piece of wire about a foot long be fastened vertically into the table so that the sun casts its shadow on the table. The direction of this shadow when *it is shortest*, which will be about noon, will approximately give the geographical meridian of the place. Let this direction be marked by a line, NS (Fig. 114), drawn on the table.

To determine the magnetic meridian, let a light bar magnet be suspended over the table by a single silk fibre. To each end of this magnet let a short piece of fine brass wire be attached by shellac or sealing-wax, in such a position that, when the magnet is properly

suspended, the pieces of wire are, as accurately as possible, vertical (Fig. 115).

Let a needle be stuck into the table at any point, a, through which the line, ww, joining the pieces of wire at the ends of the magnet passes. This point is readily determined by one observer sighting along the lower points of the wire and directing another observer where to place the needle. In the same way another needle can be placed at b, so that $a\ o\ b$ represents the direction of the line joining the wires fixed at the ends of the magnet.

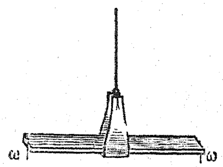


Fig. 115.

If $w\ w$ coincided with the magnetic axis of the magnet, then the line $a\ b$ would represent the trace of the magnetic meridian on the table, and the angle $N\ o\ a$ would be the required angle of declination. In all probability, however, $w\ w$ is not the magnetic axis of the magnet; but whatever the position of this axis, we can eliminate the error due to its non-coincidence with $w\ w$ by turning the magnet upside down, so that the top face becomes the bottom face, and then determining the line $a'\ o\ b'$ in exactly the same way as $a\ o\ b$ was found. The true direction of the trace of the magnetic meridian will now lie midway between the lines $a\ o\ b$ and $a'\ o\ b'$, and may be drawn by bisecting the angle between them. The line $N'\ S'$ represents this direction, and the angle $N\ O\ N'$ measures the declination.

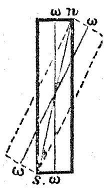


Fig. 116.

To prove that $N'S'$, bisecting the angle between $a\ b$ and $a'\ b'$, gives the true direction in which the magnetic axis of the magnet points, consider Fig. 116. Taking an extreme case, let $n\ s$ represent the position of the magnetic axis of the magnet. Then if the full outline represent the position of the magnet in the first case, on inverting the magnet it will take up the position indicated by the dotted outline, for in this position the magnetic axis has the same direction as before. From the figure it is evident that the magnetic axis $n\ s$, and, therefore, the magnetic meridian, lies midway between the two positions of $w\ w$.

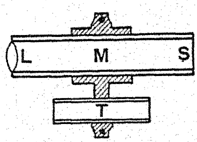


Fig. 117.

52. The Kew Magnetometer and Method for the Determination of Declination.—The magnet consists of a hollow magnetised steel tube M (Fig. 117), one end of which is closed by a piece of glass S, on which a scale is etched;

the other end is fitted with a lens L, and the whole is suspended in the box shown in Fig. 118 by silk fibres from the top of the vertical tube. The scale S is at the principal focus of the lens L, so that the rays from S, after passing through L, are parallel; hence, if the telescope, shown on the right of Fig. 118, be first focussed for infinity, and then be placed in line with the magnet so as to receive these parallel rays from L, a clear image of the scale will be seen in the telescope. The whole instrument can be rotated about a vertical axis over the horizontal circular scale, and the position at any instant noted by means of this scale and verniers.

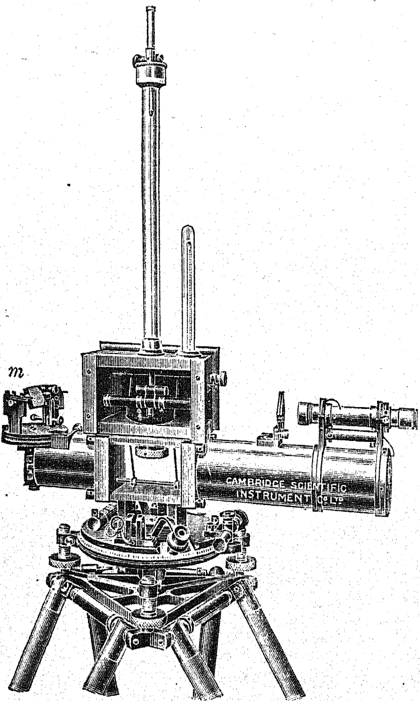


Fig. 118.

To find the magnetic meridian the suspension is first freed from torsion, as indicated in Art. 38. When the magnet is suspended, the instrument is rotated until the image of the centre of the scale S coincides with the centre of the cross wires of the telescope, and the readings on the horizontal scale are noted: let the mean

reading be θ_1. The magnet is then turned over for the reason given in Art. 51, and the instrument again slightly rotated if necessary until the centre of the image of S again coincides with the centre of the cross wires, and the readings on the horizontal scale are again taken; let the mean be θ_2. The magnetic axis of the magnet bisects the angle between the two directions just obtained, and the scale reading for the magnetic meridian is, therefore, $\frac{1}{2}(\theta_1 + \theta_2)$.

To find the geographical meridian the mirror m is adjusted so that its axis is horizontal, the normal perpendicular to the axis, and such that when m is vertical the normal passes along the optic axis of the telescope. The instrument is then rotated until an image of the sun, produced by the mirror m, just passes the centre of the cross wires. Knowing the time of this transit, the latitude, and the north polar distance of the sun, the angular distance of the sun from the geographical meridian at the time of the transit can be calculated; let this be α°. The instrument is now turned through α°, and the readings on the horizontal scale are again taken: let the mean be θ°_3. This is the scale reading for the geographical meridian, and the declination is the angle between this reading and the reading for the magnetic meridian, *i.e.* the declination is $\frac{1}{2}(\theta_1 + \theta_2) - \theta_3$.

53. Determination of Dip. The Kew Dip Circle.—If the magnetic meridian be found as in the preceding article, and a magnetic needle free to move in a *vertical plane* (Fig. 119) be placed with its axis in the meridian, it will be found that it does not set horizontally, but dips, in England, at an angle of about 68° with the horizontal. This angle is the angle of Dip, and a magnetic needle mounted so as to indicate it is called a *Dipping Needle.* The needle shown in Fig. 119 is, however, useless for purposes of measurement.

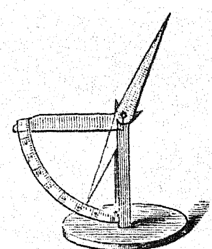


Fig. 119.

A proper instrument for measuring the angle of Dip is known as a *Dip Circle*. It consists, as shown in Fig. 120, of three essential parts—the needle mounted so as to move with as little friction as possible in a vertical plane, the vertical circular scale in front of which the needle moves, and the horizontal circular scale on which the rotation of the frame carrying the vertical scale and needle can be read. The needle is mounted by means of a short horizontal axis of hard polished steel passing, as nearly as possible, through the centre of gravity of the needle. When mounted on its bearings, this axis should pass through the centre of the vertical scale at right angles to the plane of the scale. The scale itself is usually graduated so that both ends of the needle can be read, and, since the Dip is measured from the horizontal, the *zero line* is horizontal.

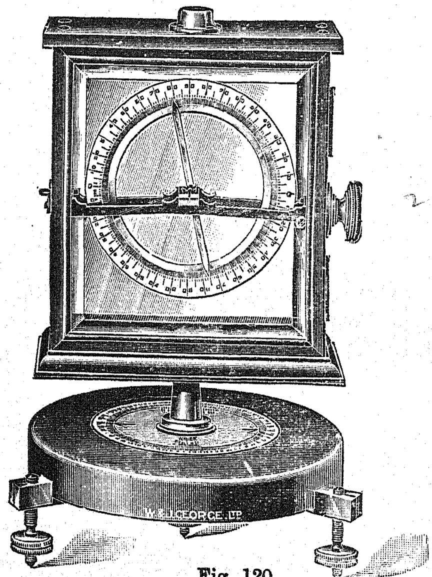


Fig. 120.

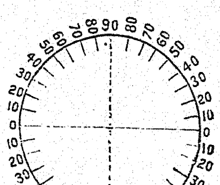


Fig. 121.

Fig. 121 shows generally how the scale is graduated: in

the best forms of the instrument, the positions of the ends of the needle are read by small reading microscopes, which carry verniers round the vertical scale (Fig. 125). The horizontal scale may be graduated continuously from 0° to 360°, commencing at any point, or it may conveniently be graduated in the way indicated in Fig. 121—its use will appear in the sequel.

Exp. To determine the Dip by a Dip Circle, it is obviously necessary, first, to set the plane of the needle in the magnetic meridian—this might be done approximately by determining the meridian by means of a magnetic needle, but it is more conveniently and accurately done by the indications of the Dip Circle itself. The instrument is carefully levelled by means of the screw feet and the spirit-level attached to it. The needle and vertical scale are then moved round the horizontal scale until the needle sets exactly vertically.

When this is the case, if the instrument be perfect, the plane of the needle is *at right angles* to the magnetic meridian, and hence on turning it through 90°, measured by the help of the vernier attached to the horizontal scale, the needle is set in the magnetic meridian.

The truth of the above is readily seen. In Fig. 113 let the plane of the paper represent the magnetic meridian. Imagine a magnet with its north pole at A pivoted to move in a vertical plane at right angles to the plane of the paper. The force acting on the pole will be in the direction of I, but only the vertical component V of this force will have any effect on the needle, for the horizontal component H can only produce a pressure on the bearings. Hence, when the plane of motion of the needle is at right angles to the meridian only V is effective, and the needle, therefore, sets vertically.

To eliminate errors in the instrument the following readings are taken in setting the vertical circle in the meridian. The vertical circle is adjusted until the needle stands vertically with its lower end at 90°, and the horizontal circle is read. Another adjustment is made, if necessary, until the upper end of the needle is at 90°, and the horizontal circle is again read. The vertical circle is now turned through 180°, and two more readings on the horizontal circle are obtained. The needle is next reversed in its bearings, and two more readings on the horizontal taken. Finally, the vertical circle is turned back again through 180°, and two more readings on the horizontal taken. If the mean of the eight readings be $\theta°$, and the vertical circle be rotated through 90° from this, the plane of the needle will be in the magnetic meridian.

The following observations are now made to obtain the dip. The reading is taken, *on the vertical circle* of each end of the needle.

The vertical circle is then turned through 180°, and each end is again read. The needle is now reversed in its bearings and two more readings taken. The vertical circle is next turned through 180° to its original position, and each end is again read. Finally, the needle is magnetised in the opposite direction and the above eight readings repeated. The mean of the sixteen angles gives the true value of the dip.

The various readings are necessary to eliminate the following possible errors:—

(1) The error due to eccentricity, that is, the error which results if the axis of suspension of the needle should not pass through the centre of the vertical scale.

(2) The error which results if the zero line of the vertical scale is not truly horizontal.

(3) The error due to non-coincidence of the magnetic axis of the needle with its geometrical axis.

(4) The error which results from non-coincidence of the point of suspension of the needle with its centre of gravity.

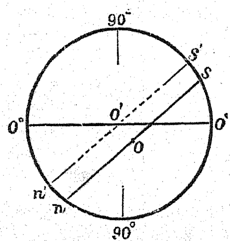


Fig. 122.

Error (1) is eliminated by reading both ends of the needle and taking the mean of the two readings. This is evident from Fig. 122, for if O represent the point of suspension of the needle and O' the centre of the circular scale, then the true Dip $o\,n'$ or $o\,s'$ is evidently the arithmetical mean of $o\,n$ and $o\,s$, the actual readings of the two ends of the needle.

Error (2) is eliminated by taking the mean reading for two positions of the needle in the magnetic meridian. When the needle is first set at right angles to the magnetic meridian, it can evidently be put in the meridian by turning through 90°, either to the right or to the left of the observer. Let it be turned first to the right, say, and let the position of the needle be read; then let it be turned through 180° in either direction, and it will be again in the magnetic meridian, but reversed relative to the observer. The mean of the readings of both ends of the needle, taken in these

two positions, gives the Dip corrected for the sources of error (1) and (2). For in Fig. 123, where the right-hand drawing represents the reverse (or obverse) view of the left-hand one, if $o'o'$ represent the true horizontal, and oo the

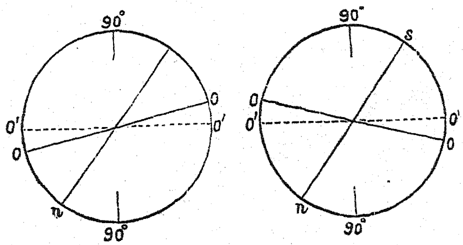


Fig. 123.

actual position of the zero-line, then the true Dip $o'n$ or $o's$ is evidently the arithmetical mean of the reading $o\,n$ or $o\,s$ in the left-hand drawing, and $o\,n$ or $o\,s$ in the right-hand drawing.

The source of error indicated in (3) has already been considered in the determination of Declination. It is eliminated in the same way by reversing the needle relative to the scale, that is, by lifting the needle off its bearings, turning it round back for front, and replacing it on the bearings. The four readings indicated above are now repeated, and the mean of the eight readings thus obtained gives the Dip corrected for errors (1), (2), and (3).

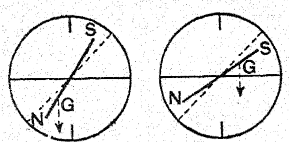


Fig. 124.

To correct for (4) the needle is remagnetised in the opposite direction. Thus in Fig. 124 (left) the centre of gravity is nearer the north pole and the readings are too large. When the polarity is reversed the centre of gravity is nearer the south pole and the readings are too small. In the mean the error is practically eliminated. It should be noted that if the centre of gravity is along the neutral

line off to one side, reversing the needle in its bearings will correct the error.

The Kew Dip Circle is a more elaborate and accurate form of instrument than that shown in Fig. 120. The essential points will be understood from Fig. 125.

The angle of Dip is sometimes determined from observations of the apparent dip taken in *any* two vertical planes at right angles. Let PA and PB (Fig. 126) be

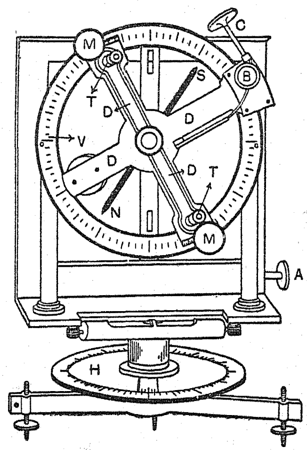


Fig. 125.

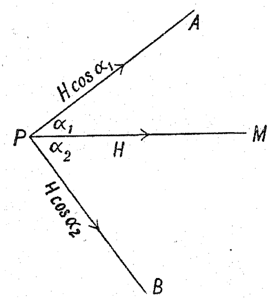


Fig. 126.

the traces in a horizontal plane of the two planes, making angles α_1 and α_2 with the magnetic meridian PM.

The components of H, the horizontal component of the earth's field, along PA and PB are evidently $H \cos \alpha_1$ and $H \cos \alpha_2$ and V, the vertical component at P, is the same for both planes. Hence if δ_1 and δ_2 denote the apparent dip in the planes of PA and PB we have

$$\frac{V}{H \cos \alpha_1} = \tan \delta_1 \text{ and } \frac{V}{H \cos \alpha_2} = \tan \delta_2.$$

Since α_1 and α_2 are complementary, we may write $\sin \alpha_1$ for $\cos \alpha_2$ and the result may be put in the form

$$\frac{H \cos \alpha_1}{V} = \cot \delta_1, \text{ and } \frac{H \sin \alpha_1}{V} = \cot \delta_2.$$

This gives

$$\frac{H^2 \cos^2 a_1}{V^2} + \frac{H^2 \sin^2 a_1}{V^2} = \cot^2 \delta_1 + \cot^2 \delta_2,$$

$$\therefore \frac{H^2}{V^2}(\cos^2 a_1 + \sin^2 a_1) = \cot^2 \delta_1 + \cot^2 \delta_2,$$

i.e.

$$\frac{H^2}{V^2} = \cot^2 \delta_1 + \cot^2 \delta_2.$$

Hence, since $\frac{H}{V} = \cot \delta$, where δ is the true dip in the plane of PM, the magnetic meridian, we get

$$\cot^2 \delta = \cot^2 \delta_1 + \cot^2 \delta_2.$$

Thus δ_1 and δ_2 are determined in any two planes at right angles to each other and the true dip δ calculated.

54. Determination of H. The Kew Magnetometer and Method.—The method of determining the horizontal component of the earth's field has been fully explained in Art. 43, and the errors, and corrections for exact work in Art. 44. The reader should revise these before proceeding further.

The Kew magnetometer arranged for the *oscillation experiment* is that shown in Fig. 118 in dealing with declination (Art. 52), the suspended magnet (which is to be used as the deflecting magnet in the deflection experiment) being of the hollow cylinder type with scale and lens (Fig. 117). The suspended magnet is given a small oscillation, and the time taken for 100 transits of the image of the centre of S across the centre of the cross wires *in the same direction*, *i.e.* the time for 100 complete vibrations is noted and from this the time of one vibration is calculated. Corrections for torsion, amplitude, temperature, and the inductive action of the earth must be applied as explained in Art. 44.

The moment of inertia K is found as follows. A brass cylinder which exactly fits the tube T is inserted in that tube and the time of a vibration is again determined.

If t_1 be the time without the cylinder and t_2 the time with it

$$t_1^2 = 4\pi^2 \frac{K}{MH} \quad \text{and} \quad t_2^2 = 4\pi^2 \frac{K + K_1}{MH},$$

$$i.e. \quad K = K_1 \frac{t_1^2}{t_2^2 - t_1^2},$$

where K_1 = moment of inertia of the cylinder

$$= m\left(\frac{l^2}{12} + \frac{r^2}{4}\right)$$

and K = moment of inertia (required) of the magnet. The variation of moment of inertia with temperature may necessitate a correction.

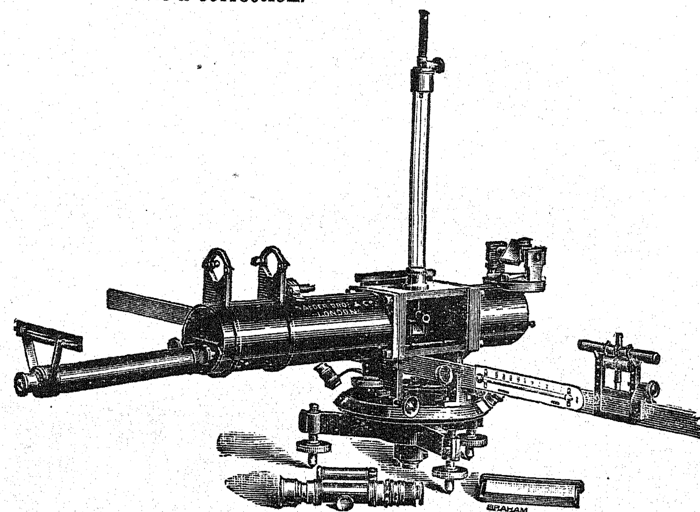


Fig. 127.

The magnetometer is now modified (Fig. 127) for the deflection experiment. The box in which the magnet has been hanging is removed and the vertical tube is screwed into the lower compartment. A small magnet with a mirror attached to its centre and perpendicular to its axis

is suspended from the top of the vertical tube. The telescope seen in the left of the figure has a scale fixed above it and the instrument is rotated until the image of this scale reflected from the mirror is seen in the telescope, the centre of the image coinciding with the centre of the cross wires, and the readings of the horizontal scale are then taken.

The graduated brass bar is then clamped in position (at right angles to the axis of the telescope) and the magnet used in the oscillation experiment is placed at a convenient distance on the carrier shown. The body of the instrument is then rotated until the image of the centre of the scale again appears on the cross wires and the readings of the horizontal scale are taken. The magnet is reversed end for end, the scale centre again brought to the cross wires and the readings of the horizontal scale again taken. Finally, readings are taken with the magnet at the same distance on the other side.

If β be the mean of the readings and α the reading before the deflecting magnet was placed in position, the deflection will be the difference between β and α. Another distance for the deflecting magnet is then chosen and the experiment repeated. The formula employed is of course the sine formula (in which l is eliminated) given in Art. 44. Correction of magnetic moment for temperature must be made as in the vibration experiment, and a further correction may be necessary for the effect of temperature on the bar which carries the deflecting magnet Theory also shows that errors are reduced if the two distances chosen for the deflecting magnet have the ratio 1·3 : 1, and if the length of the deflected magnet be ·467 of that of the deflecting magnet.

It may be noted that special instruments are necessary for the measurement of the magnetic elements at sea, and in the magnetic polar regions.

55. Miscellaneous Points.—If I and I_1, d and d_1 be the *total intensities* and the *angles of dip* at two places we have

$$\frac{I}{I_1} = \frac{\dfrac{H}{\cos d}}{\dfrac{H_1}{\cos d_1}} = \frac{H}{H_1}\frac{\cos d_1}{\cos d} = \frac{n^2}{n_1^2}\frac{\cos d_1}{\cos d},$$

where n and n_1 are the number of vibrations in equal times of an oscillating needle at the two places. Thus I and I_1 are compared. Similarly,

$$\frac{V}{V_1} = \frac{H}{H_1}\frac{\tan d}{\tan d_1} = \frac{n^2}{n_1{}^2}\frac{\tan d}{\tan d_1}.$$

Thus the *vertical components*, V and V_1, are compared.

Example. *A dipping needle of mass W gm. and magnetic moment M is pivoted not at its centre of gravity, but at a point nearer the north end. If the coordinates of the point at which it is pivoted be x along the magnetic axis, and y perpendicular thereto, and if the total intensity be T and the observed dip θ, find an expression for the true dip in terms of the given quantities.*

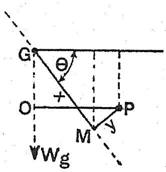


Fig. 128.

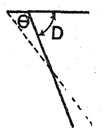


Fig. 129.

The conditions are shown in Fig. 128, where P is the pivot and G the centre of gravity.

The moment of the weight about the point P is

$$Wg \times PO$$
$$= Wg\,(x \cos \theta + y \sin \theta).$$

If D be the true dip the angle between the true line of dip and the magnetic axis of the needle is $(D - \theta)$; hence the couple due to the earth's total field T is (Fig. 129):—

$$MT \sin (D - \theta).$$

Since these balance

$$MT \sin (D - \theta) = Wg\,(x \cos \theta + y \sin \theta).$$

If $(D - \theta)$ be small

$$MT\,(D - \theta) = Wg\,(x \cos \theta + y \sin \theta),$$

$$\therefore D = \frac{Wg}{MT}(x \cos \theta + y \sin \theta) + \theta.$$

56. Magnetic Maps.—To express the results of observations of the magnetic elements at various places, it is usual to draw lines joining those points on a map where the values of the elements are equal; thus we have the following:—

(1) *Isogonals, i.e.* lines joining points at which the declination is the same: the isogonals which pass through points having zero declination are termed *agonic lines*.

(2) *Isoclinals, i.e.* lines joining points at which the inclination or dip is the same: the line which passes

through points of zero dip is called the *aclinic line* or *magnetic equator.*

(3) *Isodynamic Lines* joining points at which the horizontal force has the same value. Other lines joining points at which the vertical force has the same value are also employed.

In addition to the above we also have:—

(4) *Isodynamic Lines* of equal total force.

(5) *Duperrey's Lines* or *Lines of Magnetic Longitude, i.e.* lines which indicate the direction of the magnetic meridian.

The lines are drawn as smooth curves, but in reality they are very irregular, due to local circumstances; the general "run" of the smooth curves is, however, that of the actual irregular lines.

The north magnetic pole of the earth (discovered by Sir James Ross in 1831) is situated in the vicinity of Boothia Felix, in the far north of America, lat. 70° 5′ N., long. 96° 46′ W. The "Southern Cross" expedition of 1898-1900 placed the south magnetic pole at lat. 72° 40′ S., long. 152° 30′ E., the "Discovery" expedition of 1902-1904 placed it at lat. 72° 51′ S. long. 156° 25′ E., and Sir E. Shackelton's expedition of 1908-1909 placed it at lat. 72° 25′ S. long. 155° 16′ E. The magnetic poles are, of course, those regions where the dip is 90°, but they are not *fixed* points (Art. 58).

Fig. 130 is a map showing the *isogonals* and the *agonic lines*, the thick lines denoting the agonic lines, the continuous lines the isogonals on which the declination is west (*i.e.* north pole of compass points west of true north), and the dotted lines those on which the declination is east. Starting from the north magnetic pole and selecting an isogonal above, about 19° W. or 7° E., we are brought back to the north geographical pole. Starting from the same pole and selecting one below these values we travel from the north magnetic to the south geographical pole. A careful examination of the map will, in fact, show that the isogonals converge towards four points, viz. the two geographical poles and the two magnetic poles of the earth.

The agonic lines are, however, of more special interest,

and of these there are three, viz. the *American agonic line*, the *European agonic line*, and the *Siberian oval.* Starting at the north magnetic pole, the American agonic line passes across Canada, the eastern United States, Brazil, to the south geographical pole. A curve continues this from the south geographical to the south magnetic pole, at every point of which the compass needle also lies in the geographical meridian, but its north pole points south instead of north, so that the curve is an isogonal of 180°.

As a continuation of this the European agonic line leaves the south magnetic pole, passes through West Australia, almost east to west across the Indian Ocean, through Persia, Russia, to the north geographical pole. A curve continues this from the north geographical pole to the north magnetic pole, at every point of which the compass needle still lies in the geographical meridian, but with its north pole pointing south, so that the curve is an isogonal of 180°.

The third agonic line is an oval enclosing parts of China and Siberia. At places between the American and European agonic lines the declination is *west*, between the European agonic line and the Siberian oval it is *east*, inside the oval it is *west*, and between the oval and the American agonic line it is *east*.

The reader must carefully distinguish between the isogonals and Duperrey's lines; the latter indicate the *magnetic meridians*, are not nearly so irregular as the isogonals, and converge, not to four, but to two points only, viz. the magnetic poles.

Fig. 130 is a map showing the *isoclinals* and the *aclinic line* or *magnetic equator*. The magnetic equator lies north of the geographical equator in Asia and Africa, and south of it in America; it crosses the geographical equator at a point in the Atlantic and at another point in the Pacific Ocean. Starting at the magnetic equator, where the dip is zero, and travelling north along a meridian the dip increases, the north pole pointing downwards, and is 90° at the north magnetic pole; similarly, travelling south from the magnetic equator along a meridian the dip increases, the south pole pointing downwards, and is 90° at

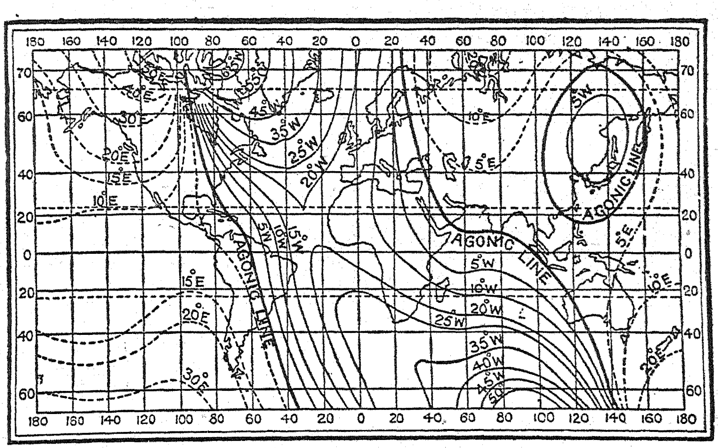


Fig. 130.—Map showing the Isogonals and the Agonic Lines.

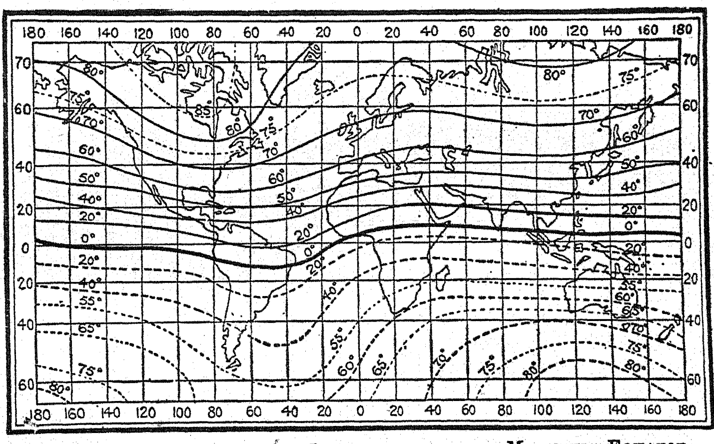


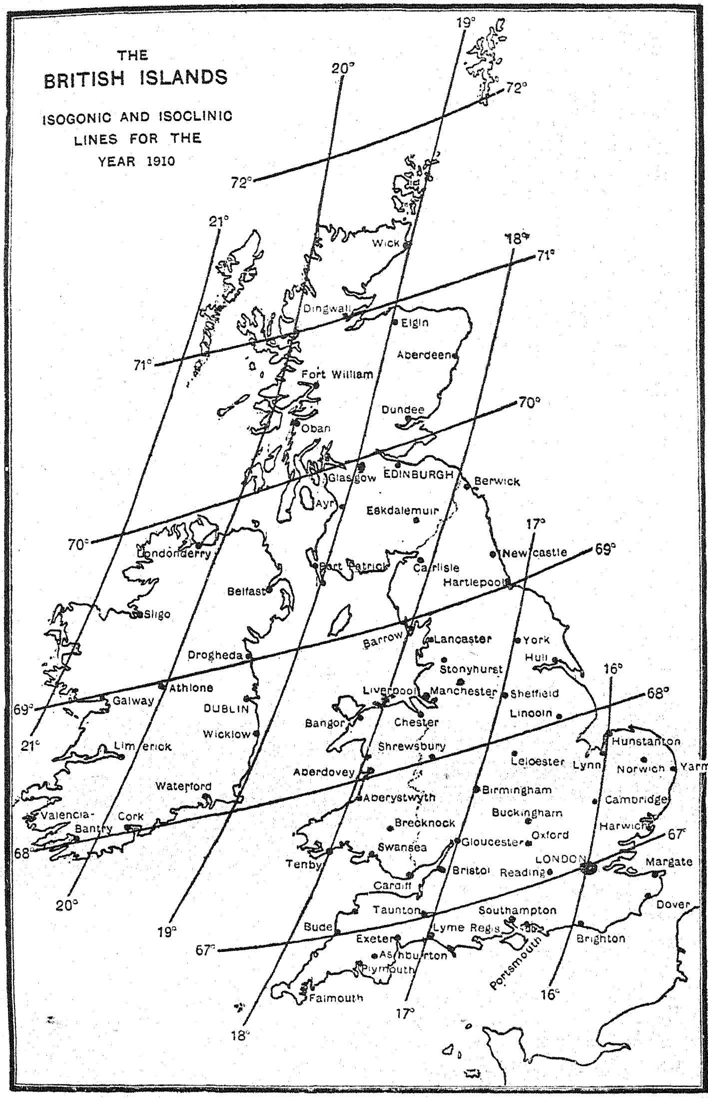
Fig. 131.—Map showing the Isoclinals and the Magnetic Equator.

the south magnetic pole. The dip is less in Europe than in the same latitudes of America, and less in South America than in the same latitudes of Africa. It will be noticed that the magnetic poles are not exactly diametrically opposite, but we may approximately take the magnetic axis of the earth as being inclined at about 17° to the geographical axis.

Maps are also drawn showing the *isodynamics* for places having equal values of the horizontal force. The horizontal force is, of course, zero at the poles, and increases towards the magnetic equator; the maximum value is about ·4 unit in an oval in the Sea of China. Maps giving the lines through all places having equal values of the vertical force are also drawn; the maximum values of the vertical force occur in the regions of the magnetic poles, and the vertical force is, of course, zero at the magnetic equator.

Maps showing the *isodynamic lines* joining places having equal values of the total force indicate that the latter reaches its minimum value in equatorial regions, and its maximum values in two regions in the northern hemisphere and in two regions in the southern hemisphere: these latter four regions are called **magnetic foci.** One of these is in Canada (lat. 52° N., long. 90° W.), and another in the north-east of Siberia (lat. 70° N., long. 115° E.), the former having the higher value of the total force; in a general way the isodynamic lines of the northern hemisphere are more or less two sets of closed ovals surrounding these foci. In the southern hemisphere the two foci are near together to the south of Australia, one being 65° S. 140° E., the other about 50° S. 130° E.; these foci have higher values of the total force than those in the northern hemisphere.

Sir Arthur Rücker and Professor Thorpe made in 1891-2 a magnetic survey of the British Isles, and their results indicate great irregularities over this area, due to local disturbances. Masses of magnetic rock (basalt) occur in various places, notably in Skye, Mull, Antrim, North Wales, and the Scottish coalfield, and these affect the needle.


THE
BRITISH ISLANDS
ISOGONIC AND ISOCLINIC
LINES FOR THE
YEAR 1910
19°
20°
72°
72°
21°
18°
71°
71°
70°
70°
17°
69°
69°
21°
16°
68°
68°
67°
67°
20°
19°
18°
17°
16°
Wick
Dingwall
Elgin
Aberdeen
Fort William
Dundee
Oban
Glasgow
EDINBURGH
Berwick
Ayr
Eskdalemuir
Londonderry
Port Patrick
Carlisle
Newcastle
Hartlepool
Belfast
Sligo
Barrow
Lancaster
York
Drogheda
Hull
Stonyhurst
Athlone
Liverpool
Manchester
Sheffield
Galway
DUBLIN
Bangor
Chester
Lincoln
Wicklow
Hunstanton
Limerick
Shrewsbury
Leicester
Lynn
Norwich
Yarmouth
Aberdovey
Waterford
Aberystwyth
Birmingham
Cambridge
Valencia
Cork
Bantry
Brecknock
Buckingham
Harwich
Oxford
Swansea
Gloucester
LONDON
Margate
Tenby
Bristol
Reading
Cardiff
Taunton
Southampton
Dover
Bude
Lyme Regis
Brighton
Exeter
Portsmouth
Ashburton
Plymouth
Falmouth

As an illustration, consider a mass of magnetic rock rising above the surface and, say, the upper end is of south polarity; this will attract the north pole of a needle, so that on the west side the declination will be less, and on the east side greater, than the normal value; whilst south of it the horizontal force will be greater, and north of it less than the normal value. An example of this kind is the Malvern Hills; at certain points on the west the declination is from 8′ to 25′ less than the normal, and at certain points on the east from 12′ to 22′ greater than the normal.

In other cases the magnetic rocks are underground, *i.e.* do not rise to the surface; examples of these are found near the Wash, and near Reading. Fig. 132 gives the isogonals and isoclinals (smoothed) for the British Isles.

57. Magnetographs.—These are instruments for recording the variations in the elements, the three selected being the declination, the horizontal component, and the vertical component. A satisfactory instrument to register directly variations in dip has not yet been devised, but variations in dip and total intensity can be deduced from the variations in the elements named. In each case the moving system carries a mirror, from which a beam of light from a lamp is reflected on to sensitised paper, the latter being attached to a revolving drum; thus any movement of the system is indicated by the trace on the sensitised paper.

(1) *Declination.*—The magnetograph consists of a magnet suspended by a long fibre, and carrying a mirror (usually concave). Just below this mirror is a second one fixed to the base of the instrument. Light from a lamp passes through a slit, falls upon these two mirrors, and is reflected therefrom through a plano cylindrical lens to the sensitised paper on the revolving drum. The spot of light produced by reflection from the fixed mirror traces a straight line on the paper, which serves as a zero or base line. The spot of light produced by reflection from the magnet mirror traces a line, the distance of which from the base line will vary with any variation in the position

of equilibrium of the magnet, *i.e.* with any variation in the declination. In another form, the magnetic system consists of nine small magnets attached to aluminium, and the suspension is a phosphor bronze strip, the latter resulting in the system being almost free from temperature errors. The magnetic system is surrounded by a ring of copper to damp the vibrations. Frequently arrangements are such that a millimetre on the record represents one minute of arc.

(2) *Horizontal Component.*—One form of magnetograph consists of a magnet with a bifilar suspension, and arranged so that the magnet is at right angles to the meridian. In this case (see Art. 20) the couple due to the earth is MH, and this is balanced by the couple due to the bifilar suspension; hence any change in H will result in a slight rotation of the magnet. To correct for any temperature error a system of bars is attached to the upper end of the bifilar, so that when the temperature rises the fibres are brought nearer together. The movements of the magnet are recorded on sensitised paper, as indicated in the case of declination above. The scale of the record may be calibrated by causing a deflection with a small magnet of known moment at a known distance (worked example, p. 158). Frequently arrangements are such that 1 mm. on the record represents a variation in H of ·00005 unit.

(3) *Vertical Component.*—In Watson's apparatus (Fig. 133) the magnets NS, $N'S'$, are arranged to move about a horizontal axis, and in the meridian. Q is a plate of fused quartz, the projecting arms of which carry the magnets, and the upper surface of which acts as a mirror. The fibres qq are of quartz, one being attached to the spring S, and the other to the torsion head T. The counterpoise c is adjusted until the ends SS' are brought below the

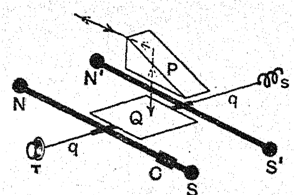


Fig. 133.

horizontal, and T is then turned until the magnets are brought into the horizontal. The totally reflecting prism P enables light in a horizontal direction to be utilised. It is clear that any variation in V will result in a movement of the system, which is recorded photographically as before. The construction of Watson's eliminates temperature errors. The scale of the record can be calibrated as in (2).

Although in all cases the variations in the elements are recorded photographically, the following mathematical investigations are instructive :—

Examples. (1) *By means of a bifilar suspension a magnet hangs in the earth's field at right angles to the meridian, the angle between the top of the suspension and the axis of the magnet being θ. A slight increase h in the earth's horizontal field H causes the magnet to move through a small angle α. Estimate h in terms of the given quantities.*

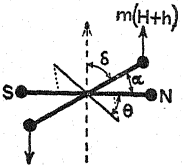


Fig. 134.

By Art. 23 the equation of equilibrium is

$$MH = \frac{Wg\,xy}{h}\sin\theta,$$

i.e. $$MH = K\sin\theta.$$

In the second case the restoring couple due to the earth is (Fig. 134) $M(H+h)\sin\delta$ and the deflecting couple due to the suspension is $K\sin(\theta+\alpha)$; hence

$$M(H+h)\sin\delta = K\sin(\theta+\alpha),$$

i.e. $$M(H+h)\cos\alpha = K\sin(\theta+\alpha).$$

$$\therefore\ M(H+h) = K\frac{\sin\theta\cos\alpha+\cos\theta\sin\alpha}{\cos\alpha} = K(\sin\theta+\cos\theta\tan\alpha).$$

But $$MH = K\sin\theta,$$

$$\therefore\ \frac{H+h}{H} = 1+\cot\theta\tan\alpha,$$

i.e. $$h = H\frac{\tan\alpha}{\tan\theta}.$$

(2) *A small magnet placed end on at a metre from a declination magnetometer deflects it through* $1°\,30'$ (*tan* $1°\,30' = \cdot 0262$), *and when placed end on at* 2 *metres from a bifilar magnetometer deflects it through* 7 *divisions of its scale. Calculate the percentage of change in the horizontal component of the earth's magnetic intensity that is represented by each division of the scale of the bifilar, assuming the average value to be* H = ·1811. (B.Sc.)

In Case 1 we have $\dfrac{M}{H} = \dfrac{d^3 \tan \theta}{2}$ approx.

$$\therefore\ M = \frac{100^3 \times \cdot 0262 \times \cdot 1811}{2} = 2372 \cdot 41.$$

In Case 2 the change in the field in the position occupied by the bifilar suspension is $\dfrac{2M}{d_1{}^3}$ when the magnet is placed as indicated; *i.e.*

$$\text{Change of Field} = \frac{2 \times 2372 \cdot 41}{200^3} = \frac{4744 \cdot 82}{8000000} \text{ units.}$$

This change produces a deflection of 7 divisions, *i.e.* one division deflection corresponds to a change in the field of $\dfrac{4744 \cdot 82}{8000000 \times 7}$ units. Expressing this as a percentage of the earth's field ·1811, we have

$$\left.\begin{array}{r}\text{Percentage change in earth's field } H \\ \text{to produce one division deflection}\end{array}\right\} = \frac{4744 \cdot 82 \times 100}{8000000 \times 7 \times \cdot 1811}$$

$$= \cdot 0468.$$

(3) *A magnet capable of moving about a horizontal axis which is at right angles to its length is so weighted that it lies horizontally. The centre of gravity is nearer the south pole than the axis of rotation by a distance x, and it is also a distance y below that axis. The mass is W and the magnetic moment M. A slight increase v in the earth's vertical field V causes the magnet to tilt through a small angle a. Estimate v in terms of the given quantities.*

In Case 1 (Fig. 135) the moment of the weight about the axis of rotation is Wgx, and the opposing moment due to the earth's vertical field is MV.

$$\therefore\ Wgx = MV.$$

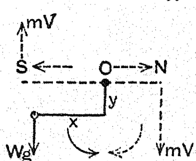


Fig. 135.

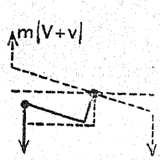


Fig. 136.

In Case 2 (Fig. 136) the centre of gravity moves through a horizontal distance ya and the above moments become $Wg(x + ya)$ and $M(V + v) \cos a$.

$\therefore\ Wg(x + ya) = M(V + v) \cos a = M(V + v)$, as a is very small.

But $$Wgx = MV,$$

$$\therefore\ v = \frac{Wgy}{M} a.$$

58. Magnetic Variations.—The magnetic elements vary from place to place on the earth's surface, as indicated by the magnetic maps of Art. 56, and they also vary from time to time at the same place. The following are the chief facts in connection with the latter:—

(1) *Secutar Changes.*—The elements are found to be undergoing gradual changes extending over a very long period of time, and these are referred to as the secular changes.

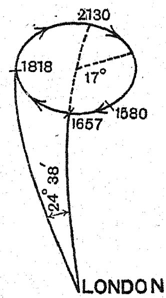


Fig. 137.

Table I. gives the declination at London for the years stated. From it we see that the declination was 11° 15′ E. in 1580, *i.e.* the north pole of the compass pointed east of the true north. Year by year the declination became less, and in 1657 the compass pointed geographically north. The declination then became west, reached its westerly limit in 1818, and is now decreasing at the rate of about 6′ per year, and somewhere about the year 2130 it will again be zero. The magnetic system is, in fact, slowly rotating as shown in Fig. 137, the time of a complete revolution being about 950 years.

Table II. illustrates the secular variations in dip, and Table III. the secular variations in the horizontal force at London. The former is decreasing at the rate of about 1·1′ per year, and the latter increasing at the rate of about ·00002 unit per year. (For Tables see opposite page.)

(2) *Daily Changes.*—Periodic daily variations in the elements are also observed. In England the north pole of the compass starting from its mean position moves to the west from about 10 A.M. to 1 P.M., at which time the declination has increased about 5′. The north pole then moves to the east, crosses its mean position about 7 p.m., moves still to the east (but very slowly) until the early hours of the morning, then moves rapidly to the east until between 7 A.M. and 8 A.M., at which time the declination has decreased about 4′. The north pole then moves to the

TABLE I.—**Declination at London.**

Date.	Declination.	Date.	Declination.	Date.	Declination.
	° ′		° ′		° ′
1580	11 15 E.	1787	23 19 W.	1865	20 58·7 W.
1622	6 0	1795	23 57	1870	20 18·3
1634	4 6	1802	24 6	1875	19 35·6
1657	0 0	1805	24 8	1880	18 52·1
1665	1 22 W.	1817	24 36	1885	18 19·2
1672	2 30	1818	24 38	1890	17 50·6
1692	6 0	1819	24 36	1895	17 16·8
1723	14 17	1820	24 34	1905	16 32·9
1748	17 40	1860	21 38·9	1909	16 10·8
1773	21 9				

TABLE II.—**Inclination (northerly) at London.**

Date.	Inclination.	Date.	Inclination.	Date.	Inclination.
	° ′		° ′		° ′
1576	71 50	1821	70 3·4	1870	67 58·6
1600	72 0	1830	69 38	1874	67 50
1676	73 30	1838	69 17·3	1891	67 33·2
1723	74 42	1854	68 31·1	1895	67 25·4
1773	72 19	1857	68 24·9	1900	67 11·8
1786	72 9	1860	69 19·8	1905	67 3·8
1801	70 36	1865	68 8·7	1909	66 59·7

TABLE III.—**Horizontal Intensity at London.**

Date.	H.	Date.	H.
1857	·17474	1891	·18193
1860	·17550	1895	·18278
1865	·17662	1900	·18428
1870	·17791	1905	·18510
1874	·17903	1910	·18526

The above data, from 1857, apply to *Kew*: the *Greenwich* figures for 1914 are:—Declination = 15° 6·3′, Dip = 66° 51′, H = ·18534. A change in the magnitude of the secular variations seems to have taken place in more recent years. During 1908 *H diminished* in Europe, except in the extreme west.

west, reaching the mean position about 10 A.M. This is shown by the curve *D* in Fig. 138. On the same figure the curve *I* gives the daily changes in dip; these are much smaller than the changes in declination, and the maximum is reached between 10 A.M. and 11 A.M. and the minimum between 7 P.M. and 8 P.M. The curve *H* gives the changes in the horizontal force; the maximum is between 7 P.M. and 8 P.M. and the minimum between 10 A.M. and 11 A.M. The changes are less in winter than in summer.

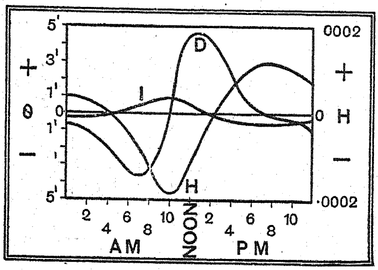


Fig. 138.

(3) *Annual Changes.*—Periodic annual variations also take place; thus the declination has its greatest (westerly) value in February and its least (westerly) value in August. There is a small periodic monthly variation in the elements also.

(4) *Pulsations.*—By using very small magnets with small moments of inertia it is readily seen that the curves such as are shown in Fig. 138 are really composed of very small undulations due to the magnetic system being constantly subject to vibrations of very small amplitude; these are spoken of as pulsations.

(5) *Irregular Changes.*—Sometimes the recording instruments indicate sudden, irregular and often large disturbances in the elements, such disturbances occurring more or less simultaneously all over the world; these are referred to as *magnetic storms*, and frequently accompany volcanic eruptions, earthquakes, earth currents and abnormal displays of the aurora. Magnetic storms often accompany the appearance of unusually large sun spots, and the "sun spot eleven year period" seems to agree with a period of change in the extent of the daily variations.

59. Causes of the Earth's Magnetic Field.—Gilbert's book, "De Magnete," in 1600 first stated that "the earth is a great magnet," and by assuming only the solid portions as being magnetic he gave some explanation of the variations of the compass. In 1676 Bond put forward the suggestion that the earth was surrounded by a magnetic sphere whose axis was inclined at about 8° 30′ to the axis of the earth, and that this sphere revolved at a slightly different rate from that of the earth.

In 1692 Halley conceived the earth as consisting of two magnetic shells, the variations being due to their different rates of revolution; he also pictured the earth as having four poles, two in each shell. In 1819 Hansteen propounded the theory that by the influence of the sun or that of its satellites a planet may have one more magnetic axis developed in it than it has moons; thus the earth would have two magnetic axes and four poles.

Gauss, in 1839, in describing the magnetisation of the earth indicated a general method of determining the distribution of magnetism to which its magnetic field is due. Whatever this distribution may be, equipotential surfaces may be imagined as drawn in the field resulting from it. These surfaces would cut the earth's surface in lines which would at every point be at right angles to the magnetic meridian for the point. The lines, which Gauss called *magnetic parallels*, would therefore be equipotential lines for the horizontal component of the earth's field, and their distance apart would therefore vary at any point inversely as the intensity of the horizontal component at that point. At any point or points at which the equipotential surfaces touch but do not cut the earth's surface the direction of the force, being at these points normal to both surfaces, is vertical, and the points correspond to the magnetic poles of the simpler symmetrical distribution considered above.

From a magnetic survey of the earth's surface it is possible to fix the position of Gauss' magnetic parallels and to determine, with a degree of approximation depending upon the number and accuracy of observation data and the magnitude of local disturbances, the nature of the magnetic

distribution to which the earth's field is due. Gauss came to the conclusion that the magnetic system is probably altogether within the surface of the globe, that the earth in fact is composed of internal magnetic masses to which the magnetic field is due.

In 1849 Grover contended that the earth's magnetism was due to the influence of electric currents circulating in and around it, these being primarily due to the action of the sun and modified by the earth's movements; the warm air ascending in equatorial regions, and thence passing northwards and southwards in upper regions, is electrified, and will have an effect on the earth similar to that of a current on a magnetisable body round which it circulates.

Atmospheric currents of electricity may be explained on the assumption that the sun is emitting a radiation somewhat similar to kathode rays (Chapter XXIII.), which rays render the atmosphere a conductor of electricity, so that atmospheric currents flow under certain conditions; modern work certainly seems to indicate that the "variations" at least are due to some such cause.

We have seen that Gauss' observations led him to believe that the earth's field is mostly due to *internal* magnetism, but an extension of Gauss' method, carried out by Schuster in 1870, led to the conclusion that at least the diurnal variations in the elements are undoubtedly due to some source of magnetisation *external* to the earth, "probably to electric currents in our atmosphere."

The origin of the earth's magnetism is therefore not a settled question at present; probably several causes contribute, *e.g.* magnetic masses in the earth, electric currents in the earth, electric currents in upper regions of the atmosphere, radiation emitted from the sun, action of moon, etc.

We have seen that the magnetic field at the surface of the earth is, in general detail and neglecting local irregularities, similar to that due to a very small magnet with its mid point at the centre of the earth, its north pole pointing to the earth's south pole, and its axis making an angle of about 17° with the earth's axis. From the theory of a small magnet it is possible on this supposition to give a

general expression for the magnetic force at any point on the surface of the earth taken as a sphere of radius r.

For any point, P, on the earth's surface let λ denote the latitude of the place from the magnetic equator and δ the angle of Dip. Then $(90 + \lambda)$ and $(90 + \delta)$ correspond as indicated in Fig. 139 to α and β of Art. 31, and we get

$$\tan \delta = 2 \tan \lambda,$$

that is the tangent of the angle of Dip is twice the tangent of the magnetic latitude.

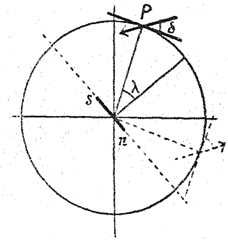


Fig. 139.

Also the magnetic force at a point on the magnetic equator is given by M/r^3 and, by Art. 31, the force at any point of latitude λ from the magnetic equator is $\dfrac{M}{r^3}\sqrt{(1 + 3\sin^2\lambda)}$.

That is, if T_e is the **total** intensity at the equator, then T, the intensity at any other point, is given by

$$T = T_e \sqrt{(1 + 3\sin^2\lambda)}.$$

From the relation $T_e = M/r^3$ we get $M = T_e r^3$, a result from which M can be calculated.

If we pass from the supposition that M is the magnetic moment of a small imaginary magnet equivalent magnetically to the earth as a magnet, and consider M to be the moment of the earth as a uniformly magnetised sphere, we get the intensity of magnetisation given by $\dfrac{T_e r^3}{\frac{4}{3}\pi r^3}$ or $\dfrac{3\,T_e}{4\pi}$.

From the known value of T this gives the intensity of magnetisation as roughly equal to ·08 unit.

60. Magnetisation by the Inductive Action of the Earth's Magnetic Field.—This has already been dealt with in Chapter I., Art. 3. Fig. 140 depicts a rod of iron held in the line of Dip, and therefore magnetised inductively by the total intensity of the earth's field. The magnetisation is assisted by tapping the rod when in this position with a wooden mallet. The figure shows the polarity

(in the northern hemisphere) and the preference which the lines show for the iron.

If placed horizontally in the meridian the bar would be magnetised in the direction of its length by the horizontal component of the earth's field, the end pointing northwards being the north pole. Similarly if placed vertically it would be magnetised in the direction

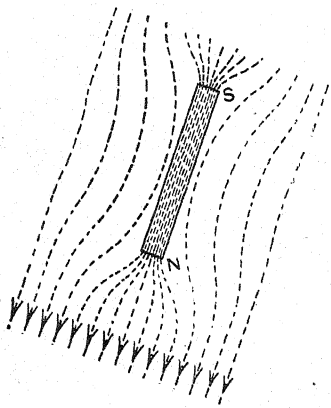


Fig. 140.

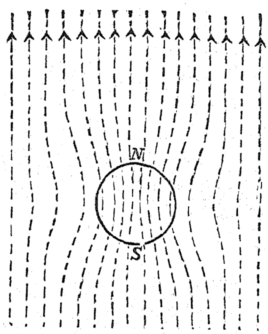


Fig. 141.

of its length by the vertical component of the earth's field, the lower end being the north pole. If placed with its length *at right angles* to the direction of the lines of force the rod is *not* magnetised in the direction of its length, so that the two ends are *not* north and south poles respectively as in the cases above; Fig. 141 shows the result in this case.

60a. The Ship's Compass. Compass Errors and Corrections.—The main features desirable in a ship's compass are:—(*a*) Permanence and intensity of magnetisation in the magnetic system. (*b*) Steadiness under shock and vibration, which necessitates a long, natural period of vibration when disturbed. (*c*) Coincidence of the magnetic axis of the suspended system and the line joining the N and S points of the card. (*d*) Quick damping of any vibration. The magnetic moment should be large, because the effective field on a warship, say, is a small fraction of the earth's true horizontal field (sometimes only about 0 2 of it), and the time of vibration should be large in order that it shall never synchronise with the rolling of the

ship. The magnets should be short, or they will induce magnetism in neighbouring correcting pieces of soft iron. Kelvin's Compass and the Liquid Compass most nearly satisfy these conditions.

(1) *Kelvin's Compass.*—This consists of eight small parallel needles, suspended by silk threads from an aluminium ring, which carries the card, the weight being supported by a sapphire centre. The small needles have a greater magnetic moment for their weight than a single bar would have, and the moment of inertia of the system is made the same about all diameters. The period of vibration is something over half a minute. This compass meets all the conditions fairly well, except that vibrations are not rapidly damped.

(2) *Liquid Compass.*—This is now the standard for many vessels. The card, which is of mica, is immersed in a vessel containing a mixture of water and alcohol, expansion being provided for by an elastic diaphragm. This compass has good directive power and great damping, but there is sometimes trouble with the quadrantal correctors (see below).

It may be noted in passing that the problem of getting a compass to point to the true north has been solved by the introduction of the *Gyro Compass*, an application of the well-known gyroscope. The axis of a gyroscope set working with freedom of movement in two planes gradually takes up a direction parallel to the axis of the earth, and thus indicates the geographical north and south. In the gyro-compass the gyro is floated in a mercury bowl with the compass card attached to it, and it is driven by a motor at a speed of about 20,000 revolutions per minute.

In the *Crough-Osborne Aeroplane Compass* the bowl is spherical and contains liquid to buoy up the card, take part of the weight, and damp the vibrations. The card has several magnets and is fitted with a vertical mica ring on which the compass points are marked in luminous paint.

Ship Compass errors are due to the inductive effect of the earth's magnetic field. Magnetisation by the inductive action of the earth's magnetic field has been dealt with in Chapter I. and in Art. 60, and the student should again note these facts before proceeding further.

Similar effects apply to the case of an iron or steel ship, so that such a ship on being launched is more or less a *permanent* magnet depending on her position during construction, whilst subsequently the masses of iron will be constantly subject to the earth's inductive action such *transient* magnetism varying with the whereabouts and position of the ship at the time of observation.

In practice, matters are more intermingled and complex than is indicated above. The iron or mild steel constituents of a ship consist, as a rule, of three classes of magnetic material, viz. hard iron,

soft iron, and iron neither hard nor soft (intermediate iron), any one bar of iron or mild steel possessing, usually, all three properties. The *hard iron* retains *permanent magnetism*. The *soft iron* is magnetised by the earth's field according to the position of the ship at any instant, and such magnetism is determined solely by the ship's direction and position on the globe; this is termed *temporary magnetism*. The *intermediate iron* retains some of its magnetism for a time, and thus the amount in it at any given instant depends not only on the position of the ship at the time, but partly on the history of the ship's movements; this is termed *sub-permanent magnetism*.

Practically, then, the following situation arises:—The ship is a weak permanent magnet due to the hard iron present, and the strength depends upon the direction in which she was built and upon her subsequent experiences. There is, in addition, a magnetic effect which gradually changes with time and vibration, heavy seas and the like, and which is due to the intermediate iron present. Finally, there is an instantaneous effect depending upon the position of the ship at any instant, and due to soft iron present.

Let then, such a ship be turned through 360° ("swinging the ship"), starting with the bow pointing magnetic north, and remaining on an even keel all the time. The compass deviations due to the above effects may be briefly classified in an elementary manner as follows:—

(1) **Semicircular Error.**—Consider a **vertical mass of soft iron** situated as in Fig. 141*a*, and magnetised by the earth so that

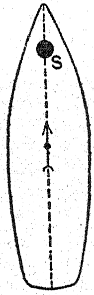


Fig. 141*a*.

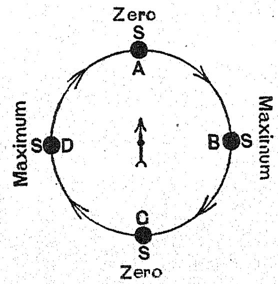


Fig. 141*b*.

the upper end (supposed on a level with the compass) is a south pole. The effect on the compass is *nil* in Fig. 141*a*, but, as the ship swings round, the effect becomes a *maximum* at *B* (Fig. 141*b*), *nil* at

C, a *maximum* at D, and again *nil* at A; hence this is referred to as *semicircular variation*, and the error could, in this case, evidently be corrected by suitably placing a vertical bar of soft iron on the opposite side of the compass ("Flinders' bar").

A little consideration will convince the student that semicircular variation may also be caused by **longitudinal and transverse masses of hard iron**, due to permanent and subpermanent magnetism. Thus in Fig. 141*c*, if NS be the permanent magnetic axis of the ship, say (due to its position during construction), then, whenever the ship is in such a position that this line lies in the magnetic meridian the deviation effect on the compass is *nil*, but in all other positions of the ship a deviation is produced; hence the error is semicircular. A diagram will readily indicate that the error due to *transverse hard iron* is also semicircular, vanishing when the ship's head is east and west (Fig. 141*d*). Semicircular

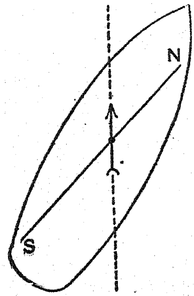


Fig. 141*c*.

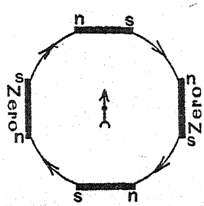


Fig. 141*d*.

errors due to hard iron are corrected by placing magnets fore and aft in the binnacle and by others athwartships.

(2) **Quadrantal Error.**—Consider a **horizontal mass of soft iron**, situated as in Fig. 141*e* and magnetised by the earth as shown. As the ship swings the effect on the compass is evidently *nil* in the positions A, C, E, G, and a *maximum* in the positions B, D, F, H; hence this is referred to as *quadrantal variation*. Clearly the effect of D (Fig. 141*e*) is opposite to that of B, so that if B be the disturbing mass the error could evidently be eliminated by suitably placing a soft iron mass D in the adjacent quadrant (*i.e.* so that $\theta = 90°$). In practice, however, the correction is made by means of two soft iron spheres, one on each side of the compass. The variation due to these is evidently quadrantal, and it may, therefore, be made to cancel the other.

(3) **Heeling Error.**—In addition to the above, which are found when the ship is on an even keel, there is an important error due

to *heeling*; this is due mainly to vertical hard iron or intermediate iron, to vertical soft iron beneath the compass, and to horizontal soft iron. The error is mainly corrected by adjustable vertical magnets under the compass card. That part of the heeling error due to horizontal soft iron is corrected by the quadrantal spheres.

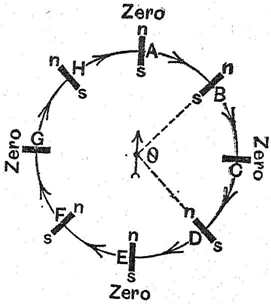


Fig. 141*e*.

(4) **Various.**—In addition to the above there are "cargo errors," specially noticeable in Telegraph Steamers, in which are tanks of metallic sheathed cable constantly varying in position and quantity. The lighting of ships by electricity also necessitates careful arrangements to avoid compass errors. Errors may also be due to faulty fixing of the magnets of the compass with respect to the card. For further information the student should consult the *Admiralty Manual for the Deviation of the Compass.*

Exercises IV.

Section A.

(1) In connection with the magnetism of the earth, define:—Declination, Dip, Magnetic Meridian, Geographical Meridian, Horizontal Component, Vertical Component, Total Intensity, Isogonals, Isoclinals, Isomagnetics, Agonic Lines, Aclinic Line, Duperrey's Lines.

(2) Describe the Dip Circle and explain how you would determine by means of it (*a*) the Magnetic Meridian, (*b*) the Dip.

(3) Give a list of the various errors which may arise in the determination of Dip by means of a Dip Circle, and show how these errors are eliminated.

(4) Describe the Kew Magnetometers, and the method of using them for the determination of Declination and Horizontal Intensity.

(5) Write an essay on "The Variations in the Magnetic Elements."

Section B.

(1) If the magnetism of the earth be represented by a magnet at its centre, show that the tangent of the dip is twice the tangent of the magnetic latitude. (B.E.)

(2) If D_1 and D_2 be the angles of dip observed in two vertical planes at right angles to each other and D be the true angle of dip, show that

$$\frac{1}{\tan^2 D} = \frac{1}{\tan^2 D_1} + \frac{1}{\tan^2 D_2} \qquad \text{(B.E.)}$$

(3) What is meant by (1) declination, (2) agonic lines, (3) magnetic equator. Draw a map showing the general position of the agonic lines, and indicate the direction of the declination in the regions into which these lines divide the earth's surface. (B.E.)

(4) Describe what observations are necessary for the determination of the total intensity of the earth's magnetic field at any given place. (B.E.)

The vertical components of the earth's magnetic force at two places are to be compared. How would you do it? (B.E.)

(5) Write an essay on terrestrial magnetism, keeping in view more especially its probable causes. (B.E.)

(6) Describe in detail the method of determining the magnetic dip at any point, explaining the reasons for each operation you describe. (B.E.)

Section C.

(1) Define the Horizontal Intensity, Vertical Intensity, and Dip, as applied to terrestrial magnetism, and state the relations between them. (Inter. B.Sc.)

(2) Define the Moment and Intensity of Magnetisation of a magnet. Explain the difference in behaviour of soft iron and steel in respect to magnetisation. What happens to a bar of soft iron if held in a vertical position in the latitude of London. (Inter. B.Sc.)

(3) Describe some method of comparing H and V, the horizontal and vertical components of the earth's magnetic field.

Show that the ratio $H \div V$ would be equal to $\frac{\cot \delta}{2}$ where δ is the latitude if the magnetic field were due to a magnet at the centre of the earth with its axis pointing north and south. (B.Sc.)

(4) Describe the method adopted to determine the variation of the horizontal component of the earth's magnetic field, and find an expression for the deflection produced in terms of the variation of the component. (B.Sc.)

(5) A magnet weighing 15 grammes has a small straight stem (length = 4 mm.) fixed centrally at right angles to the magnetic axis. The whole is suspended by a silk fibre attached to the upper end of the stem at a place where the dip is 60°. Calculate the angle which the magnet makes with the horizontal, its magnetic moment being 100 units. (B.Sc.)

(6) Describe the construction of a dip circle. Show how, by loading a dip needle with known weights, comparative measures may be obtained of the earth's magnetic force at different places. (B.Sc.)

(7) Describe the principal features of the Kew dip circle.

To eliminate the error in the determination of the dip, due to the unsymmetrical position of the centre of gravity of the needle, the polarity of the needle is reversed. Under what conditions may the mean of the readings, before and after reversal, be taken as the correct value of the dip?

Explain how the Kew dip circle is used for measuring variations of the total force of the earth's magnetism. (B.Sc. Hons.)

CHAPTER V.

ELECTROSTATICS.—FUNDAMENTAL PHENOMENA.

61. Introduction. Simple Experiments.—It is known from the writings of Thales of Miletus that as far back as 600 B.C. the fact was recognised that pieces of amber and jet possessed, when rubbed, the property of attracting light bodies, and it is from the Greek name *ēlektron* (amber) that our word "electricity" is derived. In 1600 A.D. Dr. Gilbert, physician to Queen Elizabeth, showed that many other substances are similarly affected by friction, and it is now known that, with proper precautions, *all* substances, suitably rubbed, will attract to some extent such light articles as pieces of paper, bran, cork, pith, etc.; glass rubbed with silk, sealing-wax with flannel, and vulcanite with fur do so in a marked degree, and particularly the last named.

A substance endowed with this property is said *to be electrified, to be excited, to be charged, to possess a charge, or to be in a state of electrification*, and the unknown mysterious agent which is the cause of this state is named "*electricity*." Bodies which are in their normal condition and do not exhibit the property are said to be *neutral*.

The question as to the real nature of this agent electricity need not be discussed at this stage, for we shall be chiefly concerned with simple experimental facts; but the student cannot too soon learn that, unlike heat, light, and sound, electricity is not in itself a form of energy; reasons for this statement will appear later.

Exps. (1) Electrify a vulcanite rod by rubbing it with fur, and bring it near some small pieces of cork lying on the table. The pieces of cork will jump up to the rod, remain in contact for a moment, fall to the table again, jump up to the rod again, and so on. Other light bodies will behave similarly. Again, bring the electrified rod near a ball of elder pith suspended by a *dry silk* thread from a suitable stand (Fig. 142); the pith ball will be attracted by the rod, and after being in contact for a moment will be strongly repelled. Repeat the experiments, using a glass rod rubbed with silk, a rod of sealing-wax rubbed with flannel, and a sheet of warm, dry brown paper rubbed with a clothes brush.

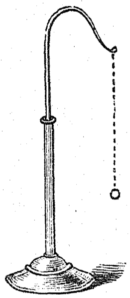


Fig. 142.

These experiments indicate that *electrified bodies attract neutral bodies*, and, further, since the cork and the pith ball, after momentary contact with the rod, are repelled, we may surmise that they become charged on touching the rod, and that *when a neutral body takes a portion of the charge from an electrified one, the two no longer attract but repel.*

(2) Bring an electrified rod near a wooden bar lying on the table; the bar *will not be lifted up to* the electrified rod. Balance the bar on a pivot (Fig. 143), or on the bottom of a small Florence flask, and bring the electrified rod near one end of it; it will be attracted, and will rotate so as to approach the electrified rod. The reason the wooden bar does not move towards the rod in the first case is simply that *the force of attraction is not sufficiently strong to overcome the weight of the bar.*

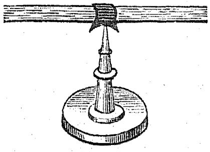


Fig. 143.

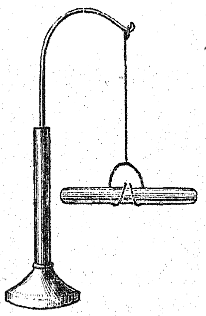


Fig. 144.

(3) Suspend the electrified rod in a wire stirrup hung by a dry silk thread (Fig. 144), and bring a wooden bar or a metal bar near it; the electrified rod will rotate so as to approach the bar. From this we infer that *the forces are mutual, i.e.* the electrified rod attracts the bar, and the bar also attracts the electrified rod.

(4) Using a fairly long rod of vulcanite, hold one end of it in the hand, rub a length of two or three inches of the other end with fur, and gradually bring various parts of the

rod, in succession, near a suspended neutral pith ball. Only that part of the rod which has been rubbed with the fur will attract the neutral pith ball, and the same will be observed if a glass rod rubbed with silk, or a rod of sealing-wax rubbed with flannel, be employed. We conclude, therefore, that *with these substances the electrification produced on one portion of the surface does not spread over the whole surface, but appears to be confined to the area on which it is produced.*

(5) Holding a brass rod in the hand, rub it with warm silk and then bring it near the suspended pith ball; the latter will not be attracted, and *the brass rod will show no signs of electrification.* Repeat with various rubbers, also with rods of copper, iron, and other metals, and the same result will be obtained.

(6) Fix the brass rod C (Fig. 145) into a handle A of vulcanite or sealing-wax or glass. Now hold A in the hand and rub C with warm silk. On bringing the rod near the pith ball the latter will be attracted, so that *the brass rod is now evidently electrified.* Similar experiments with the other metal rods will give the same result, and the general conclusion so far is, that *in order to electrify a metal rod it must not be held directly in the hand, but by means of a handle made of one of the substances which can be electrified when held in the hand.*

(7) Holding A in the hand, rub a length of two or three inches near the end of C with warm silk, and present various parts of C in turn to the suspended pith ball. *All* parts of C will attract the pith ball, and the same result will be obtained with other metals. We conclude, therefore, that *in the case of metal rods the electrification produced on one portion distributes itself over the entire surface.*

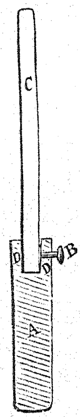


Fig. 145.

(8) Electrify the metal rod C as before. Touch it for a moment with the finger, or let it touch the floor or walls of the room for a moment. On bringing it near the suspended neutral pith ball there will be no attraction, showing that the rod has lost its electrification or, as we say, has become discharged. If the experiment be repeated with rods of glass or vulcanite, or wax, it will be found that they still remain electrified.

The above experiments illustrate important facts, which will be explained in subsequent sections.

The branch of the subject which treats of electricity at rest on bodies, and of phenomena connected with electric charges, is called *electrostatics.* The connection between

magnetism and *electrostatics* will not be apparent until current electricity is dealt with.

62. The Two States of Electrification.—We have seen that if a vulcanite rod which has been rubbed with fur be brought near a pith ball suspended by a dry silk thread, the pith ball is attracted by the rod, comes in contact for a moment, *and is then strongly repelled*; we infer that the pith ball has taken a portion of the charge from the rod—has in fact become charged by contact—and that whenever this takes place the two no longer attract but repel.

Exps. Let the pith ball be charged by contact with the electrified vulcanite rod as indicated above. Bring near the pith ball another vulcanite rod which has been rubbed with fur; *repulsion* ensues. Bring near the pith ball a glass rod which has been rubbed with silk; *attraction* ensues.

Let a suspended pith ball be charged by contact with a glass rod which has been rubbed with silk. Bring near the pith ball another glass rod which has been rubbed with silk, and the result is *repulsion.* Bring near the pith ball a vulcanite rod which has been rubbed with fur, and the result is *attraction.*

Suspend a vulcanite rod which has been rubbed with fur (Fig. 144). Bring near it a second vulcanite rod which has also been rubbed with fur; *repulsion* ensues. Bring near it a glass rod rubbed with silk; *attraction* ensues. Similarly if the electrified glass be suspended a second glass rod rubbed with silk will repel it, but the vulcanite rubbed, with fur will produce attraction.

Repeat the experiment with various electrified bodies. It will invariably be found that if A and B *both* attract or *both* repel C, then A and B will repel each other; but if A attracts and B repels C, or if A repels and B attracts C, then A and B will attract each other.

Numerous experiments of the type indicated above, with various substances and various rubbers, all lead to the same conclusions, viz.:—

(*a*) *There are two states of electrification.*

(*b*) *Bodies in similar states of electrification repel.*

(*c*) *Bodies in dissimilar states of electrification attract.*

It has been agreed to call the electrification exhibited by glass rubbed with silk *positive*, and to say that the glass is *positively charged*; it has also been agreed to call the electri-

fication exhibited by vulcanite rubbed with fur *negative*, and to say that the vulcanite is *negatively charged*. Hence—

(*a*) *Positively charged bodies repel each other.*

(*b*) *Negatively charged bodies repel each other.*

(*c*) *A positively charged body and a negatively charged one attract.*

Experiment shows that the kind of electrification developed depends both on the rubber and on the body rubbed. Glass rubbed with silk becomes positively electrified, but the same glass warmed and rubbed with fur becomes negatively electrified. Rough or "ground" glass rubbed with silk becomes negatively electrified. Vulcanite rubbed with fur becomes negatively electrified, but when excited by a piece of indiarubber tubing it becomes positively electrified. *It is useful to remember that, in general, common metals rubbed with fur become negatively electrified, and when rubbed with indiarubber they become positively electrified.*

Experiment also shows that, in electrification by friction, if the substance rubbed be positively electrified the rubber will, to an equal extent, be negatively electrified and *vice versa*. Thus if a vulcanite rod be fitted with a fur cap to which is attached a dry silk thread and if on electrifying the rod the thread, and not the fur, be held in the hand, then on separating them the rod will be found to *repel* a negatively charged pith ball, and the fur will be found to *repel* a positively charged pith ball, so that the rod is negatively, and the fur positively, electrified. Further, if before removing the cap from the rod the two be brought together near a *neutral* pith ball the latter is not affected, so that the two opposite electrifications are equal and are exactly neutralising each other.

Numerous experiments confirm the fact that *both states of electrification are produced at the same time, and in equal amounts, the one on the rubber and the other on the body rubbed.* (See page 207.)

Later work will show that when any two dissimilar metals are suitably put in contact, then, without any friction taking place, both become electrified, but the electrification developed is very feeble. Hence we surmise that *contact of dissimilar substances*, and not friction, is probably the true cause of the electrification in the

experiments previously considered. In the case, however, of glass, vulcanite, etc., intimate contact at practically every point along the surface is necessary since the substances do not allow electrification to spread over them from one or two points of contact, and this is secured by friction, the main function of which is therefore to obtain an extended contact.

Thus the degree of electrification produced in any given case depends not on the mechanical work done in friction, but rather on the nature of the substances and the conditions of contact. In fact, the work done in friction is largely dissipated in heat—heating the muscles of the arm, etc.—and the energy of the charged surfaces has its equivalent, nearly, in the work done in separating the charged surfaces against their mutual attraction.

63. Theories of Electrification.—Several theories have been developed to explain the phenomenon of electrification. These are merely briefly referred to here. Modern theory is developed in later chapters, but the explanation of electrical phenomena on strictly modern lines cannot be attempted at this stage of the reader's progress.

Symmer's Theory.—Symmer advanced the *two-fluid* theory, which assumed that there were two kinds of material but imponderable fluids corresponding to the two states of electrification. By this theory the two fluids neutralised each other when combined in equal quantities, but gave rise to two states of electrification when one or other of the fluids was in excess. The phenomena of attraction and repulsion were explained by assuming the fluids to be mutually attractive, but self-repelling.

Franklin's Theory.—Franklin proposed the *one-fluid* theory as a simplification of Symmer's theory. This theory supposes one "electric" fluid to have a normal neutral distribution in all bodies and that the two states of electrification result when this normal distribution is disturbed, so as to produce excess or defect of the normal amount in the body. A positively charged body is one which contains an excess of the fluid, and a negatively charged body is one in which the fluid is deficient. The electric fluid attracts ordinary matter but repels electric fluid, so that a neutral body is one which contains just so much fluid that the attraction between any external fluid and the matter is exactly balanced by the repulsion between the external fluid and the fluid associated with the neutral body.

There is much in Franklin's theory which approaches modern theory, although there is now perhaps reason to suppose that the names positive and negative should have been interchanged.

Simplification of Franklin's Theory.—Although it is now recognised that electricity is not a material fluid, a simplification of Franklin's

theory may be well utilised by the beginner as explaining more simply than any other theory many of the fundamental principles in electrostatics. It may be conceived for example that the vulcanite rod and the fur of the preceding experiments when neutral are imbued with a definite amount or stock of something which we call electricity. On rubbing the two together it may be further conceived that a certain amount passes from the rod to the rubber; so that the rod will have a "deficit" and may be said to have a "negative charge," and the fur will have a "surplus" or a "positive charge," and since the surplus on the rubber has come from the rod, the deficit of one and the surplus of the other, *i.e.* the "charges," will be equal and capable of neutralising. A similar explanation applies to the other rods and rubbers. Of course, since the terms "positive" and "negative" have been fixed arbitrarily, it is possible that what is called by consent positive may be the deficit and the other the surplus.

It should also be noted that this theory emphasises the point that *there are not two kinds of electricity but two states of electrification*, *i.e.* two electrical conditions, one due to a surplus, the other due to a deficit of electricity. The reader will find that this theory, which says nothing about the nature of electricity itself, will carry him over the initial ground, and he will have no difficulty in translating it into modern theory as his studies lead him into the higher parts of the subject. This simplified Franklin's Theory will be frequently used in the early chapters on Electrostatics.

Faraday's Influence on Electrical Theory.—Faraday's views on the action of the medium in magnetic phenomena have already been referred to, and his views of electrostatic phenomena were similar. Instead of fixing our attention on the "charges" on the bodies, and discussing imponderable fluids, Faraday contended that we should practically ignore the charges and concentrate on the medium surrounding the charged bodies. Electrification, he argued, may be considered as a phenomenon of the medium surrounding the "charged" or "electrified" bodies. By this theory the medium is really the seat of the energy, the charge itself being a "surface effect" appearing at the surface of separation of the medium and the material of the bodies and is associated with a state of strain in the medium; in other words the "charges" on the bodies are merely the free ends of the strained medium.

This effect of the medium is of vital importance and is dealt with in subsequent pages, but it is nevertheless interesting to note that modern research in electrical theory, whilst still upholding the part played by the medium as propounded by Faraday, have again compelled us to consider also the surface effects, *i.e.* the nature of the "charges" on the electrified bodies.

The Lorentz Theory.—This was really a modification of the two fluid theory. Lorentz imagined that the two electricities of opposite sign were caused by a large number of discrete particles scattered

throughout the material of the body. The charges on the particles were equal in magnitude, and as the particles could not be divided into smaller particles, the charge on a particle could not be divided, *i.e.* the charge on a particle represented the smallest possible charge, and might be referred to as an "atom of electricity."

The name "electron" had been suggested by Johnstone Stoney for the atom of electricity, and the name was applied to the Lorentz particles carrying this charge. A neutral body, according to this theory, is one containing an equal number of positive and negative charged particles or electrons, a positively charged body is one containing a surplus of positive electrons, and a negatively charged body is one containing a surplus of negative electrons.

The Modern Electron Theory.—As has already been mentioned (Art. 8), the existence of particles more minute than any previously known in science has been definitely established, the mass of each being about $\frac{1}{2000}$ of that of a hydrogen atom, and each being always associated with a definite negative charge—in fact, the mass is probably entirely electrical, *i.e.* solely in virtue of the charge. The name electron is now restricted to these. From whatever source these electrons are obtained, the mass and charge are always the same: charges smaller than the electron cannot be obtained, so that this charge is a natural unit of electricity.

Now the modern theory of matter is that an atom of any substance is composed of a nucleus of positive electricity surrounded by (negative) electrons in motion, the total number of free units of positive electricity (called protons) in the nucleus being equal to the total number of negative electrons outside the nucleus when the atom is neutral. (The number of electrons in an atom really determines the properties of the substance—see Chaps. XXIV., XXV.) If one or more electrons become detached from an atom, the positive nucleus predominates and the atom is positively charged: if one or more electrons be added to an atom, the negative electrons predominate and the atom is negatively charged. When vulcanite is rubbed with fur some electrons are detached from the fur and transferred to the vulcanite: the vulcanite has a surplus of (negative) electrons, and is, therefore, negatively charged, whilst the fur has a deficit of (negative) electrons and is positively charged.

64. Conductors and Insulators.—It has been shown that, if bodies are held in the hand, some (*e.g.* glass, wax, vulcanite, etc.) can readily be electrified by friction, whilst others (*e.g.* metals) give no signs of electrification. The difference is due to the fact that the first are *insulators*, the second *conductors*. **A conductor is a substance which readily allows electricity to flow along it; an insulator is a substance which does not allow electricity to flow along it.**

Consider, then, a glass rod held in the hand and rubbed with silk. The part rubbed becomes positively electrified and the "surplus" remains there, for the rod being an insulator does not allow the electricity to flow along it to the hand, from which it would readily pass to the earth, since the human body is a good conductor. Similarly, the "deficit" on the vulcanite rod, after being rubbed with fur, cannot be "made up" by a flow of electricity from the earth, through the body and hand of the experimenter, because vulcanite is an insulator.

A brass rod, held in the hand and rubbed with india-rubber, becomes positively electrified, but the electricity at once flows along it to the hand, and down the body to the earth; as has been indicated, by fitting the brass rod with an insulating handle of glass, wax, vulcanite, etc., it can be electrified. Touching an electrified brass rod with the finger immediately discharges it, but touching an electrified glass rod only removes a portion of the charge from the part touched, so that the rod is still electrified.

Similarly, if an insulated electrified conductor be placed in contact with another insulated but non-electrified conductor, it will be found that the former at once transfers a portion of its electrification to the latter; if the charged conductor be positive, some of the "surplus" passes to the uncharged conductor, so that both are positively charged; whilst if the charged conductor be negative, the "deficit" is *partly* made up by a flow of electricity from the uncharged conductor, so that both exhibit a "deficit," *i.e.* both are negatively charged. It is customary to say, in the last case, that the charged conductor shares its "negative charge" with the uncharged conductor.

The conducting and insulating powers of most bodies are influenced by temperature. Good conductors become *worse* conductors, and insulators tend to become conductors when their temperatures are raised, *i.e.* conductors decrease in conducting power, and insulators decrease in insulating power, when heated. (See Art. 158.) Further, as will be seen later, "electrical pressure" is needed to transfer electricity from one point to another, and a body may be an insulator under low pressures, but a conductor when

the pressure is sufficiently high; thus air is an insulator, but the lightning flash will pass through miles of it.

If the electron theory be used in the above explanations it will be necessary to say, in the case of the glass rod, which has a *deficit of* (*negative*) *electrons*, that electrons cannot get along it from the earth to make up the deficit, and in the case of the vulcanite, which has a *surplus of electrons*, that electrons cannot pass along it to earth to get rid of the surplus; in the case of the brass, electrons can pass along it quite readily.

In the following table the substances are arranged in the order of their conducting powers, *i.e.* the best conductors come first, and the worst conductors or best insulatos last .

Table of Conductors and Insulators.

Good Conductors. Bad Insulators.	Partial Conductors. Partial Insulators.	Bad Conductors. Good Insulators.
1. Silver (annealed).	26. Linen.	33. Oils.
2. Copper (annealed).	27. Cotton.	34. Porcelain.
3. Copper (hard drawn).	28. Wood.	35. Dry leather.
4. Silver (hard drawn).	29. Stone.	36. Wool.
5. Telegraphic silicium bronze.	30. Marble.	37. Silk.
6. Gold.	31. Paper.	38. Sealing-wax.
7. Aluminium.	32. Ivory.	39. Sulphur.
8. Zinc.		40. Resin.
9. Platinum.		41. Gutta percha.
10 Iron.		42. Indiarubber.
11. Nickel.		43. Shellac.
12. Tin.		44. Vulcanite.
13. Lead.		45. Mica.
14. German silver.		46. Jet.
15. Platinum silver.		47. Amber.
16. Platinoid.		48. Paraffin wax.
17. Manganin.		49. Glass.
18. Mercury.		50. Dry air.
19. Gas coke.		
20. Charcoal.		
21. Graphite.		
22. Acids.		
23. Metallic salts.		
24. Water.		
25. The body.		

65. The Gold-leaf Electroscope.—In the early study of the subject, this instrument is used to detect the presence of a charge, to determine the nature of a charge (*i.e.* whether positive or negative), and to compare the amounts of two or more charges; strictly, however, as will be seen later, it really measures, and its action depends on, what is called electrical "potential" (Art. 67). In recent years it has become an important instrument for the measurement of the electrical conductivity of gases under the influence of radio-active substances, etc.

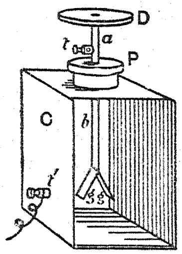


Fig. 146.

A simple laboratory form is shown in Fig. 146. It consists of a tin box *C*, of which two opposite sides are removed and replaced by sheets of glass. Into an opening in the top of the box is placed an indiarubber plug *P*, fitted with a central vulcanite rod. A vertical brass rod *ab* passes through a hole down the centre of the vulcanite and carries at its upper end a brass disc *D*, and at its lower end a horizontal brass piece, to each side of which a gold-leaf *g* is attached; in the normal position these two gold-leaves hang vertically side by side. Binding screws *t'* and *t* are fitted to the "case" and to the rod carrying the leaves.

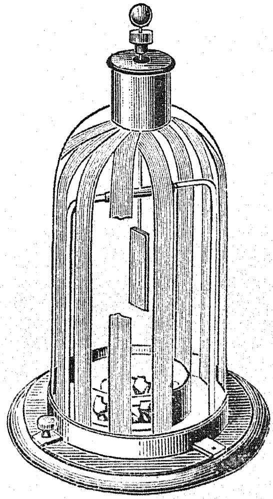


Fig. 147.

Another pattern, due to Prof. Ayrton, is shown in Fig. 147. The case is of glass and is covered with tinfoil, leaving a few narrow gaps through which the leaves may be ob-

served. The strips of tinfoil are in contact with the brass ring round the bottom of the case, and the ring is fitted with a binding screw as seen in the figure. Special precautions are taken to ensure good insulation of the leaves. The interior is kept dry by means of calcium chloride or pumice stone soaked in sulphuric acid, contained in a small vessel placed inside the instrument.

The modern electroscope of **C. T. R. Wilson** and its applications in modern work are described in Chapter XXIII.

The reader will learn later that the action of an electroscope really depends on the difference between what is called the "electrical potential" of the leaves and the "electrical potential" of the case, and that the case must be of conducting material (or coated with conducting material) if the instrument is to be efficient. The old practice of hanging two leaves in a glass bottle is worthless.

SIMPLE EXPERIMENTS WITH THE ELECTROSCOPE.—(1) *To charge the electroscope positively or negatively.* Gently beat an insulated brass ball (Fig. 148) with indiarubber, and then bring the ball into contact with the cap of the electroscope. The ball shares its (positive) charge with the cap, brass rod, and gold leaves, and the latter, being light, and both positively electrified, repel each other (Fig. 146). On removing the ball the leaves remain diverging, and the electroscope is "charged" positively. To "discharge" the electroscope touch the cap with the finger, when the electricity will immediately pass down the body to the earth, and the leaves will fall together.

To charge the electroscope negatively repeat the experiment, using an insulated brass ball rubbed with fur. In this case we may either say that the ball "shares its negative charge" with the leaves, rod, and cap, or that some of the electricity from the leaves, etc., passes to the ball, so that the former now has a "deficit" or is negatively charged. Similarly, if the cap be touched with the finger, the electroscope becomes discharged and the leaves collapse; in this case we may either say that the "negative charge has passed to earth," or (better) that electricity has passed in from the earth and made up the deficit.

(2) *To determine whether a body is a conductor or an insulator.* Charge an electroscope. Holding the body in the hand, touch with it the cap of the electroscope. If the body is a good conductor the leaves will collapse at once, if a poor conductor they will slowly collapse, if an insulator they will remain diverging without any change.

(3) *To test whether a body is electrified or not, and if electrified to determine the kind of electrification.* Take three electroscopes, one positively charged, one negatively charged, the third neutral. Bring the body near the cap of the neutral electroscope: if the leaves diverge the body is electrified, but if they do not it is not electrified. If the body *is* electrified, bring it near the *positively* charged electroscope. If the leaves *diverge more* the body is positively charged, for it is evidently causing more positive to appear at the leaves; if the leaves *diverge less* the body is negatively charged, but it is best to verify by bringing it near the negatively charged electroscope, in which case the leaves will *diverge more.* **A more scientific and accurate explanation of this experiment will be given later;** whilst a greater divergence certainly means that the leaves and the approaching body are similarly electrified, the above *explanation* is provisional only.

66. Inductive Displacement.

Exps. Bring an insulated *neutral* conductor AB near an insulated positively charged conductor C (Fig. 148). Touch the end B with a proof plane (which consists of a small, flat circular piece of brass mounted on an insulating handle), and then bring the proof plane near the cap of a positively charged electroscope. The leaves *diverge more*; hence we deduce that the proof plane and the end B of the conductor AB are positively charged.

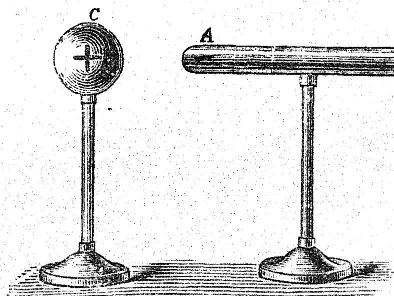


Fig. 148.

Discharge the proof plane. Touch the end A, and then bring the proof plane near the cap of a negatively charged electroscope. The leaves *diverge more,* and we deduce that the end A of the conductor AB is negatively charged.

Modify the experiment as shown in Fig. 149, where A and B are equal insulated metal spheres. Separate A and B, and then remove C. Verify that A is negatively charged and B is positively charged by bringing the latter near the cap of a positively charged electroscope, and the former near the cap of a negatively charged one; in each case there will be increased divergence.

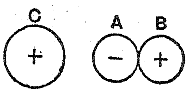


Fig. 149.

Replace the three spheres as in Fig. 149. *With A and B still in contact*, remove C to a distance and test A and B with a *neutral* electroscope. The latter is not affected, and we therefore deduce that when C is removed, A and B being still in contact, the positive charge on B and the negative charge on A neutralise each other, and are therefore equal in amount.

Fig. 150 shows a further modification, A and B being joined by a fine wire, which can be removed by insulating tongs.

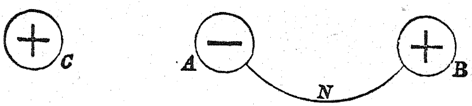


Fig. 150.

Repeat the above experiments with the conductor C negatively charged. In this case it will be found that A is positively charged and B negatively charged, and, as before, the two charges are equal.

It is not essential that C be a conductor; an electrified glass or vulcanite rod will answer the purpose.

Anticipating the matter of the next section (Art. 67), we may give an explanation of the preceding as follows. Let C be positively charged and isolated in the centre of the room. Now just as air exerts pressure, so we imagine electricity to exert an influence bearing a somewhat similar relation to it, and this influence (which tends to move electricity as air pressure tends to move air) is called "electrical pressure." Taking the case in question, it is believed that the electricity on C gives rise to this electrical pressure, extending right throughout the insulating medium (air), but diminishing at first fairly rapidly and then more slowly as we recede from C to the earth-joined bodies in the room. The result of this is that the medium

is *strained* until it sets up an opposing influence, which exactly balances the pressure due to the charge on C.

Now consider two points A and B in the insulating medium (Fig. 151). The pressure at A is greater than the pressure at B. This difference in pressure tends to move electricity from A towards B, but, as explained, the medium sets up an opposing influence, which balances the forward pressure, so that the insulating medium is strained, but there is no "flow" of electricity.

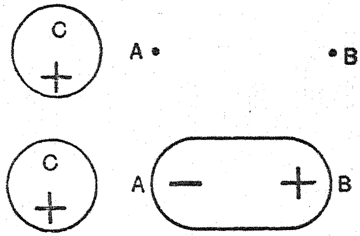


Fig. 151.

Let the conductor be now placed in position (Fig. 151). The pressure difference between A and B tends to move electricity from A to B, and the conductor, being unable to set up an opposing influence like an insulator, *permits the flow*, so that electricity flows from A to B until uniform pressure exists over the whole conductor. The end A has therefore a "deficit" or a negative charge, and the end B a "surplus" or a positive charge, and since the surplus at B has come from A the two charges are equal, and will therefore neutralise when the influence of C is removed.

If C be negatively charged, its pressure is below that of the earth, so that in the medium the pressure rises as we move away from C, and is therefore greater at B than at A. When the conductor is placed in position, electricity flows from B to A, so that the end B shows the deficit or negative charge, and A the equal surplus or positive charge.

The phenomenon above is referred to as "*electrostatic induction*" or "*inductive displacement*," and the insulating medium which transmits these effects is referred to as the "**dielectric**"; the charges on AB are often referred to as the "*induced charges*," but it must be remembered that,

taking AB as a whole, it is without charge, *i.e.* it neither contains an excess nor a deficit of electricity; its own electricity has merely been displaced. Further, it should be mentioned that all dielectrics are insulators; when we call them insulators we are referring to the fact that they do not allow electricity to flow along them, and when we call them dielectrics we are referring to the fact that they do allow these electrical influences to take place through them, and, in fact, themselves play an important part in the action.

The nearer the conductor is to the charged body the more pronounced are these inductive effects, Further, if the air dielectric be replaced by a solid or liquid dielectric such as glass, wax, mica, olive oil, etc., the effects are still more marked; for these dielectrics allow inductive influence to take place through them better than air, and are said to have a higher **specific inductive capacity**. The introduction of these substances between C and the conductor AB is, in fact, electrically equivalent to reducing the air space; thus a piece of ebonite 3·16 cm. thick is equivalent to an air thickness of 1 cm., and the specific inductive capacity of ebonite is 3·16. To put the matter generally, **if d denotes the thickness of an air dielectric, and Kd the thickness of a dielectric x which is equivalent to it, then K measures the specific inductive capacity of x** (see Art. 107). This is merely referred to here; specific inductive capacity is dealt with in later chapters.

The reader will now realise that *inductive displacement always precedes the attraction between an electrified body and a neutral conductor*. Thus when a negatively electrified vulcanite rod is held near the suspended pith ball of Fig. 142, the near side of the pith ball acquires a positive charge, and the far side a negative charge; the former charge is nearer the rod than the latter, so that, on the whole, the force is one of attraction.

67. Electrical Potential.—As a preliminary to the conception of electrical potential, the reader may consider the following simple facts; but whilst the analogies are

helpful to a beginner they are very faulty in detail, and must not be pushed too far :—

Analogy to Temperature.—It is well known that heat flows from a body at a high temperature to a body at a low temperature. If two bodies be connected together by a conductor of heat, and no heat passes from one to the other, the two bodies are at the same temperature; if the two bodies are at different temperatures, then heat will flow from the one at the higher temperature to the one at the lower temperature, and this flow will continue until the two come to the same temperature. Further, if a cupful of boiling water be mixed with several gallons of cold water, heat will flow from the former to the latter, although the latter really contains more heat; thus *the difference in temperature*, not the difference in the amount of heat, *really settles the direction in which heat will flow.*

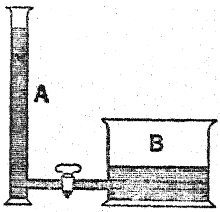


Fig. 152.

Analogy to Level.—It is also well known that water flows from a high-water level to a low-water level. Thus if the two vessels of Fig. 152 contain water as indicated, then on opening the stop-cock water will flow from *A* to *B*, and the flow will continue until the two come to the same level. The actual quantity of water in *B* may be considerably greater than the quantity in *A*; *it is the difference in level*, not the difference in the amount of water, *which settles the direction in which the water will flow.*

Analogy to Air Pressure.—Again, consider an india-rubber bag fitted with a stop-cock and a pressure-gauge. Pump air into the bag so that it contains more than its normal amount and then close the stop-cock. The gauge will indicate that the air pressure inside is greater than that outside. Open the stop-cock; *air will pass from the bag into the atmosphere, i.e.* from the region of higher pressure to that of lower pressure, and this will continue until the bag has its normal amount, and the gauge indicates that the air pressures inside and outside are the same.

Now draw air out of the bag so that it contains less than its normal amount and then close the stop-cock. The gauge will indicate that the air pressure inside is less than that outside. Open the stop-cock; *air will then pass from the atmosphere into the bag, i.e.* from the higher pressure to the lower pressure, and this will continue until the bag has its normal amount and the gauge indicates that the air pressures inside and outside are the same.

Further, take two bags and arrange matters so that the air pressure in one is higher than the air pressure in the other. Join the two cocks by indiarubber tubing and then open the stop-cocks. It will be found that *air passes from the bag at the higher pressure to the bag at the lower pressure*, and that this continues until the air pressure is the same in both bags. If, to begin with, the bags are at the same pressure, then on opening up the communication there is no flow of air. By considering a large bag and a small one, it will be readily seen that it is the difference in pressure, not in the amount of air, which settles the direction of flow.

Again, in measuring temperatures by a Centigrade thermometer, the temperature of melting ice is taken as the standard of reference or the zero; temperatures higher than this are spoken of as *positive* temperatures, and temperatures lower than this are called *negative* temperatures. Similarly in measuring heights of mountains, etc., the sea-level is taken as the zero level; distances above sea-level might well be called positive, and distances below sea-level negative.

In the study of electricity the expression *electrical pressure*, or *electrical tension*, or **potential**, has much the same meaning as temperature, level, and pressure in the three preceding considerations. Thus, consider an insulated and isolated positively charged conductor. If this be put in conducting communication with the earth, its "surplus" passes to earth, *i.e. electricity flows from it to the earth* and the conductor becomes neutral. The reason for this is that its electrical pressure is above that of the earth, and the fact is expressed by saying that *it is at a higher potential than the earth.* Since the earth is taken as the

standard of reference or zero of potential, the conductor is said to be at a *positive potential.* Of course, after the momentary flow the potential of the conductor is the same as that of the earth, viz. *zero.*

Consider now an insulated, isolated, negatively charged conductor. If this be earthed, *electricity flows from the earth to it* until its "deficit" is made up and the conductor becomes neutral. This is because its electrical pressure is below that of the earth; *it is at a lower potential than the earth,* and is said to be at a *negative potential.* After the flow the potential of the conductor is *zero.*

It should be noted that the potential of the earth is regarded as constant, and totally unaffected by any electricity which we give to it, or take from it, in our experiments; *the potential of an earthed conductor is therefore always zero.*

Again, if two conductors at different potentials be joined together, *electricity will flow from the one at the higher potential to the one at the lower potential,* thereby lowering the potential of the former and raising the potential of the latter, *until the two come to the same potential.* The actual "charge" on the body at the high potential may be *less* than the "charge" on the one at the low potential; it is not the amount of the charges, *but the relative potentials,* which settle the direction in which the electricity will flow, and, as will be seen later, the potential of a body depends on its size and other factors as well as on its actual charge.

From the preceding it follows that, if a *conductor* is in a state of electrical equilibrium, all parts of it are at the same potential; this does not follow in the case of *insulators*

Referring again to Fig. 151, *C* is at a positive potential due to its positive charge, and this produces a positive potential throughout the whole of the insulating medium (air), which latter potential decreases from its positive value at *C* (at first rapidly and then more gradually) to zero at all earth-connected bodies in the room. The medium is "strained," but there is no "flow," for the medium is an insulator and does not permit it.

When the *conductor AB* is placed in position, how-

ever, then, since the end A is at a higher potential than B, electricity flows from A to B, so that A exhibits a negative and B a positive charge. The negative charge at A lowers the potential of that end, and the positive charge at B raises the potential of that end, so that when everything is quite steady the whole conductor AB is at a uniform (positive) potential.

If C be negatively charged it is at a negative potential, and produces a negative potential throughout the insulating medium, the actual potential of the medium *rising* from its negative value at C to zero at the earth. In this case B is at a higher potential than A, and when the conductor is placed in position electricity flows from B to A, so that B exhibits a negative charge and A a positive one. The negative charge at B lowers the potential there, and the positive charge at A raises the potential there, so that when all is steady the whole conductor AB is at a uniform (negative) potential.

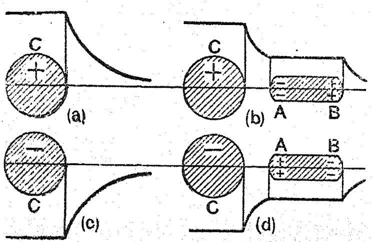


Fig. 153.

Fig. 153 represents these points graphically, the vertical distances denoting **potentials**; distances above the horizontal line denote positive potentials, and distances below the horizontal line negative potentials. It will be noted that when the conductor is in position the potential of C is slightly affected by the induced charges on AB. Thus, in Fig. 153 (b), the negative charge at A tends to lower the potential of C, the positive charge at B tends to raise it; the negative charge is nearer and has the advantage, so that on the whole the potential of C is slightly lowered. Similarly, in Fig. 153 (d), the *negative* potential of C is slightly weakened, *i.e.* the potential of C is slightly *raised* or brought nearer zero.

To summarise :—

The potential of a body is its electrical condition which determines whether electricity will flow from the body to the earth, or from the earth to the body, when they are metallically connected. This is merely

a general statement: an *exact definition* **of potential appears in Art. 81**

A body is at a positive potential if electricity flows, or tends to flow, from it to the earth, and it is at a negative potential if electricity flows, or tends to flow, from the earth to it. It is at zero potential if there is no tendency for electricity to flow from one to the other.

Similarly a body A is at a higher potential than another one B if electricity flows, or tends to flow, from A to B when they are joined, and they are at the same potential if there is no tendency for electricity to flow between them.

In considering the above from the point of view of the electron theory—the movement of electrons—the student must remember that the electrons, being negative, move from what we have called the lower to what we have called the higher potential.

68. Principle on which the Action of a Gold Leaf Electroscope Depends.—This has been incidentally mentioned in Art. 65; the following experiments will verify the statement:—

Exps. Place the electroscope on the table so that the case C (Fig. 146) is earth-connected, and therefore at zero potential. Electrify a brass ball positively, in which case it will be at a positive potential, and let it touch the cap of the electroscope. Electricity flows from the ball to the cap, rod, and leaves, and this continues until the ball, cap, rod, and leaves all attain the same positive potential, the leaves meanwhile diverging. Note that *there is a difference in potential between the leaves and the case,* the former being at a positive potential and the latter at zero potential.

Discharge the electroscope and repeat with a brass ball negatively charged, and therefore at a negative potential. Here electricity flows from the leaves, rod, and cap to the ball, and this continues until they all attain the same negative potential, the leaves meanwhile diverging. Again *there is a difference in potential between the leaves and the case,* the former being at a negative potential and the latter at zero potential.

Insulate the electroscope by placing the case C on a thick slab of paraffin wax or on a stand fitted with vulcanite legs. Earth the leaves, etc., by connecting the terminal t by a wire to the gas-pipes; the leaves will therefore be at zero potential throughout the experiment. Charge the metal ball positively, and let it touch the case C; electricity flows from the ball to the case until they acquire

the same positive potential, and the leaves will be found to diverge. Here, again, *there is a difference in potential between the leaves and the case*, the former being at zero potential and the latter at a positive potential. If the ball be negatively electrified similar results follow: the case will be at a negative potential, the leaves at zero potential, and the latter will be diverging.

Discharge the case, disconnect t and the gas-pipe, and join by a wire t_1 and t. Since the case C and the leaves are now connected, whatever potential is given to one will be also given to the other. Touch the cap or the case with a positively charged ball so that they acquire a positive charge and a positive potential: *the leaves do not diverge.* Repeat with a negatively charged ball and it will be found that the leaves do not diverge. The cap may even be connected to an electrical machine and a very strong charge be given to the electroscope, but the leaves will not diverge. Note particularly that in these tests *the leaves and case are always at the same potential.*

From the preceding experiments it follows that **the leaves of an electroscope will only diverge if there is a potential difference between them and the case,** and further experiments readily indicate that *the greater this potential difference the greater is the divergence of the leaves.* In most experiments C is earth-connected (zero potential), so that the actual potential of the leaves will be the potential difference between the leaves and case; thus, *the amount of the divergence will be a measure of the potential of the leaves and of any body to which they are joined.*

69. Potential due to Neighbouring Charges. Free and Induced Potential.—Consider a positively charged conductor, say, in the middle of the room, no other charge being near (save, of course, the induced negative charge developed on the walls, etc., of the room). The conductor is at a positive potential, and, since this is due to its own charge, it is spoken of as a *free* potential. Similarly a negatively charged conductor, no other charge being near, is at a *free* negative potential due to its own negative charge.

Consider, now, an insulated *uncharged* metal ball A, joined by a wire to the cap of an electroscope some distance away: the leaves will hang together, since the case, leaves, and ball are all at zero potential. Now bring an

insulated charged body B (Fig. 154) gradually towards A, and it will be found that the leaves diverge more and more as B approaches A. Thus, *although A has no charge, it has acquired a potential* due to the presence of the charged body B; this is spoken of as an *induced* potential, since it is due to the inductive influence of B acting through the dielectric (air). If B be positive the induced potential of A will be positive, whilst if B be negative the induced potential of A will be negative.

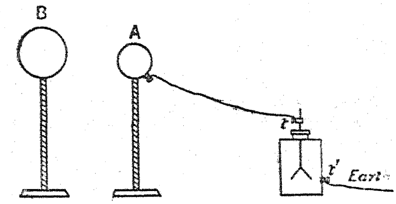


Fig. 154.

Now let A have a positive charge so that the leaves will be diverging, the amount of divergence being a measure of the positive potential of the leaves, etc., and A. Give B a strong negative charge and gradually bring it towards A; the leaves will gradually fall together, showing that the induced negative potential due to B is weakening the positive potential of A. When B reaches a certain position the leaves will collapse, so that their potential is zero, the same as the case of the electroscope; in this position the induced negative potential due to B is exactly cancelling the free positive potential of A, due to its own charge. If B be brought still nearer A the leaves will begin to diverge again, being now at a negative potential; the induced negative potential due to B is predominating.

It should be noted that, to begin with, *A has a positive charge and is at a positive potential.* When B is in such a position that the leaves collapse *A still has its positive charge, but is at zero potential.* When B is brought nearer, so that the leaves diverge again, *A still has its positive charge, but is at a negative potential.*

The phenomenon of inductive displacement (Art. 66) can now be more fully considered. If C (Fig. 153) be positively charged, the end A of the conductor AB exhibits a negative charge, and B a positive one: considered *as a*

whole the conductor *AB* has, of course, no charge, *i.e.* neither a "surplus" nor a "deficit" of electricity. From the preceding we know that *AB is at an induced positive potential due to C*, and, moreover, being a conductor all parts of it must be at the same positive potential. This latter fact can also be proved by attaching one end of a wire to the cap of an electroscope some distance away, the other end to a small brass ball, and holding the wire near the brass ball by insulating tongs, moving the ball all over the conductor: the leaves will always diverge by the same amount, showing that the potential is the same all over the surface.

If *AB* be now earthed, electricity will flow out of it until *its potential becomes zero.* It will now be found that the positive charge at *B* has disappeared and a somewhat greater negative charge has appeared at *A*; in fact, when earthed, electricity flows out of it to earth until it possesses such a negative charge that the negative potential due to this negative charge is exactly equal to the induced positive potential due to *C*, the actual potential of *AB* being therefore *zero.* If *AB* be now disconnected from the earth, and then *C* removed, *AB* will exhibit a negative charge at all parts of its surface, and it will now be *at a free negative potential* due to its acquired negative charge. These facts are graphically represented in Fig. 155.

If *C* be negative, the end *A* of *AB* exhibits a positive charge and *B* a negative one, and all parts of *AB* are at the same induced negative potential due to *C*. On earthing *AB* electricity flows from the earth until the conductor becomes at zero potential. It will be found that the negative charge at *B* has disappeared, and a somewhat greater positive charge has appeared at *A*; in fact, electricity flows to *AB* from the earth until it possesses such a positive charge that the positive potential due to this is exactly equal to the induced negative potential due to *C*, so that the actual potential of *AB* is zero. On removing the earth connection and then removing *C*, the conductor *AB* exhibits a positive charge at all parts, and is at a free positive potential due to the positive charge. These facts are graphically represented in Fig. 156.

It should be noted that when C is positive, then, to begin with, *AB has no charge as a whole, but is at a positive potential*; when AB is earthed *it has a negative charge, but is at zero potential.* If C be negative, then, to begin

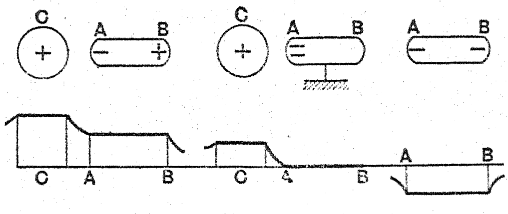


Fig. 155.

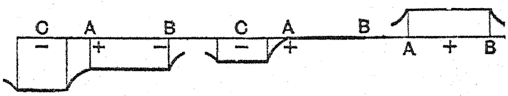


Fig. 156.

with, *AB has no charge, but is at a negative potential*, when AB is earthed *it has a positive charge, but is at zero potential.*

Caution. In connection with Fig. 155 text-books sometimes still use the expression, "the −ve charge at A is *bound* by the +ve charge on C, but the +ve charge at B is *free*; hence on earthing AB the free +ve charge at B goes to earth, leaving the bound −ve charge at A," and similarly, with Fig. 156, the +ve at A is said to be *bound*, and the −ve at B *free*; when earthed the free −ve escapes leaving the bound +ve. *These statements are not correct, and the reader should shun them*; thus, in the first case, more electricity passes to earth than is represented by the so-called free +ve, and in the second case more electricity passes in from the earth than is necessary merely to neutralise the so-called free −ve.

It should be mentioned that in the above cases the induced charges on AB are each less than the charge on C; the difference, for example, between the positive charge on C (Fig. 155) and the negative charge at A, is represented by the negative induced on the walls, etc., of the room. When the body AB is a hollow conductor entirely surrounding C (*e.g.* if AB is a can fitted with a lid and C is suspended inside), the induced charges are each equal to the

charge on C; in this case, when AB is earthed, an amount of electricity represented by the +ve charge at B only passes to earth, but even in this case the expressions "bound" and "free" should not be used.

A better explanation may now be given of the method of testing the nature of a charge by an electroscope. If the latter be positively charged the leaves, etc., will be at a positive potential. If a positively charged body be brought near the cap it will produce an induced positive potential in the leaves, etc.; thus the potential difference between the leaves and the case will be greater, and the leaves will diverge more. A similar explanation applies to a negatively charged body brought near a negative electroscope. If a negatively charged body be brought near the positive electroscope, the induced potential due to the body is a negative one; thus the positive potential of the leaves is partly cancelled, the potential difference between the leaves and case is reduced, and the leaves diverge less. A *neutral* conductor would also cause the leaves to diverge less owing to the inductive displacement in it, and the reaction on the electroscope of the induced charges.

70. Charging an Electroscope by Induction or Influence.—The method of doing this follows at once from the preceding section:—

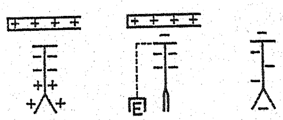


Fig. 157.

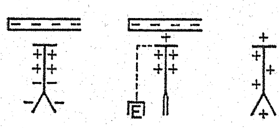


Fig. 158.

Exp. Bring a *negatively* charged rod near the cap of the electroscope, the case of which is earth-connected. Inductive displacement occurs, the cap exhibiting a positive charge, the leaves a negative charge, and the cap, rod, and leaves acquiring an induced negative potential; the leaves diverge. Touch the cap with the finger; the cap, rod, and leaves are brought to zero potential by electricity flowing from the earth, the leaves collapse since there is no difference in potential between them and the case, the negative charge has disappeared, and a somewhat greater positive charge has now appeared. Remove, *first*, the finger from the cap and *then*

the rod; the cap, rod, and leaves now acquire a free positive potential due to the positive charge, and the leaves diverge. The electroscope has been charged positively. The various stages are shown in Fig. 157.

To charge the electroscope negatively repeat the experiment, using a *positively* charged rod (Fig. 158).

71. The Charge resides on the Outer Surface of a Charged Conductor.—This statement applies both to solid conductors and hollow conductors *provided, in the latter case, there are no charged bodies inside*, and is equally true both for free charges and induced charges; the fact may be proved by numerous experiments.

Exp. 1. Fig. 159 illustrates *Biot's experiment.* *A* is an insulated brass ball positively charged. *B* and *C* are two hemispherical metal caps provided with insulating handles. The caps exactly fit the ball, so that when placed on *A* the caps and ball practically form one conductor. If this be done, and the caps be then removed, it will be found that *A* is completely discharged and that the whole of its original charge is now on the outer surfaces of *B* and *C*. The experiment may be varied by starting with *A*, *B* and *C* uncharged, *B* and *C* being in position on *A*. On charging the conductors and then removing the caps, it will be found that *A* is neutral and that the outer surfaces of *B* and *C* contain the charge.

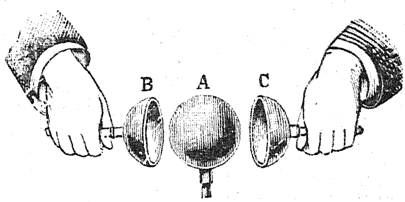


Fig. 159.

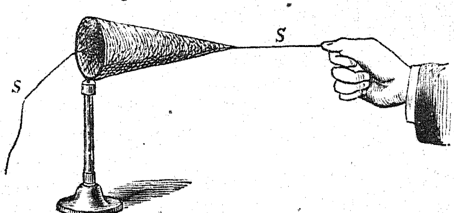


Fig. 160.

Exp. 2. Fig. 160 illustrates *Faraday's butterfly-net experiment.* The conical net is of linen gauze mounted on an insulating stand, and by means of the silk thread *SS* it can be turned inside out. The net is charged, and by means

of a proof plane and an electroscope it can be shown that the charge is on the outer surface, the inside being uncharged. The net is then turned inside out, but the charge is again found on the new outer surface.

Exp. 3. Place a metal can on an insulating stand, and charge the can, say positively. Touch the inside of the can with a proof plane, and then take the proof plane to the cap of a neutral electroscope: the leaves will not diverge, showing that the inside of the can has no charge. On similarly testing the outside of the can it will be found to be charged.

Discharge the can. Charge an insulated brass ball, say positively, and then lower it into the can allowing it to touch the inside. Remove the ball and test it: it will be found to be perfectly discharged. On testing the can, as before, it will be found that the inside is uncharged and that the outside is positively charged. When the charged ball touched the inside of the can it really became part of the latter, and the charge immediately left it and passed to the outside.

The result of the last experiment has an important application. If one conductor is required to completely give up its charge to another the latter should be made in the form of a hollow vessel and insulated; then on placing the former inside it, and in contact with its inner surface, the total charge at once passes to the outer conductor. This may be repeated as often as is desirable, and a considerable charge may be accumulated on the hollow conductor.

We may explain why the charge resides on the outer surface of a conductor as follows: The charge always distributes itself so as to be as near as possible to its induced charge, that is, in such a way as to possess minimum potential energy Just as objects always tend to fall towards the earth and to rest at the lowest attainable level, so electricity passes to its position of lowest attainable potential energy on the outer surface of the conductor. For example, if mercury be poured into a vessel full of water, it passes through the water and takes up a position at the bottom of the vessel, between the water and the vessel; similarly, if a charge be given to the interior of a hollow conductor, it passes to the outer surface, and takes up its position between the conducting surface and the insulator surrounding it. Hence, if we choose, we may say

that the charge of a conductor resides not on the conductor at all, but on the surface of the insulating medium adjacent to its "outer" surface.

72. Distribution of the Charge on the Outer Surface of a Conductor.—The general law given at the end of the preceding article may be applied to the consideration of this question. If all points on the outer surface of a conductor are positions of equal potential energy for the charge, then the distribution of the charge over that surface will be uniform. If, however, there are portions of the surface where the potential energy of the charge would be less than on other portions of the surface, then the charge will accumulate on those portions, until the distribution of the opposite induced charge is such that all points on the surface of the conductor are positions of equal potential energy for the charge. In this case the distribution of the charge on the surface will not be uniform, but the charge per unit area, or the *density* of the charge, will vary with the form of the surface, and with the position and arrangement of earth connected and other conductors in its neighbourhood.

We shall only consider how the form of the conductor affects the distribution, assuming all other conductors to be removed to a distance. On a spherical conductor the

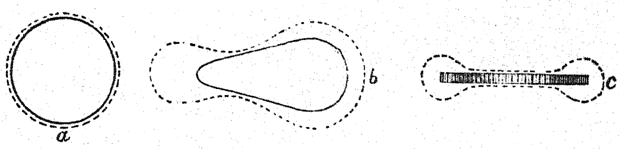


Fig. 161.

distribution is uniform, as indicated by the dotted line in Fig. 161 (a). On a conical conductor, such as shown at (b), the distribution is indicated in a general way in the figure; the charge accumulates at the pointed end, where the curvature of the surface is greater than at other points. Similarly, in the case of the circular plate (c) the charge

accumulates at the edges where the curvature is greatest, and, in general, experiment shows that the distribution varies with the curvature of the surface, that is, the greater the curvature at any point the greater will be the quantity of electricity at that point, provided the surface be *convex* at the point and the latter not within a re-entrant hollow.

The following is an important experiment bearing on the preceding :—

Exp. *To show that the potential of a charged conductor is uniform but that the distribution of the charge depends on the shape of the conductor.*

(*a*) Take the conductor shown in Fig. 162 and give it a positive charge. Place an electroscope some distance away and attach one end of a wire to the cap, the other end of the wire being fixed to a small brass ball. Holding the wire near the ball by means of insulating tongs, move the ball over the surface of the conductor. The leaves diverge by the same amount all the time, showing that the potential is the same at all parts of the surface of the conductor. It is also evident from the conception of potential that this must be so.

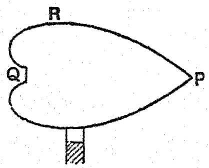


Fig. 162.

(*b*) Disconnect the wire from the electroscope and discharge the latter. Place a *small* metal can on the cap of the electroscope. Touch the point P (Fig. 162) with the proof plane, take the latter to the electroscope, lower it into the small can, allowing it to touch the inside so that all the charge passes to the can and electroscope, and note the amount of divergence. Discharge the electroscope, and repeat, touching various parts of the conductor with the proof plane. It will be found that the greatest divergence is obtained after touching the point, less after touching the flat sides, and that there is practically no divergence after the proof plane has been in the re-entrant hollow; this verifies the statements above.

In comparing (*a*) and (*b*) the reader must bear in mind that the *potential* at a point does not depend *only* on the charge at that particular point.

Further, the reader should remember that the *distribution of the charge* does not depend *only* on the shape of the

conductor. Thus, if CD (Fig. 163) be an earthed plate close to one side of the charged sphere AB, the density of the charge at B will be much greater than the density at A, and if a pin projects from the bottom (inside) of a charged can, the density at the pin will be zero. These facts are further dealt with in Arts. 73 and 77.

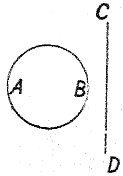


Fig. 163.

73. The Action of Points.—From the preceding articles we have learnt that the charge on a conductor tends to accumulate at the points, corners, and sharp edges where the curvature of the surface is greatest. Thus, if a needle point projects from the surface of a spherical conductor, the charge accumulates on this point, and the surface density there and the field intensity near becomes very great. Under these conditions, as will be seen in Chapter XXIII., the air, usually a very poor conductor, becomes conducting and the point loses its charge, the "carriers" of the charge being the air itself. Some of these carriers are positive and some negative: those carriers with a like charge stream away from the point and those with the opposite charge are drawn up towards it. All this will be better understood after reading Chapter XXIII. Further, owing to the great density, dust particles in contact with the point receive a charge and are then repelled carrying the charge with them, their place being then taken by another set, and the action repeated Hence, until the surface density becomes too low, there is a constant rain of charged carriers streaming away from the point, the charged body losing its charge. A similar convection discharge, as it is called, takes place from the sharp edges of a charged conductor.

This discharging action of points is readily illustrated by a number of experiments.

Exps. If a pin point be soldered to the cap of an electroscope and the instrument be charged, it will be found that it rapidly loses its charge—the divergent leaves quickly close up and soon indicate complete discharge.

If a strongly charged ebonite rod be held over the cap of an electroscope for a few seconds and then slowly removed, the divergent leaves will be seen to gradually collapse, and finally to open out

with a charge ot opposite sign. The leaves at first diverge, the charge on them being an induced negative charge, whilst the cap exhibits a positive charge. During the short time that the rod is held in position the negative charge partially escapes from the very thin edges of the gold leaf. On slowly removing the rod the leaves gradually collapse, as the positive charge on the cap neutralises the negative charge on the leaves, and finally re-diverge with the excess of positive corresponding to the negative which has escaped.

In the illustration just given the electroscope is charged by induction—the induced negative charge escapes from the thin edges of the leaves, leaving the instrument charged positively, *i.e.* opposite to that of the inducing charge. Similarly, if a pointed wire be attached to the cap of the instrument, and the inducing negative charge held for a short time over the point, the induced positive charge on the cap will escape, leaving the instrument charged negatively, *i.e.* the same as the inducing charge. Hence, if we wish to "collect" the charge from a charged body, it is only necessary to present a point or row of points attached to a conductor to it—the charge induced on the points flows as a convection discharge on to the surface of the charged body, neutralising the charge there found, and leaving the conductor charged with electrification of the same sign. The final effect is thus the same as if the points directly collected the charge from the charged body.

Thus let one end of a wire be attached to the cap of an electroscope, and the other end, carried in a split ebonite penholder, presented to any charged body—the leaves at once diverge, showing inductive action, and on removing the wire it will be found that the instrument is permanently charged. Similarly, if the point of a needle held in the hand be moved several times, close to the surface, from one end of a charged ebonite rod to the other, it will be found on testing that the rod is almost completely discharged.

Again, if a large needle, connected to the cap of a very delicate electroscope, be carefully insulated in the open air,* it will be found that the leaves of the instrument

* Some distance above the ground.

will, in general, gradually diverge. The needle point "collects" the charge from the air around it, and the divergence of the leaves indicates the nature and extent of the electrification of the air at the point where the needle is placed. When this electrification is very great, the convection discharge often gives rise to a luminous glow diverging from the discharging point. This effect is often seen from the points of ships' masts, flagstaffs, etc., and has been called St. Elmo's fire. It may be seen on a small scale in a dark room from a needle point attached to a strongly charged conductor; for example, to the newly charged plate of an electrophorous, or better, the prime conductor of an electrical machine (Chapter IX.).

Owing to this action of points and sharp edges it is necessary to have the corners and edges of all conductors intended to retain charges carefully rounded and polished. The apparatus must also be kept scrupulously clean, for every particle of dust acts as a point, and thereby admits of escape of the charge.

74. The Charge on a Body at a Given Potential depends on the Dielectric.—This may be proved by the apparatus shown in Fig. 164.

Exp. *A* and *B* are two equal brass balls fixed on insulating supports, and each carrying a brass rod (*P* and *Q*); *B* is embedded in paraffin wax, as indicated. *A* is charged, and, with *A* and *B* some distance apart, *P* and *Q* are joined by a fine wire, so that the charge is shared between them and they come to the same potential. It will be seen later that, since the balls are the same size, the charge would divide equally between them *if they were surrounded by the same medium.* Remove the wire, and then bring them in turn into the metal can standing on the cap of the electroscope (Art. 72). *B will produce the greater divergence, showing that it has the greater charge.* Similar results would be obtained if *B* were embedded in other solid (or liquid) dielectrics.

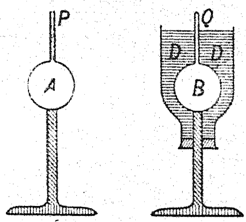


Fig. 164.

If Q be the charge on a conductor in air, and KQ the charge on the conductor when it is charged to the same potential in a dielectric x, then K measures what is called the specific inductive capacity of x. (See Art. 107.) This is merely referred to here; specific inductive capacity is dealt with in later chapters.

75. Hollow Conductor with no Charged Body Inside.—Let Fig. 165 represent an insulated metal pot, fairly deep in comparison with its diameter, and charged, say, positively; we have seen that the charge resides entirely on the outside, the inside of the pot being completely without charge. The *potential* of the pot is positive, and, moreover, the inside of the pot, where there is no charge, is at the same positive potential as the outside; this follows at once from the conception of potential and the fact that the pot is of conducting material, but it may be proved by connecting the pot by a wire to the cap of an electroscope standing on the table some distance away, as in Art. 72, when it will be found that, whether the small ball attached to the wire be allowed to touch the pot at P, Q, R, or S, the divergence of the leaves is the same.

Now consider the medium (air) inside the pot; *it can be shown, both experimentally and theoretically, that the whole of the air inside is at a uniform potential (at any rate, to within a short distance of the mouth), and that this uniform potential is identical with the potential of the pot itself.*

Exps. In the experiment referred to above, hold the ball in the centre of the pot and notice the divergence. Move the ball into various positions inside and notice that the divergence remains the same. Hence, *the potential at all points of the air region inside is the same.* NOTE.—Near the mouth of the pot the potential begins to vary; if the pot be provided with a lid, then *all* the air inside is at the uniform potential.

Place the electroscope inside the pot (it is assumed that the pot is of such a depth that the cap of the electroscope is well inside), and charge the pot strongly; the leaves do not diverge. Now the case of the electroscope is in contact with the pot, and is, therefore, at the potential of the pot, whilst the cap (and leaves) are at the potential of the air. But there is no divergence, so that the leaves and case are at the same potential; hence, *the uniform potential of the air region inside is the same as the potential of the pot.*

Various experiments may be mentioned, in addition, in proof of the theorems of this section. Thus, if a neutral metal ball, hanging by a dry silk thread, be lowered into the charged pot it takes the potential of the air region in which it is placed If it be allowed to touch the pot and then be removed, it will be without charge. This shows that the potential of the air is the same as that of the pot, for, if there had been a potential difference, electricity would have passed either from the ball to the pot, or from the pot to the ball, on contact, and the ball would have exhibited a charge on removal. Other experiments will, doubtless, occur to the reader.

Again, since the region inside is one of uniform potential, *there can be no inductive displacement on an insulated conductor completely inside*, for the necessary condition for such is that one part of the conductor must first be at a higher potential than another part (Art. 69). Further, we have the important theorem that *there is no electrical force within a closed hollow conductor which contains no charged bodies inside*; this is shown by the experiment with the electroscope above, and it also follows from the fact that the potential inside is uniform (Art. 88).

Imagine an insulated brass ball inside the positively charged pot; *the ball has no charge, but is at a positive potential.* Join the ball to the earth by touching it with a piece of wire held in the hand. Electricity flows from it to the earth, and its potential becomes zero. On removing the ball it will be found to have a negative charge (Fig. 166). If the pot be negatively charged, then, to begin with, *the ball will have no charge, but its potential will be negative.* On earthing, electricity will flow from the earth to the ball, and its potential will become zero. When the ball is removed it will be found to have a positive charge. In both these cases, as soon as

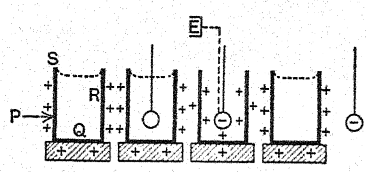


Fig. 165. Fig. 166.

the ball is earthed, we have a charged body inside the pot, and the preceding theorems of this section no longer hold good; the air inside is no longer a region of uniform potential, and charge will now be found on the inside of the pot (Fig. 166).

Example. *Discuss the potential changes of the pot and ball during the above operations.*—Taking for example Fig. 166 the potential changes are, briefly, as follows. Originally the ball has no charge, and is at zero potential; the pot has a positive charge, and is at a positive potential. When the ball is hanging inside the pot it still has no charge, but its potential is positive, the same as that of the pot; the potential of the pot is still the same. When the ball is earthed it acquires a negative charge and becomes at zero potential. The potential of the pot is now the free positive potential, due to its own charge, combined with the induced negative potential due to the negative charge on the ball. These are *very nearly* equal, so that the potential of the pot is just slightly above zero, *i.e.* the potential of the pot is still positive, but it has been considerably reduced; further, the bulk of the charge is now on the inside of the pot. On removing the ball it has a negative charge, and a negative potential; the pot has now its original positive potential, the charge being again on the outside. The reader should work out the case of a negative pot for himself.

76. Hollow Conductor with Charged Body Inside. Faraday's Ice-pail Experiment.—If a positively charged conductor be suspended by an insulating thread inside an insulated hollow conducting vessel which completely surrounds it (Fig. 167), the *total* induced charges will be found on this vessel, negative on the inside, and positive on the outside. Assuming the vessel to be neutral to start with, it is evident that these two charges must be equal to one another, for no charge has been communicated to the vessel; and, therefore, if the inducing charge is removed the vessel will again become neutral. This tells us nothing of the actual magnitude of these charges—we simply know that they are equal to one another and of opposite sign. It is important, however, to test experimentally whether there is any relation between the magnitude of the inducing

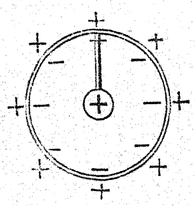


Fig. 167.

charge and that of the induced charges, and, if so, to determine the nature of this relation. With this object in view, Faraday performed the following experiment, which has now become historical.

An ice-pail, I (Fig. 168), is placed on an insulating stand and connected by the wire, w, with the cap of the electroscope, E. A positively charged body, B, suspended by a dry silk thread, is now slowly lowered into the pail. As B gradually descends, I comes more and more under its inductive action, and a gradually increasing negative

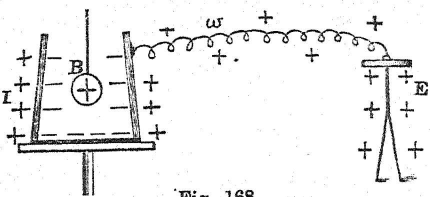


Fig. 168.

charge appears on the inner surface, while a corresponding positive charge appears on the outer surface, and on E. As a result of this action, the gold leaves of the electroscope begin to diverge as B is about to enter I, and this divergence slowly increases as B is lowered into the pail. When, however, B is well inside I, as shown in the figure, practically the whole of its inductive action is concentrated on the vessel, and the negative charge is consequently the total induced charge of that sign, while the positive charge represents the total induced positive charge. Hence, when this result is obtained, further lowering of B into I will not affect the induced charges, and consequently the divergence of the leaves of the electroscope remains unchanged.

If now B be allowed to *touch* the inner surface of I, and the leaves of the electroscope be carefully watched, it will be found that they are not in the least disturbed by the contact. Further, if B be now removed, it will be found to be completely discharged.

The significance of these results is important. When B touches the inner surface of I the positive charge on the

former tends to neutralise the negative induced charge on the latter, and, since B is completely discharged and the external charge on I and E remains quite undisturbed, it is evident that these two opposite charges have *exactly* neutralised each other, that is, the negative induced charge is exactly equal in magnitude to the positive inducing charge.

Hence, we may state generally, that the induced charges are equal to one another, but of opposite sign, and the *total* magnitude of either of the induced charges is numerically equal to that of the inducing charge. The first part of this statement applies to any case of induction; that is, whenever a body is subjected to induction, the induced charges are equal and opposite. In the second part, however, the significance of the word *total* must be noted—the magnitude of the induced charges on any body is equal to that of the inducing charge only when the body completely surrounds the inducing charge. In cases such as those illustrated in Fig. 151, the induced negative charge at A, for example, is less than the inducing positive charge on C, the difference being found in the induced negative charge on the walls, etc., of the room.

Illustrative Experiments. Let a number of metal cans of different sizes be arranged, as shown in Fig. 169, insulated from one another, and let an electroscope, E, be connected to any one of them by a wire, w. If a charged body, A, be now lowered into the inner can, B, the leaves of the electroscope at once diverge, no matter with which of the conductors it is connected. This illustrates an effect which is sometimes called *successive induction.* Suppose the charge on A to be positive; then it acts inductively on the inner can, with the result that the inner surface of this can becomes negative and the outer surface positive; this positive charge in turn acts inductively on the next can, with

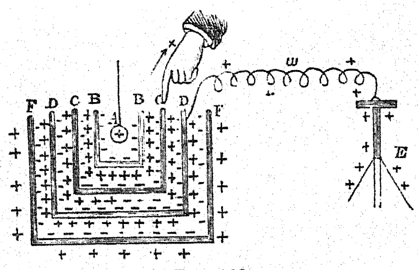


Fig. 169.

a result indicated in the figure, and by this successive action induced charges are developed on each conductor.

It will be noticed that the inductive action on any one conductor is due to the *algebraic sum* of the charges in its interior; for example, in the case of the conductor *C*, there are three *equal* charges in its interior, and the effective charge, that is, the algebraic sum of the three, may be represented by that on *A*, or by that on the outer surface of the first can. Hence the induced charges on the second can are equal to those on the first can, and each numerically equal to the charge on *A*. Similarly, the induced charges on any one of the conductors are each numerically equal to that on *A*.

If the electroscope be connected to any one of the conductors, and another *interior* to that one be touched with the finger, it will be found that the electroscope does not diverge when *A* is lowered into the inner can, thus showing that *the earth-connected conductor limits the field of induction.* This result is evident from Fig. 169: when *C* is touched the algebraic sum of the charges inside *D* is zero, owing to the fact that the odd positive charge on *C* has escaped to earth. If, however, a conductor external to that connected with the electroscope, for example, *F*, be touched, then the inductive action of *A* is at once indicated by a divergence of the leaves of the electroscope when *A* is lowered into *B*.

The fact that the inductive action on any hollow conductor is due to the algebraic sum of the charges in its interior has an important application.

In Art. 62 we have seen that when electrification is produced by friction, the rubber and the substance rubbed become oppositely electrified, and Fig. 170 depicts an experiment for showing that these opposite charges are *equal in magnitude.* Two metal cans, *A* and *B*, are arranged as shown in Fig. 170, and insulated from each other. The inner, *A*, is lined with fur, and an ebonite rod, *R*, fits closely into it. The outer can, *B*, is connected with an electroscope. On rotating *R*, the fur becomes positively and the rod negatively electrified. If these two charges are unequal, then the excess of the greater will act inductively on *B*, and the leaves of the electroscope will diverge. Experiment, however, shows that no such divergence takes place, thus proving that the algebraic sum of the charges in the interior of *B* is zero, that is, the charges produced on the rubber and the body rubbed are equal in magnitude, but of opposite sign. If, however, *R* be removed, it carries the negative charge with it, and the positive charge in *A*, acting inductively on *B*, causes an

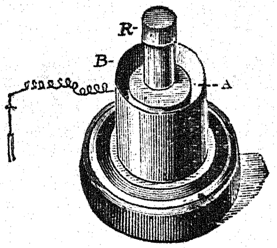


Fig. 170.

immediate divergence of the leaves. On replacing R, the leaves at once collapse, showing directly that the charge on A is equal and opposite to that on R.

If a metal pot be earthed, then whatever be the charges inside, the pot will be at zero potential, and bodies outside will be unaffected by the charges inside; the earthed pot acts as an **electric screen.** A large earth-connected metal plate, with a charged body on one side of it (not too near the edge), will act as an electric screen for bodies on the other side of the plate. An electroscope may be screened from the influence of external electrified bodies by surrounding it with an earth-connected conducting enclosure.

77. Electric Field, Lines and Tubes of Force and Induction.—The ideas dealt with in this section are somewhat similar to those treated in Art. 5 in connection with Magnetism.

An Electric Field is the space surrounding an electrified body (or system of electrified bodies) within which the influence of the charge (or charges) extends. When considering the forces in the field it is often spoken of as a **Field of Electric Force**; when concentrating on the phenomena associated with induction it is often spoken of as a **Field of Induction.**

A free, small, positive charge placed at any point in the field will be urged by a definite force in a definite direction, which latter is indicated by the line of force passing through the point in question. **A line of force is a curve such that the tangent at any point gives the direction of the electric force at that point.** The positive direction of a line is the direction in which a free *positive* charge tends to move, and as such a charge is always urged from a higher to a lower potential, *the positive direction of a line is the direction in which the potential is falling along it.* In some of the diagrams which follow, arrows indicate the positive direction of the lines.

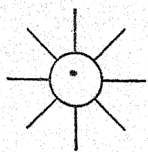


Fig. 171.

Fig. 171 shows the lines of force in the case of a positively charged sphere in the centre of a large room; the lines starting from the surface are all straight—if produced they meet at the centre of the sphere. A negatively charged isolated sphere would give the same diagram, but

any arrows would be pointing in the opposite direction. Fig. 172 represents the case of two equal conducting spheres having equal and opposite charges, whilst Fig. 173 depicts

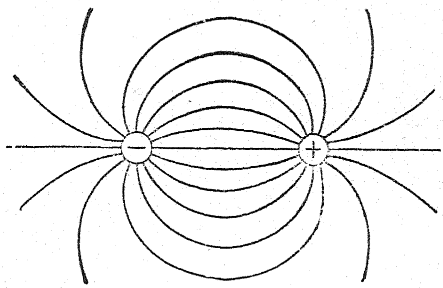


Fig. 172.

the case of two equal positive point charges. Fig. 174 shows the lines in the case of two unequal positive charges, A of 20 units, and B of 5 units, and a similar diagram

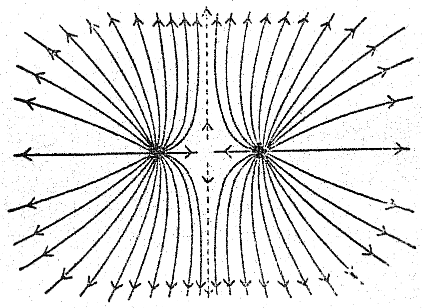


Fig. 173.

would be obtained for any two like charges in the ratio $4:1$; here N is a **null** or **neutral point** (cf. Art. 5) and it will be seen later that $AN = 2BN$. Fig. 175 represents

the lines in the case of two unequal and opposite charges, $A = +20$ and $B = -5$; here again N is the null point, and $AN = 2BN$.

In all cases it must be observed that *the lines start from a positive charge and end on a negative charge somewhere*; thus, in Figs. 171, 173 they start from the positive charges and end on the induced negative charges on the walls, etc., of the room, whilst in Fig. 175 some of the lines from A end on B, the remainder on the walls, etc., of the room. It should also be noted that lines of force do not pass *through* a conductor *but end on the surface*; the potential is uniform inside a solid conductor or inside a hollow conductor if there are no charges inside, and, therefore, the *force* there is zero. Further, if we imagine (cf. Art. 5) that each line tends to contract longitudinally, whilst lines proceeding in the same direction tend to repel each other laterally, we have an explanation of the fundamental facts of attraction and repulsion.

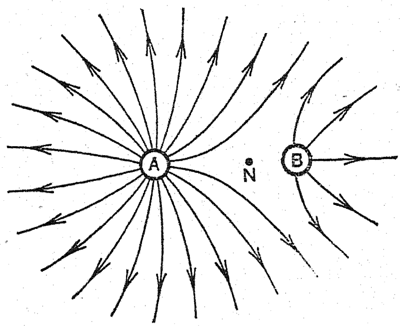


Fig. 174.

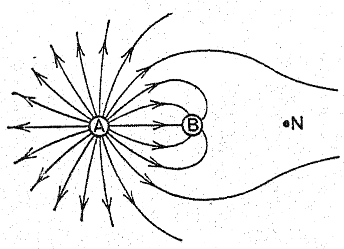


Fig. 175.

Imagine now a small closed curve (Fig. 176) to be drawn on the surface of a charged body, and lines of force to be drawn through every point of this curve out into the

medium; these lines will form a tubular surface enclosing a portion of the medium. The tube thus formed has been called a **tube of force.** If we suppose the whole surface of the charged body to be marked out by a network of closed curves, and a tube of force based on each mesh of the network, then the whole medium in which the field of force exists will be divided up into tubes of force, which touch each other laterally and fill the entire space, and, further, when these are conceived on a definite plan (to be given later) so that a definite number emanate from a definite charge they are called **unit tubes.** It is apparent that as the electric force decreases, the tubes widen; in fact, the electric force at any point in a tube of force can be shown to vary inversely as the area of cross section of the tube at that point.

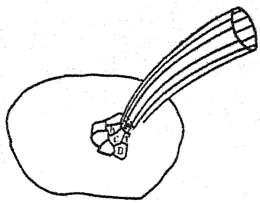


Fig. 176.

It should be noticed that, as indicated in the case of lines of force, *these tubes are not endless.* They are assumed to start from the surface of the charged body, extend out into the medium, and the outer end of each tube is found at the surface of contact of the medium with some conducting substance. Thus if a positively charged body be hung up by a silk thread in an empty room, the tubes of force starting from the surface of the charged body are terminated by the conducting walls of the room, and the medium between the body and the walls may thus be mapped out into these tubes of force. It will also be found that at each end of a tube of force where the insulating medium is in contact with the surface of a conductor electrification exists. The charge or quantity of electricity at one end of a tube is equal and opposite to the charge at the other end.

It has been indicated that the insulating medium is *strained,* and when this effect and inductive displacement in general are under consideration the field is spoken of as a *field of induction,* and the lines and tubes in the medium

as *lines and tubes of induction.* The charges at the ends of a tube have been stated to be equal and opposite, and this is another way of saying that the medium in the tube is in a state of strain, and the surface effects associated with this strain, at each end of the tube, are equal in magnitude, but opposite in sense. The energy of the electrification appearing at each end of the tube lies in the strained medium which fills the tube. The nature of the strain in the medium is such as would be produced by a tension along the length of the tube, and a pressure at right angles to its length. Faraday expressed this by saying that the tubes tend to contract and to repel each other, that is the strain in the medium is such as would be produced by each tube tending to contract, and at the same time to get thicker.

It will be evident from the preceding that corresponding to the charge or electrification on any body there is always an equal and opposite charge on the surface of the conductor or conductors at which the tubes of induction coming from the surface of the charged body terminate, and further that the force of attraction or repulsion between charged bodies is due to the state of *stress* in the intervening medium.

The reader should thoroughly grasp the idea that when an electrical field is considered as a field of force, the *stress* in the field is under consideration, but when considered as a field of induction the *strain* in the field is dealt with. In ordinary media, such as air, stress and strain are in the same direction, that is, force and induction have the same direction and tubes of force therefore coincide with tubes of induction. Strictly speaking, however, tubes of force have to do only with the electric force and the associated stress in the field, while tubes of induction have to do with the strain, and therefore with the electrical charges in the field. Thus the property referred to above that a tube in air has equal and opposite charges at its extremities is properly the property of a tube of induction, and is true for tubes of force only because in air they coincide with tubes of induction.

Let us imagine a charged insulated conductor to be

placed on a table in the middle of a room. Each surrounding object is subjected to the inductive action of this charge; if it be a conductor, then electrical displacement takes place without producing any strain, and therefore without calling up any opposing stress, in the substance; if an insulator, then slight electrical displacement takes place, until the stress produced by the strain thus induced balances the displacing electric force, and equilibrium obtains. As results, therefore, we find induced *charges* on the conductors and a *strain* in the insulators. If a conductor be insulated, then the induced charge on the side remote from the inducing charge transmits the inductive action into the space beyond; if earth-connected, then *the inductive action does not travel further*. Thus, in the case considered, the field of induction is bounded partly by the earth-connected walls of the room, and partly by the earth-connected objects which intercept the inductive action before it reaches the walls.

Figs. 177-184 will serve to illustrate many of the statements of this and preceding sections.

Fig. 177 (a) shows the tubes of induction emanating from an isolated charged sphere, and Fig. 177 (b) shows the change when the sphere is placed within an earthed can. As the nature of the strain is such that the tubes tend to shorten it is evident that the distribution of the strain will be such that the surfaces bounding the positive and negative faces of the strained medium will be as near together as possible. Thus the tubes from the ball all terminate on the inside of the can, and since the charges at the ends of a tube are equal and opposite, the charge on the inside of the can is equal to the charge on the ball. The figure also shows how the field of induction is bounded by the earth-joined can so that the space outside is screened from the effect of the ball.

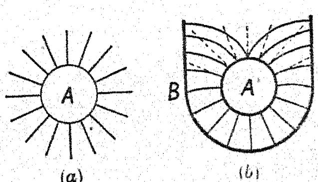


Fig. 177.

In Fig. 178 let C be positively charged and the conductor AB be placed into position; then the distribution of strain in the field is indicated in the figure. The strain in the space occupied by AB is annihilated, and in the rest of the medium the redistribution of the strain gives the field here indicated. Where tubes of induction starting from C terminate on the conductor at the end marked A there will be negative electrification. Where new tubes start from the end B of the conductor and spread out into the medium to terminate on the walls of the room there will be positive electrification. Also, since the conductor AB is initially in a neutral state the positive electrification at B must be equal in magnitude to the negative electrification at A, that is, the number of tubes terminating at A will be equal to the number starting from B.

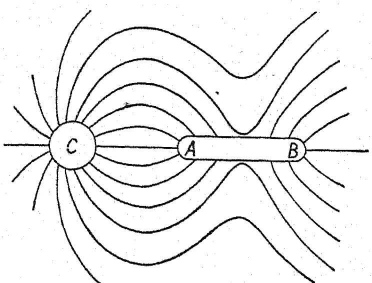


Fig. 178.

It will be seen that induction considered in this way is really a question of the redistribution of strain and strain energy in the medium round C consequent upon the annihilation of strain in the space occupied by the conductor AB. The result of this redistribution is that negative electrification is developed on AB at A, and positive electrification at B

If AB be earthed the tubes of induction which pass from B to earth disappear, and most of the tubes which in Fig. 178 pass from C to earth now terminate on A, choosing the shorter distance CA to an earth-joined body; a greater negative charge is now at A (note how this condemns the old idea of " bound and free charges ").

Fig. 179 depicts the tubes of induction in the various stages of charging an electroscope by induction; the

reader will be able to make out the explanation for himself.

The distribution of charge on charged conductors, *i.e.* the accumulation of the charge at points of greatest curvature (Art. 72) may also be inferred from a consideration of the tubes of induction indicating the distribution of the strain in the medium intervening between the inducing and induced charges. If, in a redistribution of strain in the medium, the area from which a tube of induction starts, or at which it terminates, decreases, this implies that the charge, which, before the redistribution, resided on

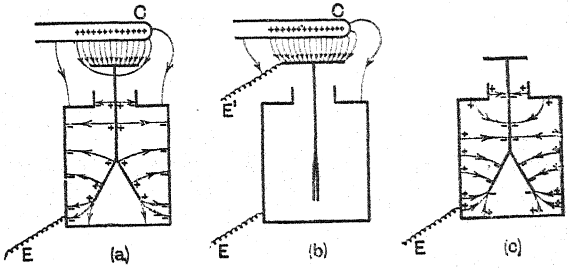
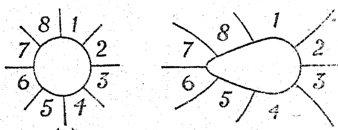


Fig. 179.

a given area of the charged surface, resides, after the redistribution, on a smaller area; that is, narrowing of the tubes of induction at any part of either surface indicates an accumulation of the charge at that part. Hence in induction diagrams wherever there is concentration of strain in the medium the tubes of induction appear to shorten and narrow, and the charges, initial and induced, tend to accumulate at those positions of the charged surfaces which are separated by regions of greatest strain. The distribution of surface charge is, in fact, only another aspect of the distribution of strain in the medium.

Thus in Fig. 180 (a) the distribution of the tubes of induction surrounding the charged spherical conductor is seen to be symmetrical. If, however, the spherical conductor be supposed to take the form shown in (b), then the

tubes of induction shown in (*a*) take up the new forms and distribution shown in (*b*), and concurrent with this change of distribution of strain there is the redistribution of the charge indicated by the changes in the terminal areas of the tubes. In (*a*) the terminal areas are all equal, that is, the charge is uniformly distributed; in (*b*) the areas are unequal, that is, the charge is no longer uniformly distributed, but is densest where the terminal areas are smallest. In Fig. 181 the redistribution of (*b*) is carried further, and it is evident that the density of the charge will be very great round the edge of the disc.



(*a*) Fig. 180. (*b*)

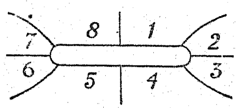


Fig. 181.

Fig. 182 gives the tube of induction diagram for a conductor with a re-entrant hollow on its surface, and indicates why there is no charge in the hollow. When, instead of a hollow, there is a protuberant point or knob on the surface of the conductor, an accumulation of charge is found on the protuberance. This accumulation is an effect of strain redistribution similar in character to the absence of charge on the re-entrant hollow; the tube of induction diagram for this is given in Fig. 183.

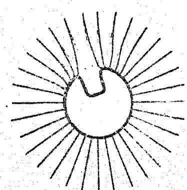


Fig. 182.

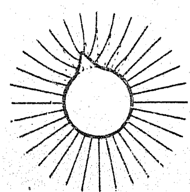


Fig. 183.

Fig. 184 is an instructive diagram, showing how the tubes of induction are modified as the two spheres charged as indicated are moved into the uniform field

between the plates charged as shown; for simplicity the other six tubes passing between the spheres have been omitted from the middle diagram.

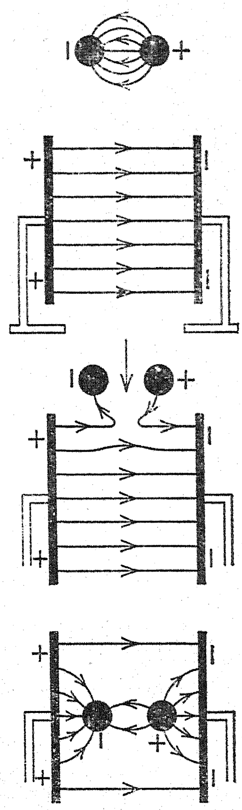


Fig. 184.

Fig. 184a shows the tubes in the case of a charged sphere A inside an insulated hollow conducting sphere B, but not situated at the centre of the latter. The distribution inside is irregular, as shown, but outside the distribution is uniform. The number of tubes leaving the outside surface is equal to the number terminating on the inside surface, *i.e.* is equal to the number emanating from A. An additional charge may be given to B, or the outside charge on it may be removed without affecting the inside distribution.

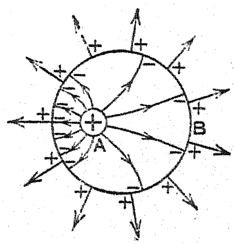


Fig. 184a.

78. Pyro-electricity and Piezo electricity.—When certain crystals (*e.g.* tourmaline) are heated uniformly one surface becomes positively electrified, and the opposite surface negatively electrified; when cooled the electrifications are reversed. The "pole" exhibiting positive electrification on heating is called the *analogous pole*, and the one

which develops negative electrification on heating is called the *antilogous pole.* This phenomenon is termed **pyro-electricity.**

In investigating the subject, *Kundt* heated the crystal and then allowed it to cool. A mixture of red lead and sulphur was then passed through muslin on to the crystal. By friction the red lead became positively electrified and the sulphur negatively electrified, and they arranged themselves on the crystal accordingly. *Gaugain* found that the degree of electrification was proportional to the change in temperature and to the cross-section of the tourmaline, but independent of the length; further, if a crystal be broken into many pieces it has been shown that each piece exhibits the properties mentioned. More recent experiments by *Riecke* show that the electrification is not strictly proportional to the change in temperature, but is expressed by an equation of the form

$$E = at + bt^2,$$

where t is the change in temperature, and a and b are constants. For some time it was considered that pyro-electricity was confined to hemi-morphic crystals, but *Hankel* showed that certain other crystals with unequal crystallographic axes exhibited the property.

The brothers *Curie*, in 1880, discovered that if tourmaline was subjected to compression in the direction of the axis of electrification observed in the phenomenon of pyro-electricity, then the same electrifications were observed as if the crystal had been cooled, whilst if it was subjected to extension the same electrifications were developed as if it had been heated; this phenomenon is termed **piezo-electricity.** The *Curies* found that the opposite charges were equal in magnitude, and that the variation in the charge was proportional to the variation in the force applied.

This has led to the introduction of the **piezo-electric balance** or **electrometer,** an instrument for the production of known but small charges of electricity. It consists of a quartz plate, suitably cut and dimensioned, with arrangements for supporting known weights. If the load be increased an additional charge is developed proportional to the additional load, and the charge can, if desired, be calculated from the dimensions and constant of the instrument. This instrument has been used by *M. and Mme Curie* in their investigations on radio-activity.

Lippmann has suggested that there should be *reciprocal effects* to those referred to above, viz. a change in temperature in the pyro-electric crystals, and a change in dimensions in the piezo-electric crystals when suitably subjected to electric strain. The latter has been definitely confirmed by the *Curies.*

Amongst the substances exhibiting the properties mentioned in this section are tourmaline, quartz, boracite, fluor-spar, etc.

79. Electrification of Gases.—When carbon burns in air the latter becomes positively electrified and the former negatively electrified, when coal gas burns in air, the air is negatively elec-

trified, when a platinum wire glows in air, the air is positively electrified, and the same result is obtained with other gases, save hydrogen, which becomes negatively electrified.

The splashing of liquids also results in the electrification of gases; thus, if water be allowed to splash on a metal plate in air, the air around is negatively electrified, and the water spray positively electrified. The air around waterfalls exhibits negative electrification, and the water spray positive. Other substances in the water may modify the result, a fact which is readily observed in the case of salt; thus the air near the sea is positively electrified and the water spray is negative.

Further details appear in subsequent chapters.

Exercises V.

Section A.

(1) Explain the following terms:—Conductor, insulator, dielectric, electric field, line of force, tube of induction, potential.

(2) Give examples of—(*a*) a body with no charge, but at a negative potential; (*b*) a body with a positive charge, but at zero potential; (*c*) a body with a positive charge, but at a negative potential.

(3) How would you prove experimentally that in the case of an electrified circular plate (*a*) the potential is the same at all points of the surface, (*b*) the density of the charge is greatest at the edges?

(4) What are the main points of difference between an insulated electrified can with no charged body inside, and an insulated can with a charged body hanging inside? Describe Faraday's Ice-pail experiment.

(5) Write a short essay on "Theories of Electrification."

Section B.

(1) Two similar deep metal jars are placed on the caps of two similar electroscopes at some distance apart, and the caps are connected by a fine wire; a positively electrified ball is lowered into one of the jars without contact. Explain the effect as to potential and divergence on both sets of leaves, and also that which occurs on breaking the wire connection by means of a silk thread and then removing the ball without allowing it to touch the jar. (B.E.)

(2) A hollow metal vessel is insulated and charged to a potential V, and the following operations are successively performed:—(*a*) An insulated metal ball is lowered into the jar without touching it; (*b*) the ball is momentarily earth-connected; (*c*) the jar is momentarily earth-connected; and (*d*) the ball is removed to a distance. State the changes of potential of the jar and the ball at each stage. (B.E.)

(3) Give a careful freehand drawing of the lines of force due to a charge of 4 units of positive electricity at A, and one of 1 unit of negative at B, if the distance between A and B is 2·5 centimetres. (B.E.)

(4) An insulated ice-pail and an insulated brass ball are both charged with positive electricity at a distance from each other, the pail to a high potential and the ball to a low potential. The ball is then brought near the pail and lowered into it without touching it until the bottom is reached. After contact the ball is entirely removed. Describe the changes in potential both of ball and pail (*a*) before contact, (*b*) on contact, (*c*) after removal. (B.E.)

(5) A positively charged conductor is brought near an insulated uncharged brass ball. Is the potential of the ball altered thereby? Would this alteration (if any) be modified by the ball having a needle sticking out of its surface, and would any such modification depend on the position of the needle? (B.E.)

Section C.

(1) A deep metal can is placed on the top of a gold leaf electroscope, and a positively charged conducting body is lowered into the can without contact. Draw a figure showing all the charges existing when the body is in the can, and state what happens to each when (*a*) the body touches the inside of the can, (*b*) the can is subsequently touched by an observer. (Inter. B.Sc.)

(2) If a metal ball, hung by a dry silk thread, be made to touch the inside of an electrified metal jar, and then carried to an electroscope, the electroscope is not affected, but if the electroscope is connected with the inside of the jar by a wire it receives a charge. Account for the difference between the two results. (Inter. B.Sc.)

(3) Define the term potential as applied to conductors in electrostatics.

Show that the potential must be the same at all points in the air space completely surrounded by a conductor. (B.Sc.) *

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* See Chapter VI. to make a complete answer to this question.

CHAPTER VI.

ELECTROSTATICS.—FUNDAMENTAL THEORY.

80. Quantity of Electricity; Unit Quantity.—The force of attraction between two oppositely electrified bodies, or the force of repulsion between two similarly electrified bodies, is found to depend on three things—the degree of electrification of each of the bodies, their distance apart, and the nature of the intervening medium. The more strongly electrified the bodies are, the greater is the force of attraction or repulsion exerted by them, and the further the bodies are apart the less is the force between them.

Imagine two small positively charged bodies separated by any definite distance, and suppose the charge on *one* to be doubled, then the force of repulsion between them will also be doubled. Similarly, if the charge on the other body be doubled, then the magnitude of the force is again doubled—that is, by doubling the charge on *each* of the bodies the force is quadrupled. Similarly, if one charge be doubled while the other is trebled, the magnitude of the repelling force will be increased to six times its original value. This result can be expressed generally by saying that *the force exerted between two charged bodies varies as the product of their charges.*

Considering the second condition on which the magnitude of the force depends, it is found *by experiment* (Art. 121) that if the distance between two charged bodies be doubled, then the force is reduced to one-fourth of its original value—that is, *the force exerted between two charged bodies varies inversely as the square of the distance*

between them. This law is rigorously true only when the charges are supposed to be concentrated at points. This involves a physical impossibility, but the law may be taken to be very approximately true when the dimensions of the charged bodies are small compared with the distance between them.

Experiment shows that the laws stated above hold for any given medium; but *the actual magnitude of the force under given conditions varies with the nature of the medium.* To state the facts mathematically, if q and q' denote the charges (strictly *point charges*), d the distance apart, and f the force between them—

$$f = \alpha\frac{qq'}{d^2},$$

where α is a constant depending only on the medium and on the units we decide to adopt.

Let the surrounding medium be air, and let f be measured in dynes and d in centimetres. It will clearly be convenient to so choose our unit quantity that the constant α becomes *unity* under these circumstances, and this will be so if *unit quantity* be defined as *that quantity which, when placed one centimetre in air from an equal and like quantity, repels it with a force of one dyne,* for if $q = q' = 1$ (unit quantity), $d = 1$ (cm.), and $f = 1$ (dyne), it follows that $\alpha = 1$. Hence, adopting this unit charge, the force *in air* between two point charges, q and q', at distance d centimetres apart, is given by

$$f = \frac{qq'}{d^2} \textbf{ dynes.}$$

For any other medium the constant α must be introduced; α is really equal to $1/K$ (Art. 95), where K is the *specific inductive capacity* of the medium. Hence, the more general formula is written—

$$f = \frac{1}{K}\frac{qq'}{d^2}.$$

As indicated, the above is strictly true only for point charges, *i.e.* two charges concentrated at two points. In the case of two charged spheres it can be shown that *if the charge on each remained* **uni-**

formly distributed *each sphere would act as if its charge were concentrated at its centre*, and the formula would apply, d being the distance between the centres. In practice, however, each charge disturbs the distribution of the other, so that the formula above does not give the actual force; but the error is small if the spheres are small compared with their distance apart.

The above unit quantity is known as the *C.G.S. electrostatic unit quantity*; hence—

The C.G.S. electrostatic unit of quantity is such that if placed one centimetre in air from an equal and like quantity it repels it with a force of one dyne. It has no name.

The **practical unit** is the **coulomb**; it is equal to 3,000,000,000, *i.e.* 3×10^9 electrostatic units.

Another quantity unit known as the "C.G.S. electromagnetic unit" is employed in current electricity; it is equal to 3×10^{10} electrostatic units; thus the coulomb is $\frac{1}{10}$ of the electromagnetic unit.

The "**electron**" $= 4{\cdot}65 \times 10^{-10}$ e.s. unit $= 1{\cdot}55 \times 10^{-20}$ e.m. unit $= 1{\cdot}55 \times 10^{-19}$ coulomb (Chapter XXIII).

The reader will remember that unit magnetic pole was defined on the assumption that the permeability μ *of air* was unity, and on this a system of units, known as the *electromagnetic system*, is based. Unit quantity of electricity has now been defined on the assumption that the specific inductive capacity K *of air* is also unity, and on this a system of units, known as the *electrostatic system*, is based. It will be seen later that these two assumptions, viz. that *both* K *and* μ *are unity for air*, are incompatible.

Example. *Two small spheres, each of mass* m *grammes, are suspended from a point by threads, each* l *cm. long. They are equally charged, and repel each other to a distance of* $2d$ *cm. If* $g = 981$, *express the charge on each in electrostatic units, electromagnetic units, and coulombs.*

The forces on the sphere A (Fig. 185) are:—

(1) The weight, mg dynes.

(2) The repulsions, viz. $\frac{q^2}{(2d)^2}$ dynes.

(3) The tension of the thread,

and these are in the directions indicated.

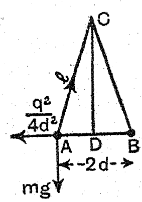

Fig. 185.

An examination of the figure will show that the triangle CDA has its sides parallel to these forces; hence the forces are proportional to the sides of this triangle to which they are parallel. Thus,

$$\frac{\frac{q^2}{4d^2}}{mg} = \frac{DA}{CD} = \frac{d}{\sqrt{l^2 - d^2}},$$

$$\therefore q^2 = \frac{4d^3mg}{\sqrt{l^2 - d^2}}, \quad i.e. \quad q = 2d\sqrt{\frac{dmg}{(l^2 - d^2)^{\frac{1}{2}}}} \text{ e.s. units.}$$

In electromagnetic units the charge is $q/(3 \times 10^{10})$, and in coulombs $q/(3 \times 10^9)$.

81. Electrical Potential. Unit Potential.—In Art. 67 potential has been defined in a general way as that electrical condition which determines the direction in which electricity will flow, or tend to flow; we proceed now to the principle upon which the actual measurement of electrical potential is based.

In practice the earth is taken as the zero of potential, because, owing to its size, none of our experiments are likely to alter its potential. Strictly, however, it is only at infinity that the force due to a charge, or system of charges, is zero, and, therefore, in mathematical investigations, the zero of potential should be taken *at infinity*. The *potential difference* (P.D.) between two points is, however, the same, whether the practical or the mathematical zero be taken, and in practice it is really only with potential differences we are concerned.

Let A (Fig. 186) be a positively charged body, no other charge being near; the potential falls from its positive value at A to zero at infinity. Imagine D to be a unit positive charge at infinity. To bring this unit charge from infinity to C, work must be done against the repulsion of A, more work must be done in bringing it to B, and still more work in bringing it to A. The work so done has its equivalent in the

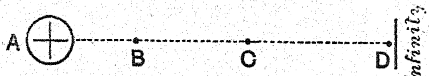

Fig. 186.

potential energy gained by the unit charge in virtue of the change in its position relative to the charge at A.

If W_1 ergs of work be done in moving the positive unit from infinity to C, W_1 electrostatic units is the potential at this point C, due to the charged body A; if W_2 ergs of work be done in moving the positive unit from infinity to B, W_2 electrostatic units is the potential at B, and the potential difference between B and C is $(W_2 - W_1)$ electrostatic units; the latter is evidently numerically equal to the work in ergs in moving the positive unit from C to B, i.e. is numerically equal to the difference between the potential energy of the unit charge at B and at C. Finally, *W_3 electrostatic units is the potential of the body A if W_3 ergs of work be done in moving the positive unit from infinity up to it.*

If the charge at A be taken as negative instead of positive, *then no work would have to be done* in bringing up the unit positive charge, but the force of attraction which the charge exerts *would do work* in so bringing it up. Hence, the potential at A, B, or C is said to be negative, for the unit charge, instead of having, as it were, to be pushed up against the force of *repulsion* exerted by the charge on A, is pulled up by the force of attraction due to that charge.

To summarise:—*The potential at any point in the field surrounding a charged body is represented numerically by the work done on or by a positive unit charge in moving from infinity, regarded as the zero of potential, to the point in question; the P.D. between two points is represented numerically by the work done on or by a positive unit charge in moving from one point to the other.* Hence—

The potential at a point is one C.G.S. electrostatic unit if one erg of work is done on or by a positive unit charge in moving from infinity to that point. It has no name.

The **practical unit** is the **volt;** it is equal to $\frac{1}{300}$ of an electrostatic unit.

The warning given in Art. 19 may be repeated here. The meaning of the definition is that, if the work done in moving unit charge

between two points is W ergs, the potential difference between the two points is W units of potential, *not W ergs*.

Another potential unit, known as the "C.G.S. electromagnetic unit," is employed in current electricity. One electrostatic unit is equal to 3×10^{10} electromagnetic units; hence the volt is equivalent to $\frac{1}{300} \times 3 \times 10^{10}$, *i.e.* 10^8 electromagnetic units.

82. Potential at a Point in an Electric Field.—Let a positive charge of Q units be placed at O (Fig. 187), and let the points $a, b, c, \ldots n$ be supposed to be very close together along the line OB.

Fig. 187.

The electric force at a, that is, the force which a unit positive charge at a would experience, is $\frac{Q}{(Oa)^2}$, and similarly the electric force at b is $\frac{Q}{(Ob)^2}$. Therefore the average force for the very short distance ab may be taken as $\frac{Q}{Oa \, . \, Ob}$, which is the geometric mean of the values at a and b. The work done when unit quantity of electricity is moved from b to a is, therefore, measured by $\frac{Q}{Oa \, . \, Ob} \, . \, ab$, that is, by

$$\frac{Q}{Oa \, . \, Ob}(Ob - Oa) \quad \text{or} \quad \frac{Q}{Oa} - \frac{Q}{Ob}.$$

That is, the difference of potentials at the points a and b is $\frac{Q}{Oa} - \frac{Q}{Ob}$. Similarly, the difference of potentials for the points b, c is $\frac{Q}{Ob} - \frac{Q}{Oc}$.

It follows from this that the difference of potential for the points a and n, separated by a finite distance an, is

$$\left(\frac{Q}{Oa} - \frac{Q}{Ob}\right) + \left(\frac{Q}{Ob} - \frac{Q}{Oc}\right) + \left(\frac{Q}{Oc} - \frac{Q}{Od}\right) + \ldots \left(\frac{Q}{Om} - \frac{Q}{On}\right);$$

this evidently reduces to

$$\frac{Q}{Oa} - \frac{Q}{On},$$

and if n be assumed to be at an infinite distance from O *the potential at* a is measured by

$$\frac{Q}{Oa} - \frac{Q}{\infty} = \frac{Q}{Oa}.$$

That is, the potential at a point P distant r from a point charge Q is given by

$$\textbf{Potential at P} = \frac{Q}{r}\ \textbf{e.s. units,}$$

and the potential difference between two points distant r and r_1 from Q is given by the expression—

$$\textbf{P.D.} = \frac{Q}{r} - \frac{Q}{r_1}\ \textbf{e.s. units.}$$

In the above the medium has been assumed to be air, for which K is taken as unity. In the case of any other medium the force at a, for example, is not $Q/(Oa)^2$, but $Q/K(Oa)^2$; making this change in the mathematics the last two expressions become—

$$\textbf{Potential at } P = \frac{Q}{Kr}; \quad \textbf{P.D.} = \frac{Q}{K}\left(\frac{1}{r} - \frac{1}{r_1}\right).$$

The work done in moving the positive unit from infinity to a given point in a field, or from one point to another, is independent of the path described. This is proved in the same way as the corresponding proof in Magnetism (Art. 19).

The reader acquainted with the Calculus will understand the neater proof—

$$Vr - Vr_1 = -Q\int_{r_1}^{r} \frac{1}{x^2}dx = -Q\left[-\frac{1}{x}\right]_{r_1}^{r}$$

$$= Q\left(\frac{1}{r} - \frac{1}{r_1}\right),$$

and if $r_1 = \infty_2\ 1/r_1 = 0$ and the potential at distance $r = Q/r$; the medium is, of course, air.

In the case of a **uniformly charged conducting sphere** of radius R cm., no other charge being near, the lines of force diverge as if from a point charge *at the centre* (Fig. 171), and in fact the sphere acts at external points as if its charge were concentrated at its centre. Hence if Q be the charge, the potential at an external point distant r cm. *from its centre* is Q/r or Q/Kr, and **the potential of the sphere itself is** Q/R **or** Q/KR. Whether the sphere be hollow or solid the latter expressions give the potential not only of the outer surface *but also of the whole interior*, provided, of course, there is no charged body inside in the case of the hollow sphere.

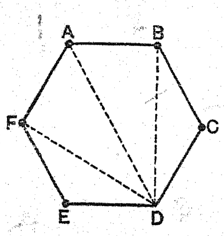

Fig. 188.

Potential is a scalar quantity; the potential at a point due to a number of charges is simply the algebraic sum of the potentials due to each.

Examples. (1) A, B, C, D, E, F are *the angular points of a regular hexagon (Fig. 188). The charges at A,B,C,E,F are $+1, +2, -5, +3, -4$ electrostatic units. If each side of the hexagon be 1 cm. and the medium be air, find the potential at D.*

$CD = 1$ cm., $ED = 1$ cm., $AD = 2$ cm.

$BD = FD = \sqrt{BC^2 + CD^2 + 2BC.CD \cos 60^\circ} = \sqrt{3}$ cm.

Hence, if V be the potential at D:—

$$V = \frac{1}{2} + \frac{2}{\sqrt{3}} - \frac{5}{1} + \frac{3}{1} - \frac{4}{\sqrt{3}}$$

$$= -(1\cdot5 + \tfrac{2}{3}\sqrt{3}) \text{ e.s. units.}$$

(2) *Within a spherical vessel of brass 1 cm. thick, the external diameter of which is 14 cm., a brass ball 8 cm. in diameter is hung by a silk thread so that the centres of the two spheres coincide. If the ball is charged with + 36 units of electricity, and if the potential of the vessel is 7 units, what is the potential of the ball?*

The three radii are 4 cm., 6 cm., and 7 cm.

If Q be the charge on the outer surface its potential will be $Q/7$; but the potential is 7—

$$\therefore \frac{Q}{7} = 7, \ i.e.\ Q = 49 \text{ units.}$$

The charge on the ball is + 36, therefore the charge induced on the inner surface of the vessel is − 36. The potential of the ball is the sum of the potentials due to its own charge and the charges on the vessel; thus if V be the potential of the ball—

$$V = \frac{36}{4} - \frac{36}{6} + \frac{49}{7} = \mathbf{10} \text{ e.s. units.}$$

Note.—In the above the outer vessel has an independent charge. If the outer vessel is merely acted on inductively by the charge on the ball the outer charge would be + 36, the *potential of the ball* would be

$$V = \frac{36}{4} - \frac{36}{6} + \frac{36}{7} = 8\tfrac{1}{7} \text{ e.s. units,}$$

and *the potential of the vessel* would be $36/7 = 5\frac{1}{7}$ e.s. units.

If the outer vessel were earthed the charge + 36 on its outer surface would disappear, its potential would, of course, be zero, and the potential of the ball would be

$$V = \frac{36}{4} - \frac{36}{6} = 3 \text{ e.s. units.}$$

83. Equipotential Lines and Surfaces.—An equipotential surface is the locus of all points having the same potential. *An equipotential line is the line in which an equipotential surface is cut by a plane;* thus the equipotential lines in Fig. 189 are the lines in which the equipotential surfaces are cut by the plane of the diagram.

Imagine a charge to be concentrated at a point A, then all points at the same distance from the charge have the same potential, that is, the equipotential surfaces in this case are a series of concentric spheres having their centre at the point A. The potential at each of the spherical surfaces is different, but for all points on any one surface it is the same; that is, work has to be done to move a positive unit of electricity from one surface to another, but no work is done in moving it from any point on a given surface to another point on the *same* surface.

From this it follows that the lines of force are at all points normal to the equipotential surfaces, for if no work is done in moving electricity from one point to an adjacent one on an equipotential surface, then the direction of motion must be everywhere perpendicular to the lines of force. In Fig. 189 the traces of the equipotential surfaces

surrounding the charge are shown as concentric circles, and the lines of force in the plane of the paper are shown as straight lines radiating from the centre.

It is convenient in drawing equipotential surfaces to represent them so that unit quantity of work must be done in conveying unit quantity of electricity from any one surface to the next. When so drawn the distance between consecutive surfaces gradually increases as the distance from the charge increases. For the greater this distance the weaker the force exerted by the charge, and therefore the greater must be the distance through which it has to be overcome to do unit work.

If a charge be uniformly distributed on a spherical conductor its action at all *external* points is the same as if the charge were accumulated at the centre of the conductor. Hence the equipotential surfaces surrounding such a sphere are concentric spherical surfaces having the centre of the sphere as their common centre. The surface of the sphere is also an equipotential surface, for with statical electricity there can be no difference of potential between any two points of a conducting body.

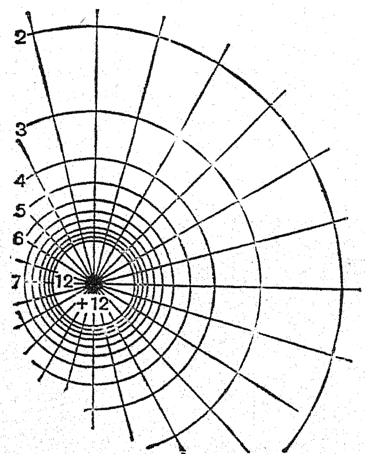

Fig. 189.

Exp. *To map the equipotential lines for a positively charged isolated sphere.* — Consider the case of a conducting sphere of 1 cm. radius (in air) charged with 12 units; the potential of its surface is $\left(V = \frac{Q}{r}\right)$ 12 units, and *this is also the potential of all points inside it.* Since $r = Q/V$ the equipotential line 11 will be a circle of radius $r = 12/11 = 1{\cdot}1$ cm. (approx.). The equipotential 10 will

be a circle of radius 1·2 cm.; the equipotential 8 a circle of radius 1·5 cm.; the equipotentials 6, 4, 3, 2 and 1 circles of 2, 3, 4, 6 and 12 cm. radius respectively. The equipotential zero is *strictly* at infinity. Fig. 189 represents this, omitting the lines inside $V = 12$.

Fig. 190 shows the general effect on the shape of the equipotentials of bringing an insulated uncharged conductor B into the field of the charged sphere A. The former is acted on inductively and acquires a certain definite potential, assumed V in the figure; thus the whole conductor B forms part of the equipotential V in the figure, and the other equipotentials are distorted as shown. The point a has now a potential lower than V_1, whereas before the introduction of B its potential was higher than V_1; this is due to the fact that there is an induced negative charge at X. The point b has now a potential higher than V_2, whereas before the introduction of B its potential was lower than V_2; this is due to the fact that there is an induced positive charge at Y. If B be earthed it forms part of the equipotential zero, the potential of A is lowered and all the equipotentials between zero and the value for A are crowded in between A and B; the positive charge on B has disappeared and an increased negative charge has appeared at X. In Fig. 190 the two horizontal lines are merely the top and bottom borders of the diagram.

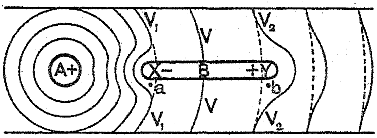

Fig. 190.

Exp. *To map the equipotential lines due to two point charges—say one positive, the other negative—at a given distance.*—The equipotential circles for each charge are first drawn as indicated above, those for the positive charge being marked + and those for the negative charge −. Where two circles intersect the actual value of the potential is the algebraic sum of the potentials indicated by the two circles, and by joining all intersections having equal values we obtain an actual equipotential line. Thus the equipotential 0 will be given by the intersection of, say, + 6, − 6; + 7, − 7; + 8, − 8, etc.; the equipotential 1 by the intersection of + 6, − 5; + 7, − 6, etc.; the equipotential − 2 by the intersection of − 8, + 6; − 9, + 7, etc. The actual values most convenient to work with in any

case will depend upon the magnitude of the charges and their distance apart.

Fig. 111 for the two poles N and S will also represent the equipotential lines for *two equal opposite charges.*

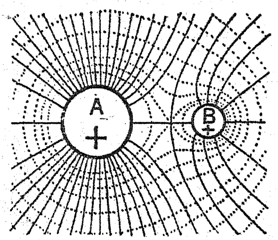

Fig. 191.

Fig. 191 gives the equipotential lines for *two unequal like charges* $A = +4$ and $B = +1$; the dotted ovals are the equipotential lines, and the continuous lines cutting the equipotentials at right angles are the lines of force which are identical with those of Fig. 174.

The dotted lines of Fig. 192 are the equipotentials in the case of a *positively charged sphere A acting inductively on an earthed sphere B,* and the continuous lines are the lines of force.

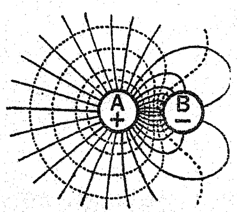

Fig. 192.

84. Electrical Capacity.—Unit Capacity.—In Art. 82 it has been noticed that the potential produced in a spherical conductor by a given charge varies with the size of the conductor; thus, if a charge of + 20 electrostatic units be given to a sphere (in air) of 10 cm. radius, the potential of the sphere will be 20/10, *i.e.* 2 electrostatic units, but if the same charge be given to a sphere (in air) of 4 cm. radius the potential will be 20/4, *i.e.* 5 electrostatic units. Further, it can be shown by experiment that if the same charge be given to any two conductors differing in size or form different potentials will be produced in these conductors.

These facts may be expressed by saying that different conductors have different *electrical capacities*, and **the electrical capacity of a conductor** may be defined **as the ratio of the charge given to the conductor to the**

potential produced in the conductor by that charge. That is, if a charge Q given to any conductor produce in it a potential V, then the capacity of the conductor is given by

$$C = \frac{Q}{V},$$

where C denotes the capacity of the conductor. Another way of expressing this definition is to say that **the capacity of a conductor is measured by the quantity of electricity necessary to raise the conductor to unit potential.** Thus, if a charge of 10 units of electricity raises the potential of a conductor to 5 units, then the capacity of the conductor is given by the ratio $\frac{10}{5} = 2$, that is, the conductor requires 2 units of electricity to raise it to unit potential. Hence—

A conductor has a capacity of one C.G.S. electrostatic unit if the electrostatic unit quantity raises its potential by one electrostatic unit. It has no name.

The **practical unit** is the **farad; a conductor has a capacity of one farad if a charge of one coulomb raises its potential by one volt.** The farad $= 9 \times 10^{11}$ e.s. units.

Another capacity unit is employed in current electricity; it is known as the "C.G.S. electromagnetic unit," and is equal to 9×10^{20} electrostatic units; thus the farad is $\frac{1}{10^9}$ electromagnetic unit, as may also be seen from the relation

$$1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}} = \frac{\frac{1}{10} \text{ electromagnetic unit}}{10^8 \text{ electromagnetic units}}$$

$$= \frac{1}{10^9} \text{ electromagnetic unit.}$$

It is advisable to remember three forms of the algebraic relation given above, viz.

$$C = \frac{Q}{V}, \textit{ i.e. } \text{Capacity} = \frac{\text{Quantity}}{\text{Potential}};$$

$$V = \frac{Q}{C}, \textit{ i.e. } \text{Potential} = \frac{\text{Quantity}}{\text{Capacity}};$$

$$Q = CV, \textit{ i.e. } \text{Quantity} = \text{Capacity} \times \text{Potential.}$$

If C be the capacity of a conductor *in air*, a charge of C e.s. units will raise its potential to 1 e.s. unit *in air*. If the conductor *with this charge* C be now immersed in a medium of specific inductive capacity K, its potential will be $1/K$ of what it was in air (Art. 82), *i.e.* its potential will be $1/K$ e.s. unit. To bring it up to unit potential in this medium the charge must be increased K times, *i.e.* a charge KC e.s. units will be necessary to raise its potential to 1 e.s. unit in this medium. *Hence, if C be the capacity of a conductor in air, its capacity in a medium of specific inductive capacity K is KC e.s. units.*

We have seen (Art. 82) that if Q e.s. units be given to a spherical conductor of radius R cm., embedded in a medium of specific inductive capacity K, the potential of the sphere is $\frac{Q}{KR}$ e.s. units. If C denote its capacity *in this medium*, its potential is also given by the expression $\frac{Q}{C}$; hence $C = KR$. With air as medium K is unity, and this reduces to $C = R$. *Hence the capacity of an isolated spherical conductor in air is, in e.s. units, numerically equal to its radius in centimetres;* in a medium other than air, the capacity, in e.s. units, is numerically equal to its radius in centimetres multiplied by K.

If two conductors charged to different potentials are brought into contact, they take up a common potential and share the combined charge in direct proportion to their capacities; for example, if two conductors of respective capacities 3 and 5 are made to share their charges, then one takes $\frac{3}{8}$ and the other $\frac{5}{8}$ of the combined charge. *To determine the common potential which two or more conductors take on sharing their charges* we must remember two things—first, that the total quantity of electricity is the same before and after sharing; second, that the capacity of two or more conductors in contact with one another is the sum of the individual capacities of the conductors. Thus, if two conductors of capacities C_1 and C_2 be charged to potentials V_1 and V_2 respectively, then on

being placed in contact they will take up a common potential V given by

$$C_1V_1 + C_2V_2 = (C_1 + C_2)V;$$

or

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}.$$

The expression $C_1V_1 + C_2V_2$ gives the quantity of electricity before contact, and $(C_1 + C_2)V$ expresses the *same* quantity after contact and redistribution of the charge. The conductors are here supposed to be put in contact in such a way that the distribution of the charge on one conductor is not influenced by the near presence of the other conductor. For example, the two conductors at a distance from each other and from other conductors might be joined by a long thin wire of negligible capacity.

The fact that *the charge divides between them in the ratio of their capacities* is readily seen. If Q_1 be the charge on the conductor of capacity C_1, and Q_2 that on the conductor of capacity C_2, and if V be the common potential—

$$Q_1 = C_1V \text{ and } Q_2 = C_2V, \quad \therefore Q_1 : Q_2 = C_1 : C_2.$$

Thus the charge on the conductor of capacity C_1 is $\frac{C_1}{C_1 + C_2}$ of the total charge on both conductors, and the charge on that of capacity C_2 is $\frac{C_2}{C_1 + C_2}$ of the total charge.

Examples. (1) *A charge of* 50 *C.G.S. units raises the potential of a spherical conductor in air from* 10 *to* 15 *units. Find the radius of the conductor.*

The capacity of the conductor is from the definition given by

$$C = \frac{50}{15 - 10} = \frac{50}{5} = 10,$$

and since the capacity of a spherical conductor in air is measured by its radius, the radius of the given conductor is 10 cm.

(2) *A spherical conductor of* 5 *cm. radius, and charged to a potential of* 100 *units, is placed inside a larger uncharged spherical conductor of* 10 *cm. radius, and made to touch its inner surface. Find the potential to which the larger conductor is raised (the medium is air).*

Here, *all* the charge from the smaller sphere passes to the larger. Hence we have

$$5 \times 100 = 10 \times V,$$

where V denotes the required potential of the larger sphere.

That is, $$10V = 500,$$

or $$V = 50 \text{ C.G.S. units.}$$

(3) *The smaller conductor of question 2, after having been discharged by contact with the inner surface of the larger conductor, is taken out and connected with the latter by a long thin wire. Find the common potential which the conductors attain, and the charge on each.*

Here, if V denote the common potential of the conductors, we have

$$V(5 + 10) = (50 \times 10) + 0.$$

That is $$15V = 500,$$

or $$V = \tfrac{500}{15} = 33{\cdot}3 \text{ units of potential.}$$

The charge on the larger conductor

$$= \tfrac{10}{15} \times 500 = 333{\cdot}3 \text{ units of quantity,}$$

and that on the smaller

$$= \tfrac{5}{15} \times 500 = 166{\cdot}7 \text{ units of quantity.}$$

(4) *Compare the forces between two small spheres charged to the same potentials (a) in air, (b) in a medium of specific inductive capacity K.*

If Q_1 and Q_2 denote the charges *in air*, and d the distance apart—

$$\textbf{Force in air} = \frac{Q_1 \times Q_2}{d^2}.$$

When in the medium the capacity of each is increased K times, so that the charge on each must be increased K times to bring it up to the *same potential* as it was in air, *i.e.* the charges must be KQ_1 and KQ_2 respectively. If the spheres were in air with these charges, the force between them would be—

$$\frac{KQ_1 \times KQ_2}{d^2};$$

but, as they are in a medium of specific inductive capacity K, the force is $\frac{1}{K}$ of this, *i.e.*

$$\textbf{Force in medium} = \frac{1}{K}\,\frac{KQ_1 \times KQ_2}{d^2} = K\frac{Q_1 \times Q_2}{d^2}.$$

Thus, *if the potentials are the same* in air and in the medium, the force is *K times greater in the medium.* If *the charges are the same* in air and in the medium the force is *K times less in the medium.* (Arts. 80, 95.)

85. Surface Density of Uniformly Charged Spheres. —Consider an isolated sphere of radius r cm., charged with Q units of electricity. The distribution is uniform; hence if ρ denote the surface density—

$$\rho = \frac{\text{Quantity}}{\text{Surface area}} = \frac{Q}{4\pi r^2},\ i.e.\ Q = 4\pi r^2 \rho,$$

and, if V be the potential,

$$V = Q/Kr,\ i.e.\ V = \frac{4\pi r\rho}{K}.$$

Again consider two spheres of radii r_1 and r_2 placed at a considerable distance apart, joined by a long fine wire, and charged. Let Q_1 and Q_2 be the charges, C_1 and C_2 the capacities, and V the common potential. Now

$$Q_1 = C_1 V,\ Q_2 = C_2 V;\ \therefore\ Q_1 : Q_2 = C_1 : C_2 = Kr_1 : Kr_2 = r_1 : r_2,$$

i.e. *the charges are directly as the radii.*

Again, if ρ_1 and ρ_2 denote the surface densities—

$$\rho_1 : \rho_2 = \frac{Q_1}{4\pi r_1^2} : \frac{Q_2}{4\pi r_2^2} = \frac{r_1}{r_1^2} : \frac{r_2}{r_2^2} = r_2 : r_1,$$

i.e. *the surface densities are inversely as the radii.*

86. Potential Energy of a Charge.—In Art. 82 we have considered the electrical potential at a point due to a given charge. This must not be confounded with the electrical potential energy of a charge placed at that point. The potential of a charged conductor is the equivalent of the work necessary to bring a unit positive charge from infinity up to the conductor, against the electric force exerted by the charge of the conductor, but the potential energy of the charge on the conductor is the equivalent of *all* the electrical work done in charging the conductor.

Consider a conductor charged with a positive charge Q to a potential V. That is, the quantity of electricity with which it is charged is denoted by Q, and the work that would have to be done to bring a positive unit of electricity from infinity up to its surface when so charged is denoted by V. Hence, the quantity of work necessary to

bring Q units of electricity from infinity up to the conductor, *supposing its potential to remain constant at* V during the process, is given by QV. This, however, is evidently not the work done in charging the conductor *up to potential* V, with Q units of electricity, for during this process the potential is not constant at V, but rises from zero to V *in proportion as the charge is increased*, the latter value being acquired when the charging is complete.

The *average* value of the potential during the process of charging is therefore $\frac{V}{2}$, and the work done is the same as if the potential remained constant at this value during the process. Hence the work done in charging is given by $\frac{1}{2}QV$, and this, therefore, expresses the potential energy of a charge Q at potential V. If Q be expressed in the electrostatic units of quantity (Art. 80), and V in the electrostatic units of potential, then the energy of the charge is expressed in *ergs*.

It is convenient to express the energy E in three forms; thus with Q, V, and C in electrostatic units:—

$$\begin{aligned} E &= \tfrac{1}{2}QV \text{ ergs} \\ &= \tfrac{1}{2}CV \times V = \tfrac{1}{2}CV^2 \text{ ergs} \\ &= \tfrac{1}{2}Q\frac{Q}{C} = \tfrac{1}{2}\frac{Q^2}{C} \text{ ergs.} \end{aligned}$$

The reader acquainted with the calculus will understand the neater proof, viz.—

$$E = \int_0^V QdV = \int_0^V CV.dV = C\int_0^V VdV. = \tfrac{1}{2}CV^2.$$

87. Loss of Energy on Sharing Charge.—We have seen that if two conductors, A and B, be charged, say, positively, A being at the higher potential, then on connecting them together charge passes from A to B until the two come to the same potential, *but the total quantity of electricity is the same before and after sharing;* we shall now show that, although there is no loss of charge, *there is always a loss of energy in such cases.*

Let C_1 and V_1 = capacity and potential of A.
Let C_2 and V_2 = capacity and potential of B.
V = common potential when joined.
Q = charge which passes from A to B.

The loss of the charge Q has lowered the potential of A from V_1 to V, and the gain of the charge Q has raised the potential of B from V_2 to V; hence—

$$C_1 = \frac{Q}{V_1 - V}, \quad i.e.\ Q = C_1(V_1 - V).$$

$$C_2 = \frac{Q}{V - V_2}, \quad i.e.\ Q = C_2(V - V_2).$$

$$\therefore\ C_1V_1 - C_1V = C_2V - C_2V_2,$$

$$i.e.\ V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}.$$

Original energy of $A + B = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$.

$$\text{Final energy of } A + B = \frac{1}{2}(C_1 + C_2)\left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)^2$$

$$= \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2}.$$

$\therefore$ Decrease in energy

$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2}$$

$$= \frac{1}{2}\frac{C_1C_2(V_1 - V_2)^2}{C_1 + C_2}.$$

This expression is always positive; hence *there is always a decrease in energy in such cases.* The loss appears as heat in the connecting wire or appears as a spark.

Example. *A conductor A charged to a potential of* 200 *units is made to share its charge with a spherical conductor B of* 20 *cm. radius. The potential of this conductor is then found to be* 100 *C.G.S. units. Find the capacity of A and the energy of its discharge before and after sharing its charge with the spherical conductor. Also find the energy lost when the charge is shared.*

If C denote the capacity of A, we have, since no electricity is lost during the sharing of the charge,

$$200\,C = 100\,C + (100 \times 20),$$

or

$$100\,C = 2000,$$

that is,

$$C = 20 \text{ units.}$$

The charge on A before sharing with the spherical conductor is given by

$$Q = 20 \times 200 = 4000 \text{ units of quantity,}$$

and the energy of its discharge would therefore be

$$\tfrac{1}{2} \times 4000 \times 200 = 400000 \text{ ergs.}$$

After sharing with the spherical conductor, A retains a charge equal to

$$\tfrac{20}{40} \times 4000 = 2000 \text{ units of quantity,}$$

and the energy of its discharge would now be

$$\tfrac{1}{2} \times 2000 \times 100 = 100000 \text{ ergs.}$$

The energy lost when the charge is shared will be given by the difference between the energy of the charge of A before sharing with the spherical conductor, and the sum of the energy of A after sharing and the energy of the charge on the spherical conductor. This latter charge is equal to $\frac{20}{40} \times 4000 = 2000$ units of quantity. Hence its energy is $\frac{1}{2} \times 2000 \times 100 = 100000$ ergs, and we therefore have

$$\begin{aligned} \text{Energy lost} &= 400000 - (100000 + 100000) \\ &= 400000 - 200000 \\ &= 200000 \text{ ergs.} \end{aligned}$$

88. Field Strength, Unit Field, Tubes of Force, and Induction.—It has been agreed that *the strength or intensity of an electric field at any point be defined as given in magnitude, direction and sense by the force (in dynes) on a positive electrostatic unit charge placed at that particular point*, it being assumed that the positive unit itself does not affect the field in which it is placed. Field is therefore a vector quantity, *i.e.* intensity can be resolved into components and follows the parallelogram law. **Unit electric field** may thus be defined as **that field which exerts a force of one dyne on a unit positive charge placed in it**, and a field of intensity, F units, is one which exerts a force of F dynes on a positive unit charge; if a charge of

q units be placed in a field of intensity F units, the force f on the charge q is given by the expression

$$f = Fq \text{ dynes.}$$

From the above it follows that the intensity of the electric field at distance r cm. from a point charge $+Q$ e.s. units, or at a distance r cm. from the centre of a uniformly charged sphere having $+Q$ e.s. units (r being greater than the radius of the sphere), is given by

$$F = \frac{Q}{r^2} \textbf{ e.s. units} \quad or \quad F = \frac{Q}{Kr^2} \textbf{ e.s. units,}$$

the medium in the first case being air, and, in the second, one of specific inductive capacity K. It will be remembered that the intensity of the field *inside* a charged conductor, *e.g.* the sphere mentioned above, is zero.

Further, to estimate the intensity of the field at a point due to a number of charges it is only necessary to imagine a *positive* unit charge placed there, to calculate the force on this unit due to each of the charges and then to find the resultant by the parallelogram law; the magnitude and direction of the resultant force in dynes will give the intensity of the field in electrostatic units. The electrostatic unit field has no particular name.

Example. *ABCD is a square of 2 cm. sides. Charges of* $+4$, -4 and $+8$ *electrostatic units are placed at the points A, C and D. The medium is air. Find the intensity of the field at B* (Fig. 193).

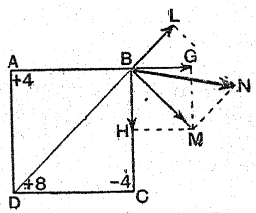

Fig. 193.

(1) Force at B due to $A = \dfrac{4 \times 1}{2^2}$

$= 1$ dyne in the direction AB.

Let BG represent this force.

(2) Force at B due to $C = \dfrac{-4 \times 1}{2^2} = 1$ dyne in the direction BC.

Let BH represent this force.

(3) Force at B due to $D = \dfrac{8 \times 1}{(\sqrt{8})^2} = 1$ dyne in the direction DB.

Let BL represent this force.

The resultant of (1) and (2) is $\sqrt{2}$ dynes represented by BM, and the resultant of this and (3) is represented by BN and is equal to $\sqrt{3}$ dynes (since $BN^2 = BM^2 + BL^2$); hence

Intensity at $B = \sqrt{3}$ e.s. units (and the direction is BN).

The warning given in Art. 18 may be repeated here; the meaning of the definition above is that if the force on a positive unit charge is *8 dynes* the intensity of the field is *8 C.G.S. electrostatic units of intensity.*

As in Art. 19, in the case of magnetic field and magnetic potential, so here the conception of electrical potential leads to another definition of the intensity of the electrical field. In Fig. 186 let V_1 = electrical potential at B, V_2 = electrical potential at C; the work required to move a positive unit from C to B is therefore $V_1 - V_2$ ergs. Suppose B and C near together, and let F = intensity of the field in the direction BC; the work required to move the positive unit from C to B is therefore $F \times CB$ ergs. Hence

$$F \times CB = V_1 - V_2,$$

$$\therefore \ F = \frac{V_1 - V_2}{CB}.$$

The right-hand side gives the **potential gradient** between B and C, hence—(a) **the intensity at a point in an electrical field is numerically equal to the potential gradient at that point,** (b) the intensity is greatest in those regions where the potential gradient is steepest, *i.e.* where the rate of variation of potential is greatest. Assuming B and C indefinitely close together, writing dV for $V_1 - V_2$, and dx for the small distance BC, we get the usual expression for the above, viz.

$$F = -\frac{dV}{dx},$$

the negative sign denoting, *e.g.*, that the potential *decreases* as the distance x from the *positive* charge at A (Fig. 186) *increases*.

Example. *A charge of* + 10 *e.s. units moves along a line of force in a uniform field from a point A to a point B. If the distance AB be* 4 *cm. and the work done be* 1500 *ergs, find the P.D. between A and B and the intensity of the field.*

Work done in moving 10 units $= 1500$ ergs.

$\therefore$ Work done in moving unit quantity $= 150$ ergs.

$$\therefore \textbf{P.D.} = V_1 - V_2 = \mathbf{150} \text{ e.s. units.}$$

$$\text{Potential gradient} = \frac{V_1 - V_2}{4} = \tfrac{150}{4} = 37{\cdot}5.$$

$$\textbf{Intensity} = F = \mathbf{37{\cdot}5} \text{ e.s. units;}$$

or again

Let $f =$ mechanical force on the charge (in dynes),

$\therefore f = 10\,F$ dynes.

Hence

work done $=$ force $\times$ distance $= (10\,F \times 4)$ ergs.

But

work done $= 1500$ ergs.

$\therefore 10\,F \times 4 = 1500$

i.e. $F = \frac{1500}{40} = 37{\cdot}5$ **e.s. units.**

The conception of lines and tubes of force and induction referred to in previous pages leads to yet another way of defining the intensity of an electrical field. Imagine an electrical field permeated with lines, and further imagine the lines to be grouped into tubes which touch each other laterally and fill the entire space. As has been indicated, when these are conceived on a certain definite plan, so that a definite number are imagined to emanate from a definite charge, they are referred to as **unit tubes.** In electrostatics the following unit tubes are employed:—

(1) *Maxwell Unit Tubes of Force.*—Consider a charge of Q electrostatic units in a medium of specific inductive capacity K, and imagine a spherical surface of radius r cm. drawn in the field, the charge Q being at the centre; the intensity of the field at any point on the surface of this imaginary sphere is Q/Kr^2 C.G.S. units. Now imagine the lines grouped into tubes in such a way that $\frac{4\pi Q}{K}$ unit tubes emanate from the charge Q (or in other words that $\frac{4\pi}{K}$ unit tubes emanate from unit charge) in this medium; these are called **Maxwell unit tubes of force.**

These tubes all pass through the surface of the imaginary

sphere, and since the area of this spherical surface is $4\pi r^2$ it follows that

$$\left.\begin{array}{r}\text{Number of Maxwell Tubes of}\\ \text{Force per unit area}\end{array}\right\} = \frac{4\pi Q}{K \, . \, 4\pi r^2} = \frac{Q}{Kr^2}.$$

But this last expression gives the intensity of the field in electrostatic units; hence, the intensity in C.G.S. units is represented numerically by the number of Maxwell tubes of force per unit area. In air K is taken as unity and the number of Maxwell tubes from a charge Q is $4\pi Q$, and the number from unit charge is 4π.

To summarise—

(a) $4\pi Q/K$ Maxwell tubes of force emanate from a charge Q (and, therefore, $4\pi/K$ from unit charge) in a medium of specific inductive capacity K. In air these expressions become $4\pi Q$ and 4π respectively, since K is unity.

(b) The intensity of a field at any point is represented numerically by the number of Maxwell tubes of force per unit area taken perpendicular to their direction at that point. Hence—

(c) *Unit electrical field may be defined as that field which has one Maxwell tube of force per unit area* taken perpendicular to the direction of the tube.

Note that the field intensity (Q/Kr^2) multiplied by the cross-section of the Maxwell unit tube at the point in question ($K \, . \, 4\pi r^2/4\pi Q = Kr^2/Q$) is *unity*; this was shown to be the case with the unit tubes of force in magnetic theory.

(2) *Unit Tubes of Induction.*—Consider now the inductive action in the field so that our lines are now referred to as lines of induction. Imagine the lines grouped into tubes in such a way that $4\pi Q$ unit tubes emanate from the charge Q *whatever the medium* (*i.e.* 4π unit tubes emanate from unit charge); these are called **unit tubes of induction.** These all pass through the surface of the imaginary sphere; hence

$$\left.\begin{array}{r}\text{Number of Tubes of In-}\\ \text{duction per unit area}\end{array}\right\} = \frac{4\pi Q}{4\pi r^2} = \frac{Q}{r^2} = K\left(\frac{Q}{Kr^2}\right).$$

But Q/Kr^2 gives the number of Maxwell tubes of force per unit area; hence

$$\left.\begin{array}{r}\text{Number of Tubes of In-}\\\text{duction per unit area}\end{array}\right\} = \left\{ K \times \begin{array}{r}\text{Number of Tubes of}\\\text{Force per unit area,}\end{array}\right.$$

i.e.

$$\text{Induction} = K \times \text{Field}$$

and

$$\text{Field} = \frac{1}{K} \times \text{Induction}.$$

In air K is taken as unity, and the number of unit tubes of induction, and the number of unit tubes of force, are the same [compare this with magnetic theory where $B = H$ (for air) and $B = \mu H$ (for other media)].

To summarise—

(*a*) $4\pi Q$ tubes of induction emanate from a charge Q and 4π tubes of induction from unit charge.

(*b*) The number of tubes of induction per unit area $= K \times$ the number of Maxwell tubes of force per unit area (perpendicular to their direction); hence, Induction $= K \times$ Field, and Field $=$ Induction $\div K$.

(*c*) The intensity of a field at any point may be said to be represented numerically by the number of tubes of induction per unit area divided by K.

(3) *Faraday Unit Tubes.*—Let the lines be grouped into tubes in such a way that Q unit tubes always emanate from a charge Q, and therefore one unit tube from unit charge; these are called **Faraday unit tubes.** These all pass through the imaginary sphere of radius r; hence

$$\left.\begin{array}{c}\text{Number of Faraday Tubes}\\\text{per unit area}\end{array}\right\} = \frac{Q}{4\pi r^2} = \frac{K}{4\pi}\left(\frac{Q}{Kr^2}\right).$$

But Q/Kr^2 gives the intensity F of the field; hence, if D denote the number of Faraday tubes per unit area—

$$D = \frac{K}{4\pi}F. \qquad \therefore F = \frac{4\pi D}{K},$$

and the intensity of a field is therefore represented numerically by the number of Faraday tubes per unit area multiplied by $4\pi/K$; in air this expression becomes 4π. Clearly also every Faraday unit tube contains $4\pi/K$ Maxwell tubes of force.

Again—

$$\left.\begin{array}{c}\text{Number of Faraday Tubes}\\ \text{per unit area}\end{array}\right\} = \frac{Q}{4\pi r^2} = \frac{1}{4\pi}\left(\frac{Q}{r^2}\right).$$

But Q/r^2 gives the number of tubes of induction per unit area; hence, denoting this by N, $D = N/4\pi$. $\therefore N = 4\pi D$, and the induction is therefore represented numerically by the number of Faraday tubes per unit area multiplied by 4π. Clearly also every Faraday unit tube contains 4π tubes of induction.

To summarise—

(*a*) Q Faraday unit tubes emanate from a charge Q and therefore one unit tube emanates from unit charge.

(*b*) Since $F = 4\pi D/K$, the *intensity* of a field is represented numerically by the number of Faraday unit tubes per unit area multiplied by $4\pi/K$.

(*c*) Since $N = 4\pi D$ the *induction* is represented numerically by the number of Faraday unit tubes per unit area multiplied by 4π.

Note that the field intensity (Q/Kr^2) multiplied by the cross-section of the Faraday unit tube at the point $(4\pi r^2/Q)$ is $4\pi/K$.

It may be mentioned that Maxwell tubes of force are connected with electrical force in the same way as tubes of induction are connected with induction, and in the same way as Faraday tubes are connected with a quantity called the **polarisation** of the dielectric or the **displacement** in the dielectric or the **electric strain.**

Maxwell used the expression "displacement" to mean the electricity which crosses *unit area* of a dielectric owing to the electric intensity at that point. In a conductor the charge would continue to move as long as the electric force acted, but in a dielectric the displacement soon reaches a limiting value proportional to the force producing it. Thus, considering any section of a tube, it will be polarised, *i.e.* opposite sides will be oppositely charged. Throughout the tube the face of each section will be neutralised by the opposite charge on the adjoining face of the next section so that only the ends of the tube where it reaches the "charged" conductors will exhibit "free charges." The amount of

electricity gathering on *unit area* of the conductors, *i.e.* the density ρ, may therefore be taken as a measure of the polarisation or displacement or strain just outside the conductors.

In the same way the polarisation, displacement or strain *at any point in a medium* is measured by the density ρ on a conducting surface placed at that point, it being assumed that its insertion there does not alter the existing polarisation, displacement or strain. But the charge ρ per unit area is equal to the number of Faraday tubes D per unit area; hence, D at any point is a measure of the polarisation, displacement or strain (P) at that point, *i.e. the polarisation, displacement or strain is represented numerically by the number of Faraday tubes D per unit area.*

In magnetic theory only Maxwell tubes are used, but in electrostatic theory Faraday tubes are most useful.

89. Normal Induction over a Surface in an Electric Field. Gauss's Theorem.—Imagine any closed surface in an electric field and consider a small area a containing a given point. Let F denote the *electrical intensity* at this point, and let α be the angle between the direction of the force and the *outward* drawn normal to the surface a at the given point (Fig. 194). The component of the force along the outward drawn normal is clearly $F \cos \alpha$, and since induction $= K \times$ intensity, the *induction* in this direction is $KF \cos \alpha$. The product $KF \cos \alpha \times a$ is the flow of induction across the small area a. The total flow of induction or the **total normal induction** over the whole closed surface is obtained by supposing the whole surface to be divided up into a very large number of small areas such as a, and summing up the values of $KF \cos \alpha \,.\, a$ for all these areas, *i.e.* **the total normal induction is the surface integral of the quantity $KF \cos \alpha \,.\, a$ over the whole surface.** Writing N' for the total normal induction we have

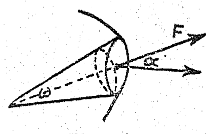

Fig. 194

$$N' = K \Sigma F \cos \alpha \,.\, a.$$

Now let Fig. 195 represent a closed surface in the field of a point charge Q at O (the surface, of course, *completely* surrounds the charge at O) and consider a small area a at P; then denoting OP by r—

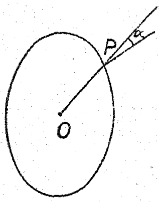

Fig. 195.

Normal Induction over a

$$= KF \cos \alpha \,.\, a$$
$$= K \frac{Q}{Kr^2} \cos \alpha \,.\, a$$
$$= Q \frac{\cos \alpha \,.\, a}{r^2}.$$

But $\cos \alpha \,.\, a/r^2$ is the solid angle w subtended at the point O by the area a; hence

Normal Induction over $a = Qw$.

$$\therefore \left.\begin{array}{c}\text{Total Normal Induction for}\\ \text{the whole closed surface}\end{array}\right\} = \Sigma Qw = Q\Sigma w,$$

$$i.e.\ N' = 4\pi Q,$$

for Σw is the solid angle subtended at O by the whole closed surface and is equal to 4π; hence the *total normal induction over the closed surface is numerically equal to* 4π *times the charge inside.*

If the point O be *without* the closed surface (Fig. 196) then a straight line, $O\,P'P$, drawn from O will cut the surface at *two* points P' and P, and if a cone with a very small solid angle ω at O be drawn with $O\,P'P$ as axis, the normal induction over the small area defined on the surface by intersection with the cone at P is measured by $Q\omega$, as above, and the induction over the corresponding area at P' is measured by $-\,Q\omega$, the minus sign being used because the direction of the normal component at P' is opposite to that of the *outward* drawn normal at that point. Hence the normal induction over the two surfaces at P and P' is zero, and this is true for all the cones that

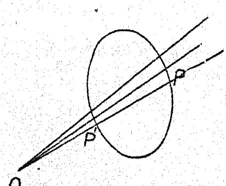

Fig. 196.

may be drawn from O through the surface. It follows that the total normal induction over the surface is zero, and as, in this case, the total quantity of electricity within the surface is zero, this result is in accordance with the theorem.

If the charge Q is not a point charge, that is, a charge supposed to be concentrated at a point, but a charge distributed over the surface of a conductor or over a number of conductors, it can be shown that the theorem still holds. For, if the charge Q is, say, on a conductor *inside* the surface, then the charge may be divided into a number of elementary charges q_1, q_2, q_3, etc., at contiguous points on the surface of the conductor, and the total normal induction over the surface for these elementary charges is $4\pi q_1$, $4\pi q_2$, $4\pi q_3$, etc., and therefore the total normal induction for the whole charge, Q, is $4\pi q_1 + 4\pi q_2 + 4\pi q_3 +$ etc. or $4\pi q_1 + q_2 + q_3 +$ etc.), or $4\pi\Sigma(q)$, that is, $4\pi Q$, which is the required result. Similarly for a charge, Q, on a conductor *outside* the surface the total normal induction over the surface is zero.

Further, if the surface be re-entrant (Fig. 197) it will be readily seen that the theorem still holds. Thus, if the charge be inside, every small cone from O cuts the surface at least once, and if it cuts more than once it will do so an odd number of times (three in Fig. 197 (a)); hence the *extra* cuts will be even (two in Fig. 197 (a)), and half of these will represent cones passing into the surface, the other half will represent cones passing out so that these will balance each other. If the charge be outside, every small cone from O cuts the surface an even number of times (Fig. 197 (b)), each entering the surface as many times as it leaves, so that the total normal induction over the surface is zero, just as in Fig. 196.

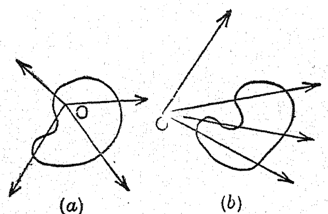

Fig. 197.

To summarise—**The total normal induction over a**

closed surface drawn in an electric field is numerically equal to 4π times the total charge inside; if there is no charge inside the normal induction over the surface is zero. This is known as **Gauss's Theorem.**

If there are both positive and negative charges the "total charge inside" refers to the *algebraic sum* of the charges inside.

We have seen (Art. 88) that induction = $K \times$ intensity. In air K is taken as unity; hence *for this medium* induction and intensity coincide, and, therefore, *for this medium* the reader will come across the statement "the surface integral *of the electric intensity* taken over any closed surface is 4π times the total charge inside." In another medium intensity = induction $\div K$, and, therefore, "the surface integral *of the electric intensity* is $\frac{4\pi}{K}$ times the total charge inside." In *all* media the "total normal induction" is, however, 4π times the total charge inside. These points are often confused even in text-books.

Again, remembering that $N = 4\pi D$ and, therefore, that the induction N must be divided by 4π to give the strain D, it follows that "*the total normal polarisation, displacement or strain over a closed surface is equal to* $4\pi Q$ *divided by* 4π, *i.e. is equal to the total charge* Q *inside.*"

In Arts. 90-96 important applications of Gauss's theorem are dealt with.

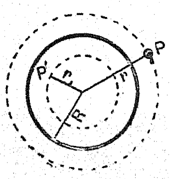

Fig. 198.

90. Uniformly Charged Sphere.—Let Q be the charge on the sphere and let P be an *external* point at which the electric force or intensity of the field due to the sphere is required (Fig. 198). Take as the closed surface a sphere through P concentric with the charged sphere. By symmetry the intensity F is the same at every point of this surface and it is along the outward normal.

Hence—

$$\text{Total Normal Induction} = KF \times \text{Area}$$
$$= 4\pi r^2 KF.$$

But „ „ „ $= 4\pi Q$

$$\therefore 4\pi r^2 KF = 4\pi Q,$$

i.e. $$F = \frac{Q}{Kr^2},$$

thus *the field at an external point* P *due to a uniformly charged sphere is the same as if the charge on the sphere were concentrated at the centre.* The potential at P is, therefore, Q/Kr and the potential of the sphere Q/KR, as previously indicated.

Again, let P' be an *internal* point (Fig. 198) and take as the closed surface a concentric sphere through P'. If F' be the intensity at P', then, as above, we have

$$\text{Total Normal Induction} = 4\pi r_1{}^2 KF'$$

and „ „ „ $= 4\pi Q = O,$

for there is no charge Q *inside* the surface (it is on the outside of the sphere); hence

$$4\pi r_1{}^2 KF' = O, \quad i.e. \; F' = O,$$

thus *the electric force inside a uniformly charged sphere is zero*; there is, therefore, no potential gradient, *i.e. the potential inside is uniform* (it is the same as the potential of the charged sphere itself, viz. Q/KR).

For a point just outside the sphere, *i.e.* indefinitely near the outer surface, we have

$$F = \frac{Q}{KR^2} = \frac{4\pi R^2 \rho}{KR^2} = \frac{4\pi\rho}{K},$$

where ρ is the surface density. This is a special case of *Coulomb's Law*, viz. that the intensity just outside a charged surface *of any form* is $\frac{4\pi}{K}$ times the surface density (Art. 94).

91. Uniformly Charged Infinite Plane.—To apply Gauss's theorem imagine a small closed cylindrical surface in the field, bounded by plane ends, parallel to the electrified surface, and having its cylindrical surface normal to the surface. If this closed surface lies wholly on one side of the electrified surface, as in Fig. 199, the total normal induction over the surface is given by $KFa - KF'a$, where F is the electric force at any point in the end, A, of the closed surface, F' the electric force at any point in the end, B, and a the area of the ends. At A the direction of the

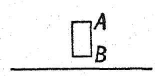

Fig. 199.

force is along the outward drawn normal, and Fa is, therefore, positive, but at B the direction of the force is along the normal inwards, and, therefore, $F'a$ is negative. The flow of induction across the cylindrical surface is zero, for the direction of the force at every point in the surface lies in the surface, being everywhere normal to the electrified plane surface. Hence the total normal induction over the whole closed surface is given, as stated above, by $KFa - KF'a$.

But, by Gauss's theorem, the total normal induction is in this case zero, for the closed surface contains no electricity. Hence we get $Fa - F'a = 0$, and, therefore, $F = F'$, that is, *the magnitude of the electric force is the same at all points in the field*, and, as has already been assumed from symmetry, *its direction is everywhere normal to the electrified surface.*

If the closed cylindrical surface lies, as in Fig. 200, with the ends A and B on opposite sides of the electrified surface, then the total normal induction over the surface is evidently given by $KFa + KFa$ or $2KFa$, where F denotes the electric force at any point in the field, and a the area of the ends of the cylindrical surface. In this case, however, the closed surface encloses the quantity of electricity distributed on the portion of the electrified plane intercepted within the cylindrical wall of the surface, and, by Gauss's theorem, the total normal induction over the surface is equal to 4π times this quantity. If ρ be the surface density, the charge inside the cylinder is ρa; hence, applying Gauss's theorem—

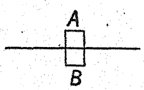

Fig. 200.

$$2KFa = 4\pi\rho a,$$

$$\therefore F = \frac{2\pi\rho}{K}.$$

It should be noted that ρ is the surface density *on both sides taken together*. If ρ be taken as the charge per unit area on one side only, the charge inside the cylinder is $2\rho a$; hence $2KFa = 4\pi \times 2\rho a$, *i.e.*

$$F = \frac{4\pi\rho}{K}.$$

92. Uniformly Charged Infinite Cylinder.—To find the magnitude of the force at a point, P (Fig. 201), outside the surface imagine a closed cylindrical surface, coaxial with the electrified surface, described through P, and having plane ends at right angles to the axis. The force at any point in the field is, by symmetry, the same for all points at the same distance from the axis, and is everywhere normal to the axis. Hence, the total normal induction over this closed surface is $2\pi rlKF$, where r denotes the distance of P from the axis, l the length of the closed surface parallel to the axis, and F the magnitude of the electric force at any point at a distance r from the axis. If Q denote the charge per unit length on the electrified cylindrical surface, then Ql denotes the quantity of electricity enclosed within the closed surface, and, by Gauss's theorem, the total normal induction over the surface is given by $4\pi Ql$. Hence, we have

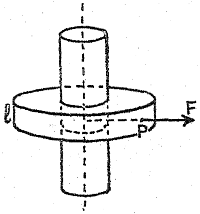

Fig. 201.

$$2\pi rlKF = 4\pi Ql,$$

$$\therefore\ F = \frac{2Q}{Kr}.$$

If P be infinitely close to the surface, and ρ be the surface density, $Q = 2\pi r\rho$; hence

$$F = \frac{4\pi\rho}{K}.$$

93. Potential Difference and Electrical Force inside a Closed Charged Conductor.—Experimentally it has been shown that the potential is *constant* (equal to that of the conductor), and the force is *zero* inside a closed charged conductor if there are no charges inside, and the same has been indicated from theoretical considerations in special cases.

The general case may be established as follows: the surface of any charged conductor is necessarily an equi-

potential surface, and therefore the direction of the electric force at any point on the surface is normal to the surface at that point. If the interior of the conductor is everywhere at the potential of the surface it is evident that the electric force inside the conductor is zero, for, as no work would be done in moving a unit charge from one point to another inside the conductor, the electric force must everywhere inside the conductor be zero.

If, however, the force be supposed not to be zero, it ought to be possible to draw equipotential surfaces inside the surface of the conductor. Imagine such an equipotential to be drawn very close to the surface of the conductor. Its potential must be higher or lower than that of the surface, and work must be done in moving electricity inwards or outwards from one surface to the other. That is, the normal electric force between the surfaces and inside the charged surface is not zero and its direction is either outwards or inwards, and therefore the total normal induction over the surface is a finite positive or negative and not a zero quantity. But the total normal induction over the surface is equal to 4π times the quantity of electricity inside the surface, and if this quantity is zero the induction must also be zero, and cannot have a finite positive or negative value. That is, *there can be no difference of potential inside a closed charged surface which contains no charge, and therefore the electric force inside this charged surface is zero.*

94. Coulomb's Law.—Special cases of this have already been established. The law states that the magnitude of the electric force at any point infinitely close to the surface of a charged conductor, *surrounded by air*, is equal to $4\pi\rho$, where ρ is the density of the charge at the point, and its direction is, at every point, normal to the surface of the conductor at that point. The surface of the conductor is an equipotential surface, and therefore the direction of the force at any point must be normal to the surface at that point.

To determine the magnitude of the force at a point, P (Fig. 202), infinitely close to the charged surface, imagine

a very small closed surface placed as AB, with its ends A and B infinitely close to and parallel to the surface at the point considered, one end inside and the other outside the charged surface, and bounded laterally by a tubular surface determined by lines drawn normal to the charged surface through all points on the boundary of each of the ends. Then if F denote the electric force at P, and a the area of the ends of the closed surface, and also, since the three surfaces are infinitely close together, of the portion of the electrified surface intercepted within the closed surface, the total normal induction over the surface is evidently Fa, and this by Gauss's theorem is equal to $4\pi a\rho$, and therefore

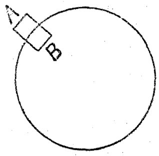

Fig. 202.

$$Fa = 4\pi a\rho,$$

or

$$F = 4\pi\rho.$$

This is a general result, true for a closed charged conductor of any form. If the surrounding medium be not air the total normal induction in the above is not Fa, but KFa, and $F = 4\pi\rho/K$.

95. Miscellaneous Problems and Theorems based on the Preceding.

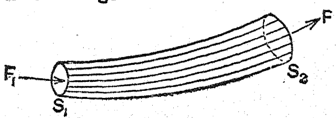

Fig. 203.

(1) *In a tube of force the intensity is inversely as the cross section taken at right angles to the tube.*

Let S_2 and S_1 be the two normal cross sections (Fig. 203). Let F and F_1 be the intensities at these sections.

Normal induction at $S_2 = KFS_2$ (outwards).
Normal induction at $S_1 = -KF_1S_1$ (inwards).
The curved sides are formed by lines of force, hence the induction *normal* to these is zero.

$\therefore$ Total normal induction for the portion of the tube shown $= KFS_2 - KF_1S_1$.

But this is equal to $4\pi Q$, *i.e. zero*, since there is no charge Q inside.

$$\therefore KFS_2 - KF_1S_1 = 0,$$

i.e.

$$FS_2 = F_1S_1 \quad or \quad \frac{F}{F_1} = \frac{S_1}{S_2}.$$

(2) *The negative charge on which a tube terminates is equal to the positive charge from which it starts.*

Let the tube be "cut off" just outside the charges.

By Coulomb's Law, at the positive end $F = 4\pi\rho/K$, *i.e.* $KF = 4\pi\rho$.

If S be the sectional area at this end

$$KFS = 4\pi\rho S.$$

Similarly, at the negative end $KF_1 = -\ 4\pi\rho_1$,

$$\therefore\ KF_1S_1 = -\ 4\pi\rho_1S_1,$$

where F_1, S_1, ρ_1, denote the intensity, sectional area, and density at the negative end.

But from (1) above $KFS = KF_1S_1$, $\therefore\ 4\pi\rho S = -\ 4\pi\rho_1S_1$,

i.e. $$\rho S = -\ \rho_1S_1,$$

and ρS is the charge at the positive end, and $-\ \rho_1S_1$ the charge at the negative end.

(3) *The intensity at any point is numerically equal to* $\frac{4\pi}{K}$ *times the number of Faraday tubes per unit area at that point.*

This follows at once from the definitions of Art. 88, but the following proof is interesting.

Let Fig. 204 represent a bundle of n Faraday tubes emanating from the area a of the charged sphere, and let a' be a normal cross section, the intensities at a and a' being F and F' as indicated.

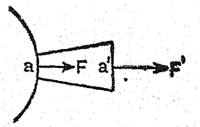

Fig. 204.

From (1) $Fa = F'a'$. But $F = \frac{4\pi\rho}{K}$,

$$\therefore\ F'a' = \frac{4\pi\rho a}{K}.$$

Now $\rho a =$ the charge on the area a, and since one Faraday tube emanates from unit charge, $\rho a = n =$ the number of Faraday tubes in the bundle,

i.e. $$F'a' = \frac{4\pi n}{K}, \quad \therefore\ F' = \frac{4\pi}{K}\frac{n}{a'}$$

or $$F' = \frac{4\pi D}{K},$$

for n/a' is the number (D) of Faraday tubes per unit area at the end a'.

(4) *There are two infinite parallel plane conductors at potentials* V_1 *and* V_2. *The medium between them consists of two parts of thicknesses* a *and* b, *and specific inductive capacities* K_1 *and* K_2. *If* ρ *be the density of the charges show that*

$$\rho = \pm\ \frac{(V_1 - V_2)K_1K_2}{4\pi(K_2a + K_1b)}.$$

By Art. 88 $4\pi\rho$ Maxwell tubes of induction emanate from a charge ρ and $4\pi\rho/K$ Maxwell tubes of force ; but ρ denotes charge per unit area, so that $4\pi\rho/K$ gives the Maxwell tubes of force per unit area, and this therefore measures the intensity.

The two fields in question have therefore intensities of $4\pi\rho/K_1$ and $4\pi\rho/K_2$, respectively. The work done in moving unit charge from one plate to the other is therefore (work = force $\times$ distance)—

$$\left(\frac{4\pi\rho}{K_1}a + \frac{4\pi\rho}{K_2}b\right) \text{ ergs.}$$

But the work in ergs also measures the potential difference in e.s. units,

$$\therefore V_1 - V_2 = \frac{4\pi\rho}{K_1}a + \frac{4\pi\rho}{K_2}b$$

$$= 4\pi\rho\left(\frac{K_2a + K_1b}{K_1K_2}\right)$$

$$\therefore \rho = \frac{(V_1 - V_2)K_1K_2}{4\pi(K_2a + K_1b)}.$$

(5) Consider, say, two charged conductors isolated from all external sources of electricity. As the potential energy of a system always tends to a minimum this system will tend to undergo any displacement which is associated with a decrease in the potential energy of the system, *the charges meanwhile remaining constant*, and the work necessary to bring about the displacement is provided by the loss of potential energy in the system. If, however, the conductors are joined to some external source of electricity, so that during any displacement *the potentials remain constant*, then, on displacement, there is an increase in the potential energy of the system, and the external source provides not only this increase but also the energy equivalent to the work necessary to bring about the displacement. Two important theorems are met with in connection with this :—

(a) *When two (or more) conductors are maintained at a constant potential by means of an external source of electricity, then, in any displacement, the energy supplied by the external source is double the increase in the energy of the system.*

Let Q_1 and Q_2 = the charges, V_1 and V_2 = the potentials,

$$\therefore \text{Energy} = \tfrac{1}{2}Q_1V_1 + \tfrac{1}{2}Q_2V_2 = E_1.$$

Let a displacement take place *at constant potentials* so that the charges become $Q_1 + q_1$ and $Q_2 + q_2$.

$$\therefore \text{Energy now} = \tfrac{1}{2}(Q_1 + q_1)V_1 + \tfrac{1}{2}(Q_2 + q_2)V_2 = E_2.$$

Hence

$$\text{Increase in energy} = E_2 - E_1$$

$$= \tfrac{1}{2}q_1V_1 + \tfrac{1}{2}q_2V_2 = \tfrac{1}{2}(q_1V_1 + q_2V_2).$$

Now the external source has supplied a charge q_1 at potential V_1 and a charge q_2 at potential V_2, and since potential is measured by the energy expended in moving unit charge—

$$\text{Energy supplied by external source} = q_1V_1 + q_2V_2.$$

This is double the increase in energy $\frac{1}{2}(q_1V_1 + q_2V_2)$, which proves the theorem.

(b) *For a very small displacement the loss of energy at constant charges is equal to the gain of energy at constant potentials.*

Let Q_1, Q_2, and V_1, V_2, be the initial charges and potentials of the conductors of the system. If, for a small displacement with constant charges, the potentials change to V'_1, V'_2, then the loss of potential energy is measured by

$$\tfrac{1}{2}Q_1(V_1 - V'_1) + \tfrac{1}{2}Q_2(V_2 - V'_2).$$

Similarly, if for the same small displacement with constant potentials the charges change to Q'_1, Q'_2, then the gain of potential energy is measured by

$$\tfrac{1}{2}V_1(Q'_1 - Q_1) + \tfrac{1}{2}V_2(Q'_2 - Q_2).$$

The difference between the expression for the loss of energy in the one case and the gain of energy in the other case is equal to

$$\tfrac{1}{2}\{\Sigma(Q - Q')(V - V')\} + \tfrac{1}{2}\{\Sigma(QV - Q'V')\},$$

where Q denotes the first charges (Q_1 and Q_2), Q' the second charges (Q'_1 and Q'_2), etc.

Now, if the displacement is very small, the quantities $(Q - Q')$ and $(V - V')$ denoting the changes in the charges and potentials resulting from the displacement are very small quantities, and therefore their product is negligible; that is, the first term of the above expression is negligibly small. Again the quantities Q, V' and Q', V are for the same conductor in the same configuration of the system, and therefore $\frac{Q}{V'} = \frac{Q'}{V}$ or $QV = Q'V'$, for the capacity of any conductor is the same when the configuration of the system is the same. Hence $QV - Q'V' = 0$, and therefore the second term of the above expression is zero—whether the displacement be large or small.

Hence, the difference between the expressions for the change of energy is negligibly small if the displacement is small, and therefore with this condition the loss of energy with constant charges is equal to the gain of energy with constant potentials, and each quantity is equal to the mechanical work done during displacement. The theory of the quadrant electrometer given in Art. 123 illustrates the application of this principle.

(6) *Explain the appearance of K in the denominator of the expression*

$$f = \frac{1}{K} \cdot \frac{qq'}{d^2}.$$

Consider a spherical surface of radius d round the point charge q. The polarisation of the medium normal to the spherical surface is uniform over the whole surface, and if P denote the normal polarisation at any point then $4\pi d^2 P$ gives the total normal polarisation over the surface. But it has been shown that the total normal polarisation over a closed surface is equal to the total quantity of electricity enclosed by the surface, and here, therefore, we have $4\pi d^2 P = q$, or $P = \frac{q}{4\pi d^2}$. But if F denote the electric force along the normal at any point on the surface, then, since $F = \frac{4\pi}{K} P$, we have

$$F = \frac{4\pi q}{4\pi K d^2} = \frac{1}{K} \cdot \frac{q}{d^2}.$$

That is, the electric force at any point at a distance d from a point charge q in a medium of specific inductive capacity K is given by $F = \frac{1}{K} \cdot \frac{q}{d^2}$, and therefore the force exerted on a charge q' placed at this point is measured by

$$F = \frac{1}{K} \cdot \frac{qq'}{d^2},$$

as stated in Art. 80.

It follows from this that if F be the electric force at any point in a field in air, then the force at the same point, when air is replaced by a dielectric of specific inductive capacity K (the charges in the field being supposed constant), is F/K, and under the same conditions, if the potential at the point is V in air it will be V/K in a dielectric of specific inductive capacity K. If, on the other hand, conductors in the field are maintained at constant potential instead of at constant charge, then the values F and q, the electric force and charge in air, change to KF and Kq in a dielectric of specific inductive capacity K. (See worked example page 236.)

96. Mechanical Force per unit of Surface Area of a Charged Conductor.—The electric force at any point in air, very near but *outside* the surface of the charged conductor, is $4\pi\rho$, where ρ is the surface density at that point. If the point be very near but *inside* the charged surface the force is zero. Now for a point outside the charged surface the force may be considered as made up of two parts, F_1 and F_2, F_1 being due to the charge on a very small area round the point, and F_2 to the charge on the rest of the surface. Similarly, if the point is just inside the charged surface the value of F_2 is practically un-

changed, but that of F_1 is reversed in direction. Hence, we get

$$F_1 + F_2 = 4\pi\rho$$

and
$$-F_1 + F_2 = 0,$$

and, therefore,
$$F_1 = F_2 = 2\pi\rho.$$

If a denote the area of the very small surface carrying the charge to which F_1 is due, then the charge on this surface is $a\rho$, and the total force experienced by this, due to the charge on the rest of the surface, is $2\pi\rho \,.\, a\rho = 2\pi a\rho^2$. That is, the mechanical force per unit of area is $2\pi\rho^2$. Since the surface is an equipotential one the direction of the force is outwards along the normal to the surface. Hence the surface of a charged conductor is subject to an outward tension, and the tension per unit area at any point where the surface density is ρ, and the surrounding medium is air, is given by

$$\textbf{Pull outwards per unit area} = 2\pi\rho^2$$

$$= 2\pi\left(\frac{F}{4\pi}\right)^2 = \frac{F^2}{8\pi} = 2\pi D^2 \text{ (dynes per sq. c.m.)}.$$

With a medium of specific inductive capacity K these expressions become

$$\frac{2\pi\rho^2}{K} = \frac{2\pi}{K}\left(\frac{KF}{4\pi}\right)^2 = \frac{KF^2}{8\pi} = \frac{2\pi D^2}{K}.$$

The greatest value of ρ possible in air is about 8 in C.G.S. units, so that the maximum value of $2\pi\rho^2$ is about 400 dynes per square centimetre, or, taking an atmospheric pressure to be 10^6 dynes per square centimetre, about the $\frac{1}{2500}$th part of an atmospheric pressure; if these be exceeded the charge begins to escape into the air. This outward pressure may be shown by gradually charging a soap-bubble, when it will be noted that electrification produces an increase in size.

97. Energy per Unit Volume of the Medium.—It has been indicated that the medium surrounding the charged bodies is really the seat of the energy, which may be assumed to exist in it as energy of strain, the

"charges" on the conductors being simply surface indications of this change in the condition of the medium (Art. 77). An expression for the energy per unit volume of the medium is readily obtained.

In the preceding section it has been shown that the force on unit area of a charged surface is $KF^2/8\pi$ dynes (outwards). Imagine this unit area to be moved in the opposite direction to this force by a small amount, dx; the work done is $(KF^2/8\pi) \times dx$ ergs; and the increase in the volume of the field, *i.e.* the volume swept out by the unit area, is dx c.cm. The work done in producing this volume of field is, therefore, $(KF^2/8\pi) \times dx$ ergs; hence the work done in producing unit volume of field, which is, therefore, the energy per unit volume of the medium, is $KF^2/8\pi$ ergs, *i.e.* denoting this by E,

$$E = \frac{KF^2}{8\pi} \text{ (ergs per c.cm.)},$$

or, since $F = 4\pi D/K$, where D is the number of Faraday tubes per unit area, we have

$$\boldsymbol{E = \frac{KF^2}{8\pi} = \frac{2\pi D^2}{K} = \tfrac{1}{2}FD} \text{ (ergs per c.cm.)}.$$

From analogy with the corresponding problem in elasticity, viz. energy per unit volume $= \frac{1}{2}$ stress $\times$ strain, if F be of the nature of stress, D is the corresponding strain.

Again, in dealing with the distribution of the energy throughout the medium, it is convenient to associate it with Faraday's tubes. Taking a complete unit tube in the field, it will have a unit positive charge at one end and a unit negative charge at the other end. Let V and V' denote the potentials of the two corresponding ends of the tube. The energy of these unit charges is measured by $\frac{1}{2}V - \frac{1}{2}V'$ or $\frac{1}{2}(V - V')$, and this energy may be supposed to be in the portion of the medium bounded by the tube. That is, the energy located in each tube is measured by one half the difference of potential between the ends of the tube. This difference of potential may be written as $\Sigma F \, . \, d$, where F is the electric force at any point on the axis of the tube, and d a very short length of the tube taken at that point, the summation to extend from one

end of the tube to the other. Hence, we may write the energy located in the tube equal to $\frac{1}{2}\Sigma F \cdot d$, which is equivalent to stating that the energy associated with a tube, *per unit length of the tube,* is at any point in the length of the tube numerically equal to one half the electric force at that point.

To determine the amount of energy per unit volume of the medium at any point in the field, consider a very short length, d, of a unit tube, taken at the point. Let F denote the electric force at that point. Then the energy located in this short length of the tube is $\frac{1}{2}Fd$. But if D denote the number of tubes per unit area at this point, the area of one tube is measured by $\frac{1}{D}$, and, since by Art. 88 $F = 4\pi D/K$, we get

$$D = \frac{KF}{4\pi} \quad \text{or} \quad \frac{1}{D} = \frac{4\pi}{KF}.$$

The volume of the short length of the tube may now be calculated, for its length is d and its area of cross section $\frac{4\pi}{KF}$, and, therefore, its volume is $\frac{4\pi d}{KF}$. The energy located in this volume was found to be $\frac{1}{2}Fd$. Therefore the energy per unit volume must be given by

$$\frac{Fd}{2}\Big/\frac{4\pi d}{KF} \quad \text{or} \quad \frac{KF^2}{8\pi},$$

that is, the energy per unit volume of the medium at a point where the intensity is F is given by the expressions

$$\boldsymbol{E = \frac{KF^2}{8\pi} = \frac{2\pi D^2}{K} = \tfrac{1}{2}FD} \text{ (ergs per c.cm.)},$$

as previously obtained.

Finally, imagine the medium to be mapped out into Faraday tubes, each crossed by equipotential surfaces (Fig. 205); it will evidently be divided into small blocks or cells, each containing a certain quantity of energy. If the equipotential surfaces correspond to unit difference of potential the cells are known as **unit cells.** Imagine a conductor with charge Q, and at potential V, entirely

surrounded by another at zero potential. Each tube will be cut into V cells by the equipotential surfaces, and, as Q tubes emanate from a charge Q, the total number of unit cells in the medium is QV. By Art. 86 the energy of the charge is $\frac{1}{2}QV$, so that the number of unit cells is twice the number of units of energy. If we assume this energy to be in the medium, then **each unit cell contributes half a unit of energy.**

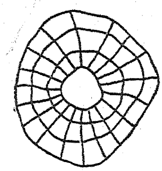

Fig. 205.

Example. In Fig. 205 let the central portion represent a conductor, with charge 18 at potential 3, and the outer surface a conductor at zero potential. The equipotentials of values 2 and 1, together with the surfaces of the conductors, divide each tube into 3 cells, and, as there are 18 tubes, the total number of cells is 54; the energy is $\frac{1}{2}QV$, *i.e.* $\frac{1}{2} \times 18 \times 3 = 27$. Thus 54 unit cells contribute 27 units of energy, *i.e.* each unit cell contributes half a unit of energy.

Now, let a be the cross section of a unit cell (say mean value), d the distance between the two consecutive equipotential surfaces forming its ends, and D the number of tubes per unit area; then, from preceding sections—

$$\text{Volume of unit cell} = ad,$$

$$a = \frac{1}{D}, \quad F = \frac{4\pi D}{K}, \quad \therefore\ Fa = \frac{4\pi}{K}, \ i.e.\ a = \frac{4\pi}{KF}.$$

Again—

$$\text{Potential gradient in the cell} = \frac{1}{d} = F, \quad \therefore\ d = \frac{1}{F},$$

$$\therefore\ \text{Volume of unit cell} = ad = \frac{4\pi}{KF^2}.$$

This volume contains half a unit of energy, hence

$$\text{Energy per unit volume} = \boldsymbol{E} = \tfrac{1}{2} \Big/ \frac{4\pi}{KF^2} = \frac{\boldsymbol{KF^2}}{\boldsymbol{8\pi}}$$

$$= \frac{\boldsymbol{2\pi D^2}}{\boldsymbol{K}} = \tfrac{1}{2}\boldsymbol{FD},$$

as previously obtained

If a small uncharged conductor be placed in a field, then a loss of energy results to the medium, due to the disappearance of the state of strain within the space occupied by the conductor.' In accordance with the principle that the potential energy of a system always tends to a minimum, it is evident that if the conductor is free to move it will tend to move towards the strongest part of the field, where the loss of energy due to its presence in the medium is a maximum. The force urging the conductor in any direction is measured by the rate of decrease of the potential energy of the medium with displacement in the given direction.

It should be noted that the loss of energy to the medium is here due, not only to the disappearance of the strain in the space occupied by the conductor, but also, in some measure, to the readjustment of the strain necessary to set the lines of force in the medium external to the conductor at right angles to the surface of the conductor. For example, in the case of a field due to a single point charge, Q, *in air*, the force at any point at distance r from the charge is Q/r^2, and, therefore, $F^2/8\pi$, the energy per unit volume at this point, is equal to $\frac{Q^2}{8\pi r^4}$. The rate of decrease of this for displacement towards the charge is $\frac{Q^2}{2\pi r^5}$, and this gives a lower limit for the value of the force per unit volume urging a small conductor, placed at this point, towards the charge.

This result is only a lower limit because the *loss* of energy attendant on the redistribution of the strain external to the conductor has been neglected, that is, it has been assumed that the value of F is the same after the introduction of the small conductor into the field as before. From the value obtained above for the force per unit volume on the small conductor, it can only be stated that the force on, say, a very small spherical conductor of radius ρ placed at a distance r from a point charge Q is *greater than* $\frac{Q^2}{2\pi r^5} \times \frac{4\pi\rho^3}{3}$ or $\frac{2Q^2\rho^3}{3r^5}$. The real value is $\frac{2Q^2\rho^3}{r^5}$, just three times this quantity. (See Arts. 101, 102.) [Note that ρ = *radius*.]

98. Longitudinal Tension and Lateral Pressure in Faraday Tubes.—It has been proved that the tension or pull on unit area at the surface of a charged conductor is $2\pi D^2/K$ dynes. Now from this unit area D Faraday tubes emanate, and if we imagine each tube to exert a pull equal to $2\pi D/K$ the required tension will be obtained. Hence, we may regard the pull at the surface of a conductor as being due to the fact that the Faraday tubes originating at the surface are in a state of tension, the magnitude of the tension at any point in a tube being given by the value of

$2\pi D/K$ at that point; but there are D tubes per unit area at that point, so that the tension per unit area at a point in the tube where the electric force is F is given by

Longitudinal Tension per unit area

$$= \frac{2\pi D}{K} \times D = \frac{2\pi D^2}{K} = \frac{KF^2}{8\pi} \text{ (dynes).}$$

Since there is a tension along the tubes each tube must also exert a lateral pressure on its neighbours, otherwise the tubes joining, say, two opposite charges would shrink into straight lines between the charges and there would be no tubes in other parts of the medium. It can be shown that in order to maintain equilibrium in the medium the lateral pressure per unit area, *i.e.* the pressure (per unit area) at right angles to the tubes, is also given by the expressions above, viz.

$$\left.\begin{array}{c}\textbf{Lateral Pressure}\\ \textbf{per unit area}\end{array}\right\} = \frac{2\pi D^2}{K} = \frac{KF^2}{8\pi} \text{ (dynes).}$$

Let $ABCD\ A'B'C'D'$ (Fig. 206) be a section of the tube of thickness x, the faces $ABCD$ and $A'B'C'D'$ being equipotential surfaces and the edges in the direction of the field. The tensions per unit area at the faces $ABCD$ and $A'B'C'D'$ are $2\pi D_1^2/K$ and $2\pi D_2^2/K$ respectively; hence, if S_1 and S_2 be the areas of these surfaces and p_1 and p_2 the total forces on the surfaces—

$$p_1 = \frac{2\pi D_1^2}{K} \cdot S_1$$

$$p_2 = \frac{2\pi D_2^2}{K} \cdot S_2$$

$$\therefore p_2 - p_1 = \frac{2\pi}{K}(D_2^2 S_2 - D_1^2 S_1),$$

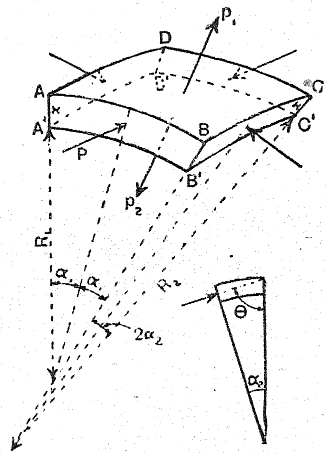

Fig. 206.

and since $D_2 S_2 = D_1 S_1$,

$$D_2^2 S_2 = D_2 D_2 S_2 = D_2 D_1 S_1;$$

so also $D_1^2 S_1 = D_1 D_2 S_2$, hence

$$p_2 - p_1 = \frac{2\pi D_1 D_2}{K}(S_1 - S_2).$$

Now
$$AB = (R_1 + x)2\alpha_1 = 2(R_1 + x)\alpha_1$$
$$A'B' = R_1 2\alpha_1 = 2R_1\alpha_1$$
$$BC = (R_2 + x)2\alpha_2 = 2(R_2 + x)\alpha_2$$
$$B'C' = R_2 2\alpha_2 = 2R_2\alpha_2,$$
$$\therefore S_1 = AB \times BC = 4(R_1 + x)(R_2 + x)\alpha_1\alpha_2$$
and
$$S_2 = A'B' \times B'C' = 4R_1R_2\alpha_1\alpha_2,$$
i.e. $S_1 - S_2 = 4(R_1 + R_2)x\alpha_1\alpha_2$, neglecting term containing x^2.

Hence *the resultant pull inwards*, viz. $p_2 - p_1$, is
$$p_2 - p_1 = \frac{2\pi D_1 D_2}{K} \times 4(R_1 + R_2)x\alpha_1\alpha_2.$$

Now suppose there is a lateral pressure P. The total force on the side $ABB'A'$ will be $P \times \text{Area} = P \times A'B' \times AA' = P \times 2R_1\alpha_1 \times x = 2PR_1\alpha_1 x$. The component of this in the opposite direction to p_2 is (Fig. 206)
$$2PR_1\alpha_1 x \times \cos\theta = 2PR_1\alpha_1 x \sin\alpha_2$$
$$= 2PR_1 x\alpha_1\alpha_2.$$

The corresponding result for the side $CDD'C'$ is also $2PR_1x\alpha_1\alpha_2$, whilst for $BCC'B'$ and $ADD'A'$ the corresponding result is $2PR_2x\alpha_1\alpha_2$. Hence, on account of the lateral pressure P, the *resultant pull upwards* is
$$4PR_1x\alpha_1\alpha_2 + 4PR_2x\alpha_1\alpha_2$$
$$= P \times 4(R_1 + R_2)x\alpha_1\alpha_2.$$

For equilibrium the resultant pull upwards must balance the resultant pull inwards, and from the above expressions this will be the case if $P = 2\pi D_1D_2/K$. But the section is thin, so that D_1D_2 may be written D^2; hence for equilibrium
$$\left.\begin{array}{c}\text{Lateral Pressure}\\ \text{per unit area}\end{array}\right\} = P = \frac{2\pi D^2}{K} = \frac{KF^2}{8\pi}\text{(dynes)}$$
as indicated above.

99. Coefficients of Potential, Capacity, and Induction.— In the case of a system of conductors in the same field the potential of any conductor is due not only to its own charge, but to all the other charges in the field. This may be expressed for a number of conductors with charges and potentials denoted by $Q_1, Q_2, Q_3 \ldots$ and $V_1, V_2, V_3 \ldots$ by writing
$$V_1 = {}_1p_1Q_1 + {}_1p_2Q_2 + {}_1p_3Q_3 + \ldots$$
$$V_2 = {}_2p_1Q_1 + {}_2p_2Q_2 + {}_2p_3Q_3 + \ldots$$
Here the quantities ${}_1p_1, {}_1p_2, {}_1p_3 \ldots$ indicate the extent to which V_1 depends upon $Q_1, Q_2, Q_3 \ldots$, and these quantities are called *coefficients of potential*.

Similarly, the charge on any conductor of the system is related not only to the potential of that conductor, but also to the potentials of all the other conductors. That is, we may write

$$Q_1 = {}_1q_1V_1 + {}_1q_2V_2 + {}_1q_3V_3 + \dots$$
$$Q_2 = {}_2q_1V_1 + {}_2q_2V_2 + {}_2q_3V_3 + \dots$$

Here the quantities ${}_1q_1$, ${}_1q_2$, ${}_1q_3 \dots$ indicate the extent to which Q_1 depends upon V_1, V_2, V_3, ... and these quantities are called *coefficients of capacity* or *coefficients of induction*, according as the suffixes are the same or different, that is, ${}_1q_1$, ${}_2q_2$, ${}_3q_3$ are coefficients of capacity, ${}_1q_2$, ${}_1q_3$, etc., coefficients of induction.

If the set of equations involving the coefficients of potential be solved for Q_1, Q_2, Q_3, .. it is evident that the second set, involving the coefficients of capacity and induction, can be obtained, and that therefore these coefficients can be expressed in terms of the coefficients of potential. It can also be proved generally that reciprocal coefficients such as ${}_1p_2$ and ${}_2p_1$, ${}_1p_3$ and ${}_3p_1$, ${}_1q_2$ and ${}_2q_1$, ${}_1q_3$ and ${}_3q_1$ are equal.

As an example of the use of these coefficients, consider the case of two insulated concentric spherical conductors of radii a and b charged with quantities Q_1 and Q_2 to potentials V_1 and V_2 for the inner and outer conductors respectively. Now the charge Q_1 on the inner conductor produces a potential Q_1/a for that conductor, and Q_1/b for the outer conductor, that is,

$${}_1p_1 = \frac{1}{a} \text{ and } {}_2p_1 = \frac{1}{b}.$$

Similarly the charge Q_2 on the outer conductor produces a potential Q_2/b for that conductor, and Q_2/b also for the inner conductor, that is,

$${}_2p_2 = \frac{1}{b} \text{ and } {}_1p_2 = \frac{1}{b}.$$

We may therefore write for

$$V_1 = {}_1p_1Q_1 + {}_1p_2Q_2$$
$$V_2 = {}_2p_1Q_1 + {}_2p_2Q_2$$

the equations

$$V_1 = \frac{1}{a}Q_1 + \frac{1}{b}Q_2,$$

$$V_2 = \frac{1}{b}Q_1 + \frac{1}{b}Q_2.$$

Solving these equations for Q_1 and Q_2 we get

$$Q_1 = \frac{ab}{b-a}V_1 - \frac{ab}{b-a}V_2,$$

$$Q_2 = -\frac{ab}{b-a}V_1 + \frac{b^2}{b-a}V_2.$$

That is

$$_1q_1 = \frac{ab}{b-a};\ _1q_2 = \frac{-ab}{b-a} = {_2q_1},\ _2q_2 = \frac{b^2}{b-a}.$$

An interesting example of the use of coefficients of capacity and induction is found in the explanation of the action of **electric screens.** Consider the three conductors A, B, C (Fig. 207), and imagine A to be enclosed in C, and B outside it. Let Q_1, Q_2, Q_3, and V_1, V_2, V_3 be the charges and potentials of these conductors, then we may write

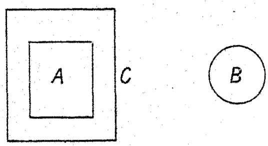

Fig. 207.

$$Q_1 = {_1q_1}V_1 + {_1q_2}V_2 + {_1q_3}V_3$$
$$Q_2 = {_2q_1}V_1 + {_2q_2}V_2 + {_2q_3}V_3$$
$$Q_3 = {_3q_1}V_1 + {_3q_2}V_2 + {_3q_3}V_3.$$

If now C be connected to earth its potential will be zero, and if A be also supposed to have no charge its potential in the interior of C must also be zero. That is, if V_3 and Q_1 are zero V_1 must also be zero, and the first equation becomes $0 = {_1q_2}V_2$, which shows that ${_1q_2} = 0$ for all values of V_2. Hence, if A and B be charged, and C earth-connected, the above equations reduce to

$$Q_1 = {_1q_1}V_1$$
$$Q_2 = {_2q_2}V_2$$
$$Q_3 = {_3q_1}V_1 + {_3q_2}V_2,$$

for V_3 is zero in value, and it has just been proved that ${_1q_2} = 0$ and ${_2q_1} = {_1q_2}$.

From these simplified equations it is evident that when C is earth-connected the charge on A (Q_1) depends upon its own potential V_1 only, and similarly the charge on B (Q_2) depends upon its potential V_2 only. That is, the action between A and B is completely stopped by the earth-connected conductor C, which may thus be said to screen A from the inductive action of the charge on B.

The same result is obvious from a consideration of the distribution of the tubes of force in the field of A, B, and C. The tubes starting from A must all terminate on C; those from B will terminate on C and on other adjacent earth-connected objects, but no tubes cross from A to B or from B to A.

The method of coefficients is somewhat roundabout for simple problems in electrostatics, just as the use of equations may be for simple algebraic problems, but in general problems of the electrostatic field they are found serviceable.

100. Electrical Images. Application to an Earthed Plane Conductor in the Presence of a Point Charge.—Problems on electrical distribution can often be readily

solved by a method due to Lord Kelvin, and known as the "method of electrical images." A brief treatment only can be given here.

Let CE (Fig. 208) be an infinite conducting plane earth connected, and, therefore, at zero potential, and let $+ Q$ be the point charge at A; let $CA = d$. The plane is, of course, acted on inductively and acquires a negative charge, the combined effect of this negative charge and the charge $+ Q$ at A being to make each point of the plane at zero potential.

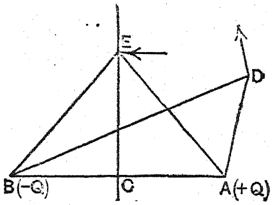

Fig. 208.

Now, imagine the conductor removed and a charge $- Q$ placed at B, where $BC = CA$. The potential at any point E in the plane, previously occupied by the conductor, is $Q/AE - Q/BE = 0$. Thus the effect on the plane of Q at A and $- Q$ at B is the same as the effect of Q at A and the actual induced distribution on the plane, and this applies to all points in front of the plane. The charge $- Q$ at B is called the "electrical image" of the charge $+ Q$ at A.

Intensity at E.—The actual intensity at E is, from the above, the same as would be produced by Q at A and $- Q$ at B. The field due to Q at A is Q/AE^2 along AE, and that due to $- Q$ at B is Q/BE^2 along EB; since $AE = BE$, these component fields are equal and equally inclined to CE, so that the resultant is perpendicular to the plane. If F be the resultant

$$\frac{F}{\frac{Q}{AE^2}} = \frac{AB}{AE} \quad \text{(See Art. 30)}$$

$$\therefore F = \frac{Q}{AE^2} \cdot \frac{AB}{AE} = \frac{2Qd}{r^3}.$$

Surface Density at E.—If $- \rho$ be the surface density at E, then by Coulomb's Law

$$F = -4\pi\rho, \text{ i.e. } \rho = -F/4\pi,$$

$$\rho = -\frac{Qd}{2\pi r^3}.$$

Attraction between Plane and Point Charge at A.—This is obtained by substituting the image for the plane, hence

$$f = \frac{Q \times -Q}{(2d)^2} = -\frac{Q^2}{4d^2}.$$

Potential and Field at any Point D.—The potential and field at D are due to Q at A and the distribution on the plane, and these are equivalent to Q at A and $-Q$ at B. Thus the potential at D is $Q/AD - Q/BD$, and the field is the resultant of Q/AD^2 along AD, and Q/BD^2 along DB.

From the preceding we may define an electrical image as follows:—*An electrical image is an electrified point (or system of points) on one side of an electrified surface which produces on the other side the same electrical action which the actual electrification of that surface does produce.*

101. Electrical Images. Application to an Earthed Conducting Sphere in the Presence of a Point Charge.—In Fig. 209 let R be the radius of the earthed sphere, and $OA = d$. Take the point B so that $OA \times OB = R^2$; hence

$$\frac{OA}{R} = \frac{R}{OB},$$

and the triangles OAP and OPB are similar; thus

$$\frac{BP}{AP} = \frac{OP}{OA} = \frac{R}{d};$$

and, since R and d are constant, the ratio BP/AP for any point P on the sphere is constant.

Now imagine a charge $-\frac{BP}{AP} \cdot Q$ placed at B and the spherical conductor removed,

$$\text{Potential at } P = \frac{Q}{AP} - \frac{BP}{AP} \cdot \frac{Q}{BP} = 0;$$

thus, from the preceding section the electrical image required is a charge $-\frac{BP}{AP}Q$, *i.e.* $-\frac{R.Q}{d}$ at the point B.

Potential and Field at any point D.—The potential and field at D due to Q at A and the distribution on the sphere are the same as the potential and field at D due to Q at A and $-R.Q/d$ at B, and are obtained in the usual way.

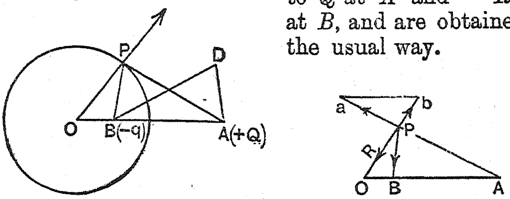

Fig. 209.

Force between the Sphere and the Point Charge at A.—This is the same as the force between the charge and the image, *i.e.*

$$f = -\frac{Q \times \frac{R.Q}{d}}{AB^2} = -\frac{R.Q^2}{d\,(d-OB)^2}$$

$$= -\frac{R.Q^2}{d\left(d-\frac{R^2}{d}\right)^2}, \text{ since } OA\,.\,OB = R^2,$$

$$f = -\frac{Q^2Rd}{(d^2-R^2)^2}.$$

Intensity and Surface Density at P.—The intensity at P due to A is Q/AP^2 in the direction Pa; if Pa represent this, and if it be resolved into two components, one along the normal OP and the other parallel to OA, then Pb will represent the normal component,

$$\therefore \text{ normal component} : Q/AP^2 = bP : Pa,$$

$$i.e. \text{ normal component} = \frac{Q}{AP^2}\cdot\frac{bP}{Pa} = \frac{Q}{AP^2}\cdot\frac{OP}{AP}$$

$$= \frac{Q\,.\,R}{AP^3}$$

The intensity at P due to RQ/d at B is $R.Q/d.BP^2$ in the direction PB, and if this be resolved in the same two directions, OP will represent the normal component,

$$\therefore \text{ normal component} = \frac{R.Q}{d.BP^2} \cdot \frac{OP}{BP}$$

$$= \frac{Q.R^2}{d.BP^3} = Q\frac{R^2}{BP^2} \cdot \frac{1}{d.BP},$$

and since
$$\frac{R}{BP} = \frac{d}{AP},$$

$$\therefore \text{ normal component} = \frac{Qd^2}{R.AP^3}.$$

$$\text{Total normal inwards} = \frac{Qd^2}{R.AP^3} - \frac{Q.R}{AP^3}$$

$$= \frac{Q}{R.AP^3}(d^2 - R^2).$$

If the components parallel to OA be similarly estimated it will be found that they are equal and opposite; hence

$$F = \frac{Q}{R.AP^3}(d^2 - R^2).$$

But by Coulomb's Law $F = -4\pi\rho$,

$$\therefore \rho = -\frac{Q}{4\pi R.AP^3}(d^2 - R^2).$$

In the case of an *insulated* sphere and a point charge it can be shown that

$$f = \frac{Q^2R^3(2d^2 - R^2)}{d^3(d^2 - R^2)^2} \quad \text{and} \quad \rho = \frac{Q}{4\pi R}\left\{\frac{d^2 - R^2}{AP^3} - \frac{1}{d}\right\}.$$

If R be very small compared with d the expression for f may be written $\frac{2Q^2R^3d^2}{d^7}$, *i.e.* $\frac{2Q^2R^3}{d^5}$, as indicated at the end of Art. 97.

Exercise. O is the centre of an earthed sphere of radius 5 cm., and A is a point 8 cm. from the centre of the sphere. The point charge at A is 5 e.s. units. Find the surface densities at the two points where the line AO cuts the surface of the sphere, and find the force of attraction between the sphere and the point charge at A.

102. Insulated Conducting Sphere in a Uniform Field.—This is most readily investigated by the method used in Art. 32 for the magnetised sphere, which the student should again read before proceeding further.

The uniform field, F, is in the direction of the arrow (Fig. 210), so that the hemisphere on the left is negative and that on the right positive, and these must be such that they produce a field inside the sphere equal and opposite to F, for the total field inside the conducting sphere is zero.

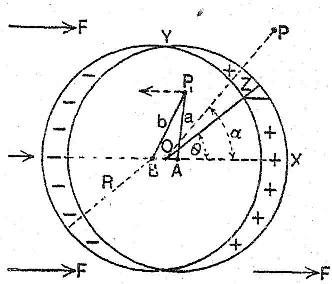

Fig. 210.

Imagine a positive sphere of uniform density, $+\rho$, to have its centre at A, and a negative sphere of uniform density, $-\rho$, to have its centre at B, where AB is very small. Consider an internal point P_1 at distance a from A, and b from B. In writing down the intensity at P_1, due to the positive sphere, it is only necessary to take into account the sphere with centre A and radius a, for the portion of the positive sphere *external* to P_1 has no effect. The electricity in this effective sphere is $\frac{4}{3}\pi a^3\rho$, and this may be assumed collected at A. The intensity at P_1 due to this is, therefore, $\frac{4}{3}\pi a^3\rho/a^2$, *i.e.* $\frac{4}{3}\pi\rho a$ along AP_1. Similarly, the intensity at P_1 due to the negative sphere is $\frac{4}{3}\pi\rho b$ along P_1B. An examination of the triangle P_1BA will indicate that the resultant intensity at P_1 is $\frac{4}{3}\pi\rho AB$, parallel to AB. If this be equal to F, which is the necessary condition,

$$\tfrac{4}{3}\pi\rho AB = F.$$

$$\therefore \rho = \tfrac{3}{4}\frac{F}{\pi \,.\, AB}.$$

Now consider the surface density (negative) on the left, and the surface density (positive) on the right. At any point Z, where the line drawn from it to the centre makes

an angle θ with the direction of the uniform field F, the depth of the surface layer is $AB \cos \theta$, and the surface density is, therefore, $\rho AB \cos \theta$; hence

$$\text{Induced surface density} = \rho AB \cos \theta$$
$$= \frac{3}{4} \frac{F \cos \theta}{\pi},$$

which determines the surface density at any point on the sphere; thus at X the surface density is $3F/4\pi$, since $\theta = 0°$, and at Y it is zero, since $\theta = 90°$.

The *intensity*, due to the sphere, at an external point P is calculated as follows:—The sphere acts externally as if a charge $+\frac{4}{3}\pi\rho R^3$ were collected at A, and a charge $-\frac{4}{3}\pi\rho R^3$ at B. These form an **electric doublet** (compare Art. 32), the moment M of which is $\frac{4}{3}\pi\rho R^3 \times AB$, *i.e.* $\boldsymbol{FR^3}$, and the intensity at the external point P *due to the sphere* is given by the formula of Art. 30 for a small magnet, the moment M having the value FR^3, *i.e.*

$$\text{Intensity due to sphere} = \frac{M}{d^3} \sqrt{1 + 3 \cos^2 a}$$
$$= \frac{FR^3}{d^3} \sqrt{1 + 3 \cos^2 a}.$$

The actual field at P is, of course, the resultant of this and F.

Examples. (1) *Show that the total field at X is $3F$. What is the total* **normal** *intensity at Z?*

By the above the field at X due to the sphere is $\frac{FR^3}{R^3} \sqrt{1+3\cos^2 O}$ $= 2F$ (compare the "end on" formula in magnetism, *i.e.* field $= 2M/d^3 = 2FR^3/R^3 = 2F$). The inducing field F is parallel to this and in the same direction. The *total* field at X is, therefore, $3F$. Verify this by an application of Coulomb's Law.

To find the *normal* intensity at Z, due to the sphere, resolve the moment FR^3 into two components, one $FR^3 \cos \theta$ *along* OZ, the other, $FR^3 \sin \theta$, perpendicular to OZ. The field due to the latter is perpendicular to OZ, and may, therefore, be neglected. The field due to the former is $\frac{2M}{d^3} = \frac{2FR^3 \cos \theta}{R^3} = 2F \cos \theta$ along OZ. The component of F along OZ is $F \cos \theta$. Hence the total normal intensity at Z is $\boldsymbol{3F \cos \theta}$. Verify this by an application of Coulomb's Law.

(2) *Prove, as mentioned at the end of Art.* 97, *that the force between a small insulated spherical conductor of radius* ρ, *and a point charge* Q *at distance* r, *is* $\frac{2Q^2\rho^3}{r^5}$.

From the preceding,

$$\text{Intensity due to sphere at distance } r = \left[\frac{2M}{r^3}\right] = \frac{2F\rho^3}{r^3}.$$

But
$$F = \frac{Q}{r^2},$$

$$\therefore \text{Intensity at distance } r \text{ from sphere} = \frac{2Q\rho^3}{r^5}.$$

$$\text{Hence force on charge } Q \text{ at this point} = \frac{2Q\rho^3}{r^5} \times Q$$
$$= \frac{2Q^2\rho^3}{r^5}.$$

What assumptions are made in the above? (Note that $\rho = radius$.)

103. Refraction of Tubes of Force.—When a field of force contains more than one dielectric medium it is of interest to determine the conditions which must obtain at the boundary of two media of different specific inductive capacities. Let AB (Fig. 211) represent the trace of a small portion of the boundary between two media of specific inductive capacities K_1 and K_2. The electric force in each medium at a point in the boundary surface may be resolved into two components, one parallel to or tangential to the surface at the points and the other normal to the surface.

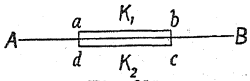

Fig. 211

The tangential components must be equal; otherwise it would be possible to obtain an infinite supply of energy by moving a charge of electricity round the cycle indicated by $a\,b\,c\,d$, the cycle being so taken that the work done on the charge along $a\,b$ is greater than that done by the charge along $c\,d$, if we assume the tangential force in the upper medium greater than that in the lower medium. The cycle is infinitely small and the algebraic sum of the work for the path da, bc, of the cycle may be taken as zero. One condition which must obtain at the surface of separa-

tion is, therefore, that **at any point in the surface the tangential component of the force in one medium must equal the tangential component of the force in the other medium.** That is, if T_1 and T_2 denote the two tangential components, then

$$T_1 = T_2.$$

Again, if there is to be no free charge at the surface of separation of the two media, it is evident from the definition of polarisation that **at any point the normal polarisation in one medium must be equal to the normal polarisation in the other medium.** That is, if F_1 and F_2 are the *normal components of the electric force* in the upper and lower media, then the corresponding normal polarisations, P_1 and P_2, are

$$P_1 = \frac{K_1}{4\pi} F_1 \quad \text{and} \quad P_2 = \frac{K_2}{4\pi} F_2,$$

and the second condition which must obtain at the bounding surface is given by the relation $P_1 = P_2$ or

$$\frac{K_1}{4\pi} F_1 = \frac{K_2}{4\pi} F_2.$$

When a tube of force passes from one medium to the other the law of its refraction can be determined from the conditions specified above. Let F_1 and F_2 denote the magnitudes of the electric force in the two media, and let the directions of these forces make angles ϕ_1 and ϕ_2 with the normal to the surface. Then the tangential components and normal components are $F_1 \sin \phi_1$, $F_2 \sin \phi_2$, and $F_1 \cos \phi_1$, $F_2 \cos \phi_2$, and the necessary relations between these quantities are

$$F_1 \sin \phi_1 = F_2 \sin \phi_2$$

$$\frac{K_1}{4\pi} F_1 \cos \phi_1 = \frac{K_2}{4\pi} F_2 \cos \phi_2.$$

This gives
$$\frac{\tan \phi_1}{K_1} = \frac{\tan \phi_2}{K_2}$$

or
$$\frac{\tan \phi_1}{\tan \phi_2} = \frac{K_1}{K_2},$$

a relation which determines the refraction of a tube of force in passing from a medium of specific inductive capacity K_1 to one of specific inductive capacity K_2. From this relation it is evident that if K_1 is greater than K_2, then ϕ_1 is greater than ϕ_2, that is, when a tube passes from one medium to another of smaller specific inductive capacity the tube is bent towards the normal, as in Fig. 212.

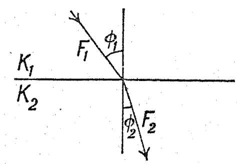

Fig. 212.

This question of refraction of the tubes of force may also be associated with the distribution of energy in the media of the field. It has been shown that, for a given intensity of polarisation, the energy per unit volume of a medium is smaller the greater the specific inductive capacity of the medium. Hence it follows that since the energy in the field always tends to a minimum, the tubes of force will pass as far as possible through the media of greatest specific inductive capacity, and the law of refraction from one medium to another is that each tube of force is refracted so as to take the path of minimum potential energy possible for it.

104. Dielectric Sphere in a Uniform Field.—Fig. 213 illustrates the preceding for the case of a ball of sulphur or other dielectric of high specific inductive capacity placed in the initial uniform field in air between two charged parallel plates. The tubes of force are refracted so as to crowd into the dielectric of high specific inductive capacity, and each one seeks a path of less energy than it initially possessed in the uniform field in air. There is a limit to the number of tubes which are drawn through the

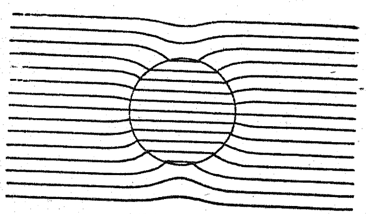
Fig. 213.

ball, because for tubes some distance out the necessary increase in air path, in order to pass through, would involve a greater increase in energy than would be compensated for by the decreased energy of the path through the ball. It will be seen that the crowding of the tubes of force through the ball causes, for a portion of their path, an increase in the area of cross section of the adjacent external tubes. This evidently means that at points of increased area of cross section the intensity of polarisation of the medium has decreased, and, therefore, the energy per unit volume has decreased, that is, although the volume of the tube has increased for a portion of its path, the energy for that portion can be less than before.

By an extension of the method of Art. 102 in the case of a *conducting* sphere in a uniform field it can be shown that in the case of a *dielectric* sphere in a uniform field, if F_1 be the intensity of the uniform field outside and F_2 the field in the sphere, K_1 the specific inductive capacity of the medium outside and K_2 that for the sphere—

$$F_2 = \frac{3K_1}{K_2 + 2K_1} F_1,$$

or, if the outside medium be air as in Fig. 213,

$$F_2 = \frac{3}{K + 2} F_1,$$

where K is the specific inductive capacity of the sphere.

Let Fig. 210 now refer to a dielectric sphere in a uniform field. In Art. 102 it was shown that if the sphere was equivalent to a doublet of moment FR^3, the field inside was changed from F to zero; we will, therefore, assume that in the present case to change the field inside from F_1 to F_2 the doublet must have a moment $(F_1 - F_2)R^3$.

Consider any point Z at the surface of the sphere. The normal intensity F, *just outside*, is given by

$$F = \frac{2(F_1 - F_2)R^3 \cos\theta}{R^3} + F_1 \cos\theta \quad \text{(See example, Art. 102.)}$$

$$= 2(F_1 - F_2)\cos\theta + F_1\cos\theta = (3F_1 - 2F_2)\cos\theta,$$

and the normal intensity F^1, *just inside*, is given by

$$F^1 = F_2 \cos\theta,$$

since the field inside is F_2 parallel to AB and therefore at an angle θ with the normal OZ.

If P_1 and P_2 be the normal polarisation, displacement or strain in the two media,

$$P_1 = \frac{K_1}{4\pi} F, \qquad P_2 = \frac{K_2}{4\pi} F,$$

and (Art. 103) these are equal; hence

$$K_1 (3F_1 - 2F_2) \cos\theta = K_2 F_2 \cos\theta,$$

$$\therefore F_2 = \frac{3K_1}{K_2 + 2K_1} F_1.$$

When a piece of conducting material is placed in an electric field it has been assumed that there is no energy in the space occupied by the conductor. That is, the conductor acts as if its surface enclosed a dielectric of infinite specific inductive capacity. Thus, if a conducting sphere be supposed to take the place of the ball of sulphur in the field shown in Fig. 213, the distribution of the tubes of force will still be of the same nature, but, in accordance with the tangent law of refraction and with the assumption that the conducting surface acts as if it enclosed a dielectric of infinitely large specific inductive capacity, the lines of force as shown in Fig. 214 cut the surface of the conductor everywhere in a direction normal to that surface.

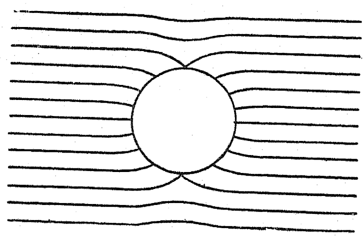

Fig. 214.

The explanation given in Art. 97 of the force acting on a neutral conductor placed in an electric field is evidently capable of extension. If a piece of dielectric of greater specific inductive capacity than that of the surrounding medium be placed in a non-uniform electric field it is evident that the piece will, like the conductor, be urged towards the region where the intensity of the field is a maximum. Similarly, if the specific inductive capacity of the piece of dielectric be less than that of the surrounding medium it will be urged towards the region of minimum

electric force. The lower limit of the force acting in each of these cases can evidently be determined by the method indicated in Art. 97 for the case of a small conductor in the field.

Exercises VI.

Section B.

(1) A small brass sphere is charged with 30 units of positive electricity, and is then made to touch another equal sphere having a charge of 10 units of negative electricity. Find the force exerted between the charged spheres when they are separated by a distance of 10 cm.

(2) Two charges of + 10 and - 10 units are placed at two corners of an equilateral triangle of 10 cm. side. Find the magnitude and direction of the resultant force acting on a charge of + 10 units placed at the third corner of the triangle.

(3) Six equal charges are placed at the corners of the base of a hexagonal pyramid. If the slant edge of the pyramid is equal to the diagonal of its base, find the intensity of the field at the apex due to the charges at the base.

(4) Four equal charges, each of 10 units, are placed one at each of the four corners of a square of 5 cm. side. Find the potential at the centre of the square and at the middle point of either of the sides.

(5) A sphere of 10 cm. diameter charged with 50 C.G.S. units of electricity is placed in contact with an insulated tin can, and the potential of the conductors after contact is found to be 5 C.G.S. units. Find the capacity of the tin can.

(6) Find an expression for the energy lost when a charged conductor is made to share its charge with another exactly equal and similar conductor. In what sense is the energy lost? What becomes of it?

(7) A small pith ball weighing one decigram suspended by a silk fibre and charged with positive electricity is repelled when a charged glass rod is brought near it. If the direction of the electric field of the glass rod near the ball is horizontal and its magnitude is equal to 20 C.G.S. electrostatic units, when the deflection of the fibre is 45° what is the charge on the ball?

(8) An insulated soap bubble 10 cm. in radius is charged with 20 C.G.S. electrostatic units. Taking the atmospheric pressure as 10^6 dynes per sq. cm., find the increase in radius due to the charge.

(9) What is meant by an electric image? A charge of electricity q is situated at a distance l from an earthed conducting sphere. Find the distribution of the induced charge in terms of the distance from the point, etc.

(10) A circular metal plate A of radius 10 cm. is earthed. At a distance of 1 mm. from it is placed another plate B of the same size, which is insulated and charged with 100 units. Find approximately the charges on the four surfaces and the force on the plate A. [The capacity of a charged circular disc of radius a at a large distance from all other conductors is $\frac{2a}{\pi}$.]

(11) A brass ball 7 cm. in radius is suspended concentrically inside a spherical brass vessel of internal radius 9 cm. and external radius 10 cm. If the charge on the ball is 56 units and the potential of the outer vessel is 5, what is the potential of the ball?

(12) A metal ball of mass one gramme suspended by a dry silk fibre forms a simple pendulum whose period of vibration is two seconds. The ball is now charged with 100 units of electricity, and a large earthed metal plane is held two cm. below it. The pendulum being now set swinging, find its period. (Take $g = 1000$ cm./sec.^{2})

(13) A conducting sphere of diameter 6 is electrified with 105 units; it is then enclosed concentrically within an insulated and unelectrified hollow conducting sphere formed of two hemispheres of thickness $\frac{1}{4}$ and internal diameter 7. The outer sphere is then put to earth; determine the potential of the inner sphere before and after the outer sphere is earth-connected. (B.E.)

(14) Explain the term *electric potential.* If 100 units of work must be done in order to move an electric charge equal to 4 from a place where the potential is -10 to another place where the potential is V, what is the value of V? (B.E.)

(15) Show that if the energy in the electrostatic field is regarded as distributed throughout the field the amount of energy per unit volume at any point P is $\frac{kR^2}{8\pi}$, where k is the specific inductive capacity and R the electric intensity at P. (B.E. Hons.)

(16) Explain how the forces in the electric field may be regarded as due to tension along the lines of force combined with pressure at right angles to them. (B.E. Hons.)

(17) Discuss the application of the method of images to the solution of electrostatic problems. (B.E. Hons.)

(18) Show that in passing from one dielectric to another electric lines of forces may undergo a change in direction. (B.E.)

Section C.

(1) What is the law of attraction and repulsion between small bodies charged with electricity?

If a number of insulated bodies, some charged positively and some negatively, be suspended within an insulated tin canister, what will be the condition of the outside of the canister and under what circumstances will it possess no charge? (Inter. B.Sc.)

(2) A, B and C are three conductors equal in all respects. A is charged, made to share its charge with B and afterwards to share the remainder with C—both B and C being previously without charge. The three are now separately discharged. Compare the quantity of heat resulting from each discharge with what would have been produced by the discharge of A before any sharing of its charge (*i.e.* compare the energies). (Inter. B.Sc.)

(3) What is meant by an electrical image? A charge of electricity $+ q$ is situated at a distance l from a large earthed plane conducting sheet. Find the distribution of the induced charge in terms of the distance from the point. (B.Sc.)

(4) Two spheres of radii 5 and 10 centimetres respectively have equal charges of 50 units each. They are then joined by a thin wire so that their charges are shared between them. Calculate the total energy before and after sharing. What becomes of the difference of energy? What difference would be caused by bringing the spheres into direct contact with one another? (B.Sc.)

(5) What is an electrical image?

A point charge is placed 3 cm. in front of an infinite plane conductor. Show that the total induced charge on the portion of the plane which is contained by the circumference of a circle of radius 4 cm., and whose centre is the foot of the perpendicular let fall from the point charge on the plane, is numerically $\frac{2}{5}$ of the point charge. (B.Sc. Hons.)

(6) Can you reconcile Maxwell's system of tensions and pressures in the dielectric medium with the theory which gives the energy per cubic centimetre as $\frac{k E^2}{8\pi}$? (B.Sc. Hons.)

(7) Find an expression for the force per square cm. of surface on a conductor due to its charge. What charge must there be upon a soap-bubble of radius $1\frac{1}{2}$ cm. if the air pressure is the same inside and outside the bubble, assuming the surface tension to be 27? (B.Sc. Hons.)

(8) Show how the induced electrification distributes itself on a conducting sphere placed in an uniform field. (B.Sc. Hons.)

(9) Explain the method of electrical images for the solution of problems in electrostatics.

Find the distribution of electricity produced on a conducting sphere when a point charge is placed near it. (B.Sc. Hons.)

(10) Show that in an electrified system, if equipotential surfaces be drawn corresponding to unit difference of potential, the energy of the system is proportional to the number of cells into which the dielectric is divided by these surfaces and the unit tubes of force.

What is the relation between the number of unit tubes of force which emerge from a closed surface and the total quantity of electricity within it? (B.Sc. Hons.)

(11) What is an electrical image? Show how to determine the electrical image of an external charge in a given sphere. (D.Sc.)

CHAPTER VII.

ELECTROSTATICS.—CONDENSERS AND CAPACITY.

105. Principle of Condensers.—Let A (Fig. 215) be an insulated metal plate fully charged to the potential $+V$, say of the positive pole of a Wimshurst electrical machine (Chapter IX.). Now bring near A a second insulated metal plate B; inductive displacement takes place, the near side of B exhibiting a negative charge, the far side a positive charge, and the uniform potential of B becomes $+v$, this being somewhat less than V in magnitude.

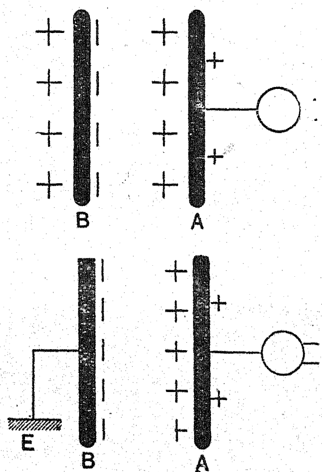

Fig. 215.

Consider now the effect of B on the potential of A. The negative charge on B tends to lower the potential of A and the positive charge on B tends to raise it; these two opposing influences nearly counteract each other, but the negative charge has a slight advantage, so that on the whole the potential of A is lowered just a little. Clearly then a very slight additional charge can be sent from the machine to A until the potential is once more equal to V; clearly also the nearer the plates are together the more marked is this effect.

Let the plate B be now earthed; as previously indicated the positive charge on B disappears, a somewhat greater negative charge appears, and the potential of B becomes zero. This increased negative charge reacting on A lowers the latter's potential, and as there is now no positive charge on B to counteract this, it is clear that the potential of A is considerably lowered, and in consequence a much greater charge can be sent from the machine to A to raise the potential to the limiting value V, *i.e.* the capacity of A is considerably increased. To summarise:—

The potential of an insulated charged conductor is considerably weakened and its capacity is considerably increased when an earthed conductor is brought near it.

Such an arrangement is called a **condenser,** the conductors being termed the *coatings*, and the insulating medium the *dielectric*, of the condenser.

It has been indicated that glass, wax, mica, etc., allow inductive influence to take place through them better than air and are said to have a higher specific inductive capacity; with one of these substances as dielectric the effects mentioned would be still more marked. For this reason, and also owing to their greater mechanical rigidity, solid dielectrics of glass, wax and mica are frequently employed in practical condensers.

If B entirely surrounds A, the induced charges are each equal to the inducing charge, and when B is earthed, only the positive charge on B passes to earth, so that the negative charge on B is as before (Art. 76); in other words, *the electricity which flows out of B is exactly equal to the charge which has passed on to A.* This is practically true also for condensers used in practice where the plates are large and the dielectrics thin; hence, looking on the condenser as a whole, there is no accumulation of electricity, since as much flows out of B as flows on to A, *but there is a large charge on A* and that is what we mainly require.

106. Capacity of a Condenser.—*The capacity of a condenser is measured by the quantity of electricity which must be given to it to establish unit potential difference between the coatings;* if one coating be earthed the capacity

of the condenser will be measured by the quantity of electricity necessary to raise the other coating to unit potential, *i.e.* the capacity of the condenser is numerically the same as the capacity of the plate A when B is earthed. Hence—

A condenser has a capacity of one C.G.S. electrostatic unit if the electrostatic unit quantity produces a P.D. of one electrostatic unit between its coatings.

The practical unit is the **farad; a condenser has a capacity of one farad if a charge of one coulomb produces a P.D. of one volt between its coatings.**

$$\text{One farad} = 9 \times 10^{11} \text{ electrostatic units.}$$

$$\text{One microfarad} = \frac{1}{10^6} \text{ farad} = 9 \times 10^5 \text{ electrostatic units.}$$

The condenser of Art. 105 was charged by earthing one coat B and connecting the other coat A to the positive pole of the machine. If C and V denote the capacity and potential of A in electrostatic units (which will therefore, in this case, be the "*capacity*" and "*potential difference between the coatings*" *of the condenser*), the charge accumulated (Q), and the energy of the charge (E), will be—

$$Q = CV \text{ e.s. units}$$
$$E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\cdot\frac{Q^2}{C} \text{ ergs,}$$

whilst if C be in farads and V in volts, Q will be in coulombs and E in joules (1 joule = ·7375 foot pound).

Imagine now both plates insulated, A connected to the positive pole at potential $+V$ and B to the negative pole at potential $-V$. The potential difference between the coatings is $2V$, and if C denote *the capacity of the condenser* the "charge" and "energy" are—

$$Q = 2CV$$
$$E = \tfrac{1}{2}C(2V)^2 = 2CV^2, \text{ etc.}$$

The reader will now have distinguished between the *capacity of the condenser*, which is obtained by *dividing the charge on A by the potential difference between A and B*, and the *capacity of the coat A*, which is obtained by *dividing the charge on A by the actual potential*

of A. **The capacity of a given condenser is constant,** depending only on dimensions and the nature of the dielectric, but the capacity of A is affected by the electrical condition of B and is only numerically the same as the capacity of the condenser when B is earthed, for then the potential of A *is* the potential difference between the coatings, since B is at zero potential.

107. Specific Inductive Capacity or Dielectric Constant.—This has already been defined in Arts. 66, 74, and these definitions should be re-read before proceeding further.

In Art. 84 it has been indicated that the capacity of a body depends upon the medium in which it is placed; it depends in fact upon the strain in the medium accompanying a given potential difference—the greater the strain the greater is the charge accumulated, and the greater therefore is the capacity of the body. **If C denotes the capacity of a body in air, and if its capacity increases to KC when it is embedded in a dielectric x, K measures the specific inductive capacity of x.**

Again, imagine two condensers exactly alike, but one with air and the other with a medium x for dielectric, and let their capacities be compared by methods indicated later. **The ratio of the capacity of the condenser with dielectric x to the capacity of the equal air condenser measures the specific inductive capacity of x, *i.e.***

$$\frac{\text{Capacity of condenser with dielectric } x}{\text{Capacity of equal air condenser}} = \left\{\begin{matrix}\text{Specific Inductive}\\ \text{Capacity of } x\end{matrix}\right\} = K,$$

$$\therefore \left\{\begin{matrix}\textbf{Capacity of condenser}\\ \textbf{with dielectric } x\end{matrix}\right\} = K \left\{\begin{matrix}\textbf{Capacity of an equal}\\ \textbf{air condenser}\end{matrix}\right\}.$$

For example, it is shown in Art. 111 that the capacity of the parallel plate air condenser of Fig. 215 is $\frac{S}{4\pi d}$, where S is the area of the plate A and d is the distance between A and B; if the air between the plates be replaced by a dielectric of specific inductive capacity K, the capacity of the condenser becomes $K\frac{S}{4\pi d}$.

It will be noticed that the capacity of a condenser depends upon three things. (1) The size of the plates—the greater the size the greater the capacity. (2) The thickness of the dielectric—the greater the thickness the less the capacity. (3) The specific inductive capacity of the dielectric—the greater the specific inductive capacity the greater the capacity.

108. The Leyden Jar.—A familiar form of condenser is called the Leyden Jar. It consists, as shown in Fig. 216, of a glass jar having an inner and outer coating of tinfoil. These coatings cover the bottom and sides up to about three-quarters of the height of the jar, and act as the plates of the condenser. The mouth of the jar is closed by an indiarubber bung, and contact is made with the inner coating by means of a brass rod, which passes through the centre of the bung. This rod terminates above in a rounded knob and carries below a loose piece of brass chain, the lower end of which rests on the tinfoil.

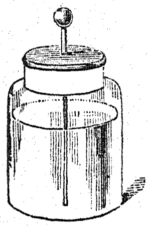

Fig. 216.

To charge the jar the inner coating is put in connection with the pole of an electric machine, and the outer coating is connected to earth, for example, by being held in the hand. The inner coating thus receives, say, a positive charge, and an equal negative charge is induced on the inner surface of the outer coating, the corresponding positive charge passing to earth.

These charges are mainly on the inner and outer surfaces of the glass where they are as near as possible to each other. That this is actually the case may be shown experimentally by means of a jar with movable coatings (Fig. 217).

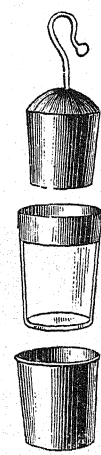

Fig. 217.

Exp. The jar is charged in the usual way and placed upon a plate of glass. The inner coating is now lifted out by means of an

insulating loop of silk thread and placed on the sheet of glass. The glass jar is now lifted out of the outer coating and also laid on the glass. If either of the coatings be now tested, it will be found to possess practically no charge; but if the jar be put together again as before, it will be found to be as strongly charged as at first. This shows that the charge may be located on the surfaces of the glass which forms the dielectric of the condenser.


Fig. 218.

If the two coatings of a charged jar be connected a discharge at once takes place. This is best done by means of the discharging tongs (Fig. 218), which are provided with an insulating handle; one knob of the tongs is placed against the outside of the jar, the other is brought near the knob of the jar and a bright spark passes between them. If the outer coat be touched with one hand, and the knob of the jar with the other, the discharge takes place through the body and an unpleasant "shock" is felt which may be dangerous.

A charged jar can be discharged by "alternate contacts." The charged jar is placed on a sheet of glass; its outside is at zero potential with a negative charge, and its inside is at a positive potential with a positive charge. *Touch the knob with the finger;* a small amount of electricity passes to earth and the potential of the inner coat becomes zero. The potential of the inner coat is zero under the combined influence of its own positive charge and the negative charge on the outer coat; hence, when the inner coat is touched only a small charge passes to earth, the bulk of it necessarily remaining on the coat. Now the outer coat was originally at zero potential under the combined influence of its own negative charge and the positive charge on the inner coat. As some of the latter has now been removed the effect of the outer coat's own charge predominates, so that the potential of the outer coat is now slightly negative. *Touch the outer coat with the finger;* a small amount of electricity comes from the earth, neutralises a small negative charge and the outer coat becomes at zero potential. The inner coat is once more touched, then the outer, and so on, until finally the jar is discharged.

109. Residual Charge and Discharge.—If a Leyden jar be charged to a given potential difference, and then be allowed to stand for a time, the potential difference will be found to have diminished. Further, if a jar be discharged so that the two coatings are brought to the same

potential, and then be allowed to stand for a time, it will be found to gradually acquire a potential of the same sign as at first, but smaller, and a second discharge can be obtained: with some Leyden jars four or five successive discharges can be obtained in this way, the jar being allowed to rest insulated after each discharge.

These effects are often said to be due to "electric absorption." Thus Faraday imagined that the opposite charges penetrated the dielectric a certain distance towards each other, the penetration being further, within limits, the longer the interval of time during which the charged jar is standing. After discharging the jar, some part of these charges, freed from the repulsion of the like charges behind, is conducted back to the plates, and a second discharge can be obtained, and so on; this explanation, although partly true, is not, as worded, wholly satisfactory, and the phenomenon is more usually explained on the supposition of strain in the dielectric.

On charging, the solid dielectric is strained to a greater extent than an air dielectric would be, for it sets up a less opposition to the forward pressure of the machine, and, moreover, the solid dielectric tends to retain this condition of strain. When the jar is discharged, the dielectric does not recover itself at once, but does so gradually, and, therefore, after a time, a charge appears on the coatings. In air condensers these effects are absent.

Maxwell has worked out a theory on the supposition that the solid dielectric is heterogeneous, parts being slightly conducting and other parts more perfectly insulating, the strain tending to persist in the latter, but to break down in the former. If the dielectric is homogeneous and slightly conducting there will be no such effects, for on the first discharge the strain will break down simultaneously throughout the whole medium.

These effects are known as "residual effects," the charges being termed "residual charges," and the discharges "residual discharges."

The capacity of a condenser will be affected by the phenomena referred to above, and, in any experiment, will depend on the time of charging; hence *the capacity is*

more exactly defined as measured by the instantaneous quantity required to produce unit potential difference between the coatings.

110. Capacity of Spherical Condenser.—Let the condenser (Fig. 219) consist of two concentric spheres, A and B, of radii R and R_2 cm. respectively, separated by a dielectric of specific inductive capacity K. Further, let B be earthed and a charge $+Q$ e.s. units be given to A so that a charge $-Q$ e.s. units is developed on the inner surface of B.

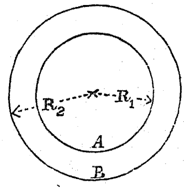

Fig. 219.

The potential *at all points inside B, due to the negative charge on B*, is $-\dfrac{Q}{KR_2}$; this is, therefore, the *induced potential of A* due to the charge on B.

The *free potential of A due to its own charge* is $\dfrac{Q}{KR_1}$.

The *actual* potential of A is, therefore, $\dfrac{Q}{K}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$,

and, since B is at zero potential, this is the potential difference (P.D.) between the coatings. Hence

$$\text{Capacity of condenser} = \frac{\text{Charge on } A}{\text{P.D. between } A \text{ and } B},$$

$$= \frac{Q}{\dfrac{Q}{K}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)}$$

$$= \boldsymbol{K\frac{R_1R_2}{R_2 - R_1}} \textbf{ e.s. units,}$$

and, for a *spherical air condenser*,

$$\text{Capacity} = \frac{R_1R_2}{R_2 - R_1} \text{ e.s. units.}$$

Again, consider the case where B is insulated so that a charge $+Q$ appears on its outer surface. Let the small

thickness of B be x cm., and, for simplicity, let the dielectric be *air*. Then

Potential of $A = \frac{Q}{R_1} - \frac{Q}{R_2} + \frac{Q}{R_2+x}$; Potential of $B = \frac{Q}{R_2+x}$

$$\therefore \text{ P.D. between } A \text{ and } B = \frac{Q}{R_1} - \frac{Q}{R_2},$$

$$\textit{i.e.} \text{ Capacity of Condenser} = Q \Big/ \left(\frac{Q}{R_1} - \frac{Q}{R_2}\right) = \frac{R_1 R_2}{R_2 - R_1}$$

as before.

Note that, although the capacity of the condenser (obtained by dividing the charge on A *by the P.D. between A and B*) is the same in both cases above, the capacity of A (obtained by dividing the charge on A *by the actual potential of A*) is *not* the same; it is *less* in the second case. Of course, the second case would be of no value used as indicated above, for the potential *of A* is not sufficiently reduced by the action of B to enable a large charge to pass on to it.

111. Capacity of Parallel Plate Condenser.—Let S square centimetres be the area of either of the plates of Fig. 215, d cm. the distance apart, $+\rho$ the surface density of the charge given to A, and let $-\rho$ be the induced surface density on B. The latter plate is earthed and the specific inductive capacity of the dielectric is K.

A positive unit in between the plates will be repelled by A with a force $\frac{2\pi\rho}{K}$, and it will be attracted by B with an equal force; hence the force on the positive unit will be $\frac{4\pi\rho}{K}$, and the work done in moving it from B to A will be $\frac{4\pi\rho}{K} \times d$. But this work in ergs measures the P.D. in e.s. units; also the total charge on A is $S\rho$. Hence

$$\text{Capacity of condenser} = \frac{\text{Charge on } A}{\text{P.D. between } A \text{ and } B}$$

$$= \frac{S\rho}{\frac{4\pi\rho}{K} d} = K \frac{S}{4\pi d} \textbf{ e.s. units.}$$

In the above we are assuming that there is no disturbance due to edge distribution on the plates, and that the field between the plates is uniform, the lines of force being parallel straight lines crossing normally from one plate to the other.

Imp.

112. Capacity of Parallel Plate Air Condenser with Intervening Slab of other Dielectric.—In Art. 111 the dielectric of specific inductive capacity K completely fills the space between the plates: if the medium be air the capacity is, of course, $S/4\pi d$.

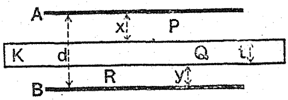

Fig. 220.

Now consider Fig. 220, which depicts a slab of dielectric (of specific inductive capacity K), with plane parallel faces, and of thickness t cm., in between the plates, the rest of the medium being air.

The forces in the three regions, P, Q, and R, are evidently $4\pi\rho$, $\frac{4\pi\rho}{K}$, and $4\pi\rho$ respectively.

The total work done in conveying unit quantity from one plate to the other is, therefore, $4\pi\rho x + \frac{4\pi\rho}{K}t + 4\pi\rho y$, and this measures the potential difference; hence

$$\text{P.D.} = 4\pi\rho\left\{(x + y) + \frac{t}{K}\right\} = 4\pi\rho\left(d - t + \frac{t}{K}\right),$$

and, since the charge on A is $S\rho$,

$$\text{Capacity of condenser} = \frac{S}{4\pi\left(d - t + \frac{t}{K}\right)} \textbf{ e.s. units.}$$

From this it follows that the capacity of this condenser is the same as if the dielectric of thickness t were replaced by a layer of air of thickness t/K; in other words, a dielectric of thickness Kt is equivalent to an air thickness t, which confirms the definition of K given in Art. 66.

113. Capacity of Cylindrical Condenser, Submarine or Concentric Cable.—For simplicity, we will first consider an air condenser. A cylindrical condenser consists of two coaxial cylinders. If the inner cylinder is charged and the outer one connected to earth, the induced charge on the outer cylinder will be equal in magnitude and opposite in sign to that on the inner one; for it is evident that all the tubes of induction emanating from the inner cylinder must terminate on the inner surface of the outer one. Let the charge *per unit of length* on the inner cylinder be Q units. Then, as shown in Art. 92, the electric force at a point at a distance r from the axis of the cylinders is $\frac{2Q}{r}$. Let the radii of the inner and outer charged cylindrical surfaces be a and b cm. respectively. Then the work done in conveying unit quantity of electricity from the outer to the inner surface is given by

$$-\int_b^a \frac{2Q}{r}dr \quad or \quad 2Q\log_e \frac{b}{a}.$$

But this is the difference of potential between the two plates Hence the capacity of the condenser *per unit length* is

$$Q/2Q\log_e \frac{b}{a} \quad or \quad 1/2\log_e \frac{b}{a},$$

and for a length l cm. we have

$$\text{Capacity of condenser} = \frac{l}{2\log_e \frac{b}{a}} \textbf{ e.s units.}$$

Approximate expressions may be used in certain cases for simplicity. Consider, for example, the capacity per unit length, viz. $1/2\log_e\frac{b}{a}$. If the difference between b and a is very small compared with a, so that $b = a + d$, where d is very small, then the result reduces to $1/2\log_e\frac{a+d}{a}$ *or* $1/2\log_e\left(1+\frac{d}{a}\right)$ *or* $1/2\frac{d}{a}$ (approximately), that is, to $\frac{a}{2d}$.

This approximate result is easily obtained more directly. If a and $a + d$ are the radii of the cylinders, and d is very

small compared with a, then the electric force between the cylinders is approximately $\frac{2Q}{a}$, and the work done in conveying unit quantity of electricity from one cylinder to the other is approximately $\frac{2Qd}{a}$, and as this is approximately the difference of potential between the cylinders, the capacity per unit length is approximately equal to

$$Q \Big/ \frac{2Qd}{a} \text{. } \textit{or} \quad \frac{a}{2d}.$$

This result may be written as $\frac{2\pi a}{4\pi d}$, and $2\pi a$ is evidently the area of unit length of the charged cylindrical surface, so that in this case also the capacity of the condenser is given by $\frac{S}{4\pi d}$, and the capacity *per unit area* is $\frac{1}{4\pi d}$, where d is the distance between the cylinders, and is assumed to be very small compared with the radii of the cylinders.

If the medium between the two cylinders be one of specific inductive capacity K it is readily deduced that

$$\text{Capacity} = K \frac{l}{2 \log_e \frac{b}{a}} \text{ e.s. units.}$$

This formula is applicable to cables, where l is the length in cm., a the radius (or diameter) of the inner conductor, b the outer radius (or diameter) of the insulating material, and K the specific inductive capacity of this material. Since common logarithms of base 10 can be converted into Napierian logarithms of base e (2·71828) by multiplying by 2·3026—

$$\text{Capacity of cable} = K \frac{l \text{ (cm.)}}{2{\cdot}3026 \times 2 \log_{10} \frac{b}{a}} \text{ e.s. units}$$

$$= K \times \frac{2{\cdot}413}{10^7} \times \frac{l \text{ (cm.)}}{\log_{10} \frac{b}{a}} \text{ microfarads.}$$

114. Capacity of Spherical Air Condenser with Inner Conductor connected to Earth.—In this case if the inner sphere be earth-connected and the outer one insulated, the question is slightly complicated. The inner sphere is at potential zero and the outer one at potential V_1, say. The potential of the surroundings of the outer sphere may also be assumed to be zero. Hence the lines of induction emanating from the outer sphere will run from its inner surface to the inner sphere and also from its outer surface to the surrounding objects, that is, there will be a positive charge on both the inner and outer surfaces of the sphere. Since all the tubes emanating from the inner surface must terminate on the inner sphere it follows that the negative charge on the latter is equal to the positive charge on the former, and, similarly, the positive charge on the outer surface is equal to the negative charge distributed over the surroundings on which the tubes of induction, emanating from the outer surface, terminate. Let R_1, R_2, and R_3 denote the radii of the charged spherical surfaces, in order from the centre. Then, since the arrangement practically consists of a spherical condenser with surfaces of radii R_1, R_2, and a spherical conductor of radius R_3, the combined capacity of the arrangement is

$$\frac{R_1 R_2}{R_2 - R_1} + R_3.$$

The potential of each of the two inner surfaces is *equally* affected by the charge on the outer surface, and, therefore, the *difference* of these two potentials is independent of the outer charge. Also the potential at the outer surface is not affected by the charges on the inner surfaces, since the algebraic sum of these charges is zero. Hence, it follows that the capacity of the condenser formed by the two inner surfaces is unaffected by the third outer charge, and also that the capacity of the outer spherical surface is unaffected by the inner charges, and, therefore, the capacity of the system is the sum of the capacities of the two constituent parts, as given in the formula above.

The capacity of the outer spherical surface is measured by R_3 only when the surrounding earth-connected objects are at an infinite distance from it. If one or more earth-connected conductors are in the near neighbourhood of the surface the magnitude of the charge on this surface will be increased by an amount depending upon the conditions of the case. As an extreme case, for example, if the system be surrounded by a fourth concentric spherical surface (Fig. 221), of radius R_4, connected to earth, the system evidently becomes a double spherical condenser, the radii of the surfaces being R_1 and R_2 for one condenser, and R_3 and R_4 for the other, and the capacity of the system is

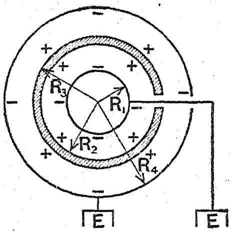

Fig. 221.

$$\frac{R_1R_2}{R_2 - R_1} + \frac{R_3R_4}{R_4 - R_3}.$$

115. Grouping of Condensers in Parallel, Abreast, or in Battery.—This arrangement is indicated in Fig. 222, and clearly the capacity of the compound condenser is the sum of the capacities of the individual condensers. For if C denote the capacity of the compound condenser, and V the difference of potential between its plates, then the total charge in it is given by $Q = CV$. But if C_1, C_2, C_3, etc., denote the capacities of the individual condensers, and Q_1, Q_2, Q_3, etc., the individual charges, we have

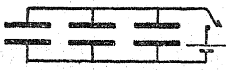

Fig. 222.

$$Q = Q_1 + Q_2 + Q_3 + \ldots,$$

and therefore

$$VC = VC_1 + VC_2 + VC_3 + \ldots,$$

that is $\quad VC = V(C_1 + C_2 + C_3 + \ldots),$

and therefore $\quad C = C_1 + C_2 + C_3 + \text{etc.},$

or $\quad C = \Sigma C.$

If the condensers are alike, say each of capacity C_1, then in Fig. 222 the joint capacity is $3C_1$, and if there are n condensers the joint capacity is nC_1.

The charge on each condenser is, if they are alike, $C_1 V$; hence the total charge in Fig. 222 is $3C_1 V$, and, in the case of n condensers, $nC_1 V$.

The energy of each condenser is, if alike, $\frac{1}{2}C_1 V^2$; hence the total energy in Fig. 222 is $3 \times \frac{1}{2}C_1 V^2$, and, in the case of n condensers, $n\frac{1}{2}C_1 V^2$.

Summary of n equal Condensers in Parallel.

1. Joint capacity = n times the capacity of one condenser.
2. Total charge = n ,, ,, charge ,, ,,
3. Total energy = n ,, ,, energy ,, ,,

116. Grouping of Condensers in Series or in Cascade. Mixed Grouping.—This arrangement is shown in Fig. 223. Since the outflow from one condenser constitutes the charge on the next, the charge Q on the positive coating of each must be the same and equal to that communicated to the first condenser. If V be the potential difference between A and B, V_1, V_2, V_3 the potential differences between the coatings of the separate condensers, C_1, C_2, C_3 the individual capacities, and C the joint capacity—

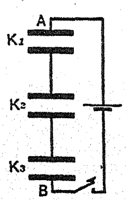

Fig. 223.

$$V = V_1 + V_2 + V_3,$$

i.e. $$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3},$$

$$\therefore \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

or $$\frac{1}{C} = \Sigma \frac{1}{C}.$$

If the capacities are the same, say each C_1, then in Fig. 223 $\frac{1}{C} = \frac{3}{C_1}$, *i.e.* $C = \frac{1}{3} C_1$, and, if there are n equal condensers, the joint capacity is $\frac{1}{n} C_1$.

Again, since the condensers are assumed alike, the potential difference for each (Fig. 223) is $\frac{V}{3}$, and the charge on each is $C_1 \frac{V}{3}$, and

the *total* charge $3 \times C_1 \frac{V}{3}$, *i.e.* $C_1 V$. This is equal to the charge which one condenser would take if used alone; the same applies to n equal condensers.

Finally, the energy of each, assumed alike, is $\frac{1}{2} C_1 \left(\frac{V}{3}\right)^2$ and the total energy is $3 \times \frac{1}{2} C_1 \left(\frac{V}{3}\right)^2$, *i.e.* $\frac{1}{3} \times \frac{1}{2} C_1 V^2$. This is only $\frac{1}{3}$ of the energy of a single condenser used alone; with n equal condensers it would be $\frac{1}{n}$.

Summary of n equal Condensers in Series.

1. Joint capacity $= \frac{1}{n}$ of the capacity of one condenser.
2. Total charge = same as on one condenser used alone.
3. Total energy $= \frac{1}{n}$ of the energy of one condenser used alone.

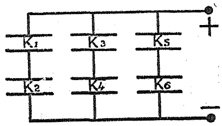

Fig. 224.

A mixed grouping is shown in Fig. 224, and the total capacity is obtained thus:—

Joint capacity of K_1 and K_2 in series $\left(\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}\right)$

$$= \frac{C_1 C_2}{C_1 + C_2}.$$

Joint capacity of K_3 and K_4 in series $= \frac{C_3 C_4}{C_3 + C_4}$.

Joint capacity of K_5 and K_6 in series $= \frac{C_5 C_6}{C_5 + C_6}$,

and, since these three sets are in parallel—

$$\text{Total capacity} = \frac{C_1 C_2}{C_1 + C_2} + \frac{C_3 C_4}{C_3 + C_4} + \frac{C_5 C_6}{C_5 + C_6}.$$

117. Types of Condensers.—One form of *standard condenser* for testing purposes consists of alternate sheets of tinfoil (thin lines in Fig. 225) and paraffined paper or mica (thick lines), the odd conducting sheets being joined

to one terminal A, the even numbers being joined to terminal B; the arrangement is thus equivalent to two large plates separated by a thin and good dielectric. They are frequently of $\frac{1}{3}$ microfarad capacity, this being approximately the capacity of one knot of the Transatlantic cable.

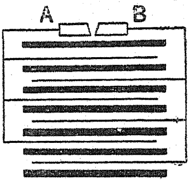

Fig. 225.

Lord Kelvin's standard condenser is shown in plan and section in Figs. 226, 227. It consists of two systems a and b of parallel metal plates, arranged, as shown in Fig. 226, so that the plates of one system alternate with those of the other.

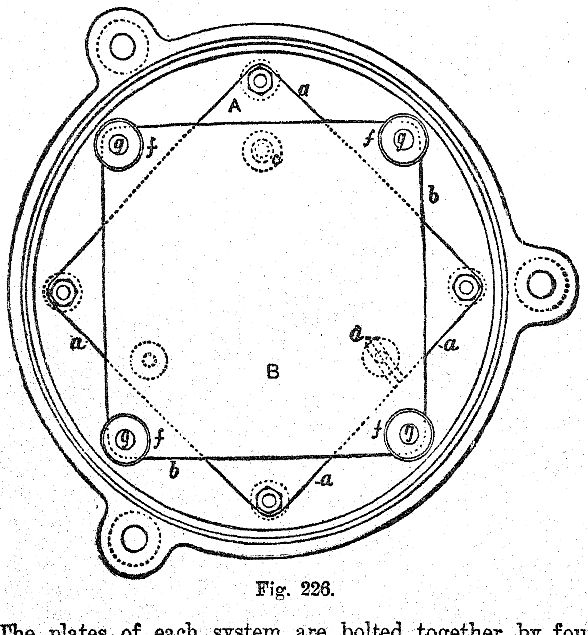

Fig. 226.

The plates of each system are bolted together by four vertical brass rods passing through the corners of the

plates, and exact parallelism of the plates is secured by means of accurately cut distance pieces or rings on the rods. One system of plates is fixed to the sole-plate of the condenser and the other set rests on three short glass pillars carried by screws working through the sole-plate. By means of these screws the plates of the system carried

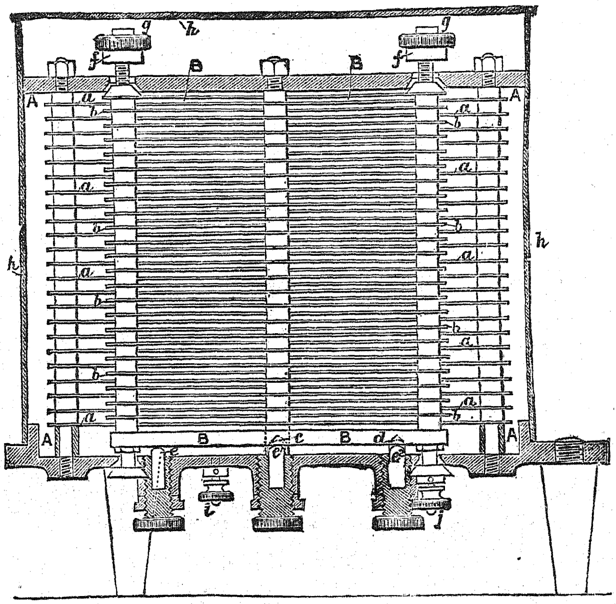

Fig. 227.

by them can be adjusted parallel to those of the other system and so that any plate of one system is midway between the two adjacent plates of the other system. The plates are covered by a dust-proof cover and the instrument rests on insulating vulcanite legs.

An *adjustable standard condenser* is shown in Fig. 228. It will readily be seen that when the plugs are in the back set of holes the component condensers are short-circuited

or connected for discharge, but when the plugs are in the front row of holes all the condensers are connected in parallel and their joint capacity is the sum of their indi-

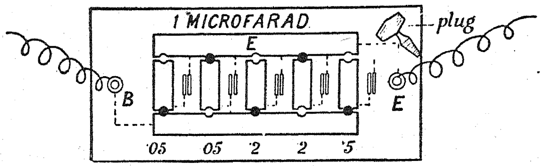

Fig. 228.

vidual capacities. It will be understood from this that by moving one or more plugs from the back to the front it is possible to combine in parallel one or more sections of the condenser. The terminals are at E and B.

The general principle of the *guard-ring condenser* will be understood from Fig. 229. In a simple parallel plate condenser the density is greatest round the edges of the plate and the field is not everywhere uniform as is assumed in developing the formula (Art. 111). This is remedied in the guard-ring condenser by surrounding the plate A by a metal ring G, the two being in the same plane. If A and G are in conducting communication and charged, the effect of G is to prevent this irregularity at the edge of A. The second plate B of the condenser is fixed to an insulating support and its distance from A can be adjusted by a micrometer screw. In using the condenser, A and G are metallically connected and charged, B being earthed; then A and G are disconnected from each other and G is earthed. The charge on A remains, and the capacity of the condenser is given by the formula $S/4\pi d$, S being the mean of the areas of A and the circular opening in G.

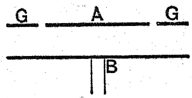

Fig. 229.

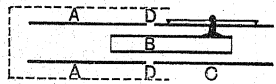

Fig. 230.

The principle of the *sliding condenser* is shown in Fig. 230. A and C are two metallic cylinders in line and separated by a small air gap D, whilst B is an inner coaxial cylinder carrying a slider which moves on C. A is insulated and B and C are earthed. If B be caused to slide into A by an additional distance l, the change in the capacity is $l/2 \log_e \frac{b}{a}$ (Art. 113). B is supported on vulcanite rests inside C and metal covers are arranged to protect A from external disturbing influences.

118. Further consideration of the Parallel Plate Condenser.—Let S be the area of the insulated plate A, ρ the surface density of the charge on it, d the distance between A and B, and, in the first place, let the medium be air.

Each unit of charge on A is attracted by B with a force $2\pi\rho$, and, since unit area contains a charge ρ units, it will be attracted with a force $2\pi\rho \times \rho$, *i.e.* $2\pi\rho^2$; hence

Mechanical force on unit area of $A = 2\pi\rho^2$,

$\therefore$ Mechanical force on the whole plate $A = 2\pi S\rho^2$, assuming the charge uniform all over the surface.

Now let the medium be one of specific inductive capacity K, and *let the charges be the same as before* so that the surface density is still ρ. The force on each unit of charge on A is now $2\pi\rho/K$ and the force on the charge per unit area is $2\pi\rho/K \times \rho$, *i.e.* $2\pi\rho^2/K$; hence,

$$\text{Mechanical force on unit area of } A = \frac{2\pi\rho^2}{K},$$

$$\therefore \text{ Mechanical force on the whole plate } A = \frac{2\pi S\rho^2}{K}.$$

Thus, if the charges remain the same, the force of attraction between the plates when the dielectric is one of specific inductive capacity K, is the Kth part of what it is in air (i.e. the force is K times less than the force in air).

Again, imagine a positive unit charge in between the plates; it will be attracted by B with a force $2\pi\rho/K$, and repelled by A with a force $2\pi\rho/K$, so that the total force

on it is $4\pi\rho/K$. The work done in moving this positive unit from B to A is, therefore, $4\pi\rho d/K$, and this measures the P.D. between the plates; if V denotes this—

$$V = \frac{4\pi\rho d}{K}, \qquad \therefore \rho = \frac{KV}{4\pi d}.$$

Now let the medium be air for which K is unity; then

$$\text{Mechanical force on } A = 2\pi S\rho^2 = 2\pi S\left(\frac{V}{4\pi d}\right)^2$$
$$= \frac{SV^2}{8\pi d^2}.$$

Substitute now the second dielectric, but let the charge be altered so that *the potential difference V is the same as before*; let ρ_1 be the new density, then

$$\text{Mechanical force on } A = \frac{2\pi S\rho_1^2}{K} = \frac{2\pi S}{K}\left(\frac{KV}{4\pi d}\right)^2$$
$$= \frac{KSV^2}{8\pi d^2}.$$

Thus, if the potential difference remains the same, the force of attraction between the plates is K times greater when the dielectric is one of specific inductive capacity K than it is with air.

Again, if f denote the force of attraction on A, we have from the preceding—

$$f = \frac{2\pi S\rho^2}{K} = \frac{2\pi S\rho \,.\, \rho}{K} = \frac{2\pi Q\rho}{K}, \quad \therefore Q = \frac{Kf}{2\pi\rho}$$

and
$$V = \frac{4\pi\rho d}{K},$$

$$\therefore \text{Energy} = \frac{1}{2}QV = \frac{1}{2} \cdot \frac{Kf}{2\pi\rho} \cdot \frac{4\pi\rho d}{K} = fd.$$

Now imagine the plates close together so that V (and d are practically zero: the energy is practically zero. Move the plate A back through a distance d; the work done is fd, and this, by the above, is equal to the energy of the charged condenser. We may deduce from this that the

energy resides in the medium as previously indicated. The reader should think this out in detail for himself.

Again, since the *energy* of a condenser is $Q^2/2C$, it follows that **if Q be constant,** the energy is *inversely* as the capacity, *i.e.* it is **less** with a medium of specific inductive capacity K than it is with air.

Further, since the *energy* is also $\frac{1}{2}CV^2$, it follows that **if V be constant,** the energy is *directly* as the capacity, *i.e.* it is **greater** with a medium of specific inductive capacity K than it is with air.

In the above K is, of course, assumed greater than unity.

The formula $C = \dfrac{S}{4\pi d}$ for a parallel plate air condenser is, as has been indicated, not strictly true, for it assumes that edge disturbances are absent and that no lines emanate from the back of the plate. Kirchhoff gives the full formula—

$$C = \frac{\pi r^2}{4\pi d} + \frac{r}{4\pi d}\left(d \log_e \frac{16\pi r\,(d+t)}{ed^2} + t \log_e \frac{d+t}{t}\right),$$

where r = radius of the circular plates, t = their thickness, d = distance apart in air, and e = base of Napierian logarithms.

119. Further consideration of the Parallel Plate Condenser with Slab of other Dielectric.—The force exerted on a unit area of either plate (Fig. 220) is, when the medium surrounding the plates is air, given by $2\pi\rho^2$ (Art. 96), and *is, therefore, unchanged* by the introduction of the slab of dielectric if ρ is unchanged, that is, *if the charges on the plates are unchanged.* If the charges remain constant with a surface density ρ the potential difference for the plates must change from $4\pi\rho d$ to $4\pi\rho d'$, where $d' = d - t + t/K$ as previously explained.

If the potential difference be maintained constant, then the charges must change, and we get

$$4\pi\rho d = 4\pi\rho' d',$$

so that

$$\rho = \frac{d}{d'}\,\rho = \frac{d\rho}{d - t + t/K}$$

and the force exerted on either plate per unit of area is given by

$$2\pi\rho'^2 \text{ or } 2\pi\rho^2\left(\frac{d}{d - t + t/K}\right).$$

Hence, if the quantity

$$\frac{d}{d - t + t/K}$$

is greater than 1 the force is increased, that is, the force is increased or decreased by the introduction of the slab of dielectric according as K is greater or less than 1, that is, according as the specific inductive capacity of the dielectric is greater or less than that of air. We are assuming K greater than 1, so that the force *is increased* in this case, *i.e.* when the potentials are kept constant.

Obviously, if the dielectric slab be replaced by a conducting plate the effect is merely to reduce the air thickness from d to $(d - t)$, and this is consistent with the assumption that the value of K for conducting material may be taken as infinite.

The effect of the slab on the energy of the condenser is important, and, here again, two cases arise.

Case 1. *Charges the same*—

(*a*) *Without the slab.* Consider unit area of the plates

$$\begin{aligned}\text{Energy} &= \tfrac{1}{2}\rho V = \tfrac{1}{2}\rho \,.\, 4\pi\rho d \\ &= 2\pi\rho^2 d.\end{aligned}$$

(*b*) *With the slab.*

$$\begin{aligned}\text{Energy} &= \tfrac{1}{2}\rho V^1 = \tfrac{1}{2}\rho\left\{4\pi\rho\left(d - t + \frac{t}{K}\right)\right\} \\ &= 2\pi\rho^2\left\{d - t\left(1 - \frac{1}{K}\right)\right\},\end{aligned}$$

that is, the energy is *less* on the introduction of the slab by an amount $2\pi\rho^2 t\left(1 - \dfrac{1}{K}\right)$.

Case 2. *Potentials the same*—

(*a*) *Without the slab.* Consider again unit area,

$$\begin{aligned}\text{Energy} &= \tfrac{1}{2} CV^2 = \tfrac{1}{2}\frac{\rho}{V}V^2 = \tfrac{1}{2}\frac{\rho}{4\pi\rho d}V^2 \\ &= \frac{V^2}{8\pi d}.\end{aligned}$$

(*b*) *With the slab.*

$$\text{Energy} = \tfrac{1}{2}C_1V^2 = \tfrac{1}{2}\frac{\rho_1}{V}V^2$$

$$= \tfrac{1}{2}\frac{\rho_1}{4\pi\rho_1\left(d - t + \frac{t}{K}\right)}V^2$$

$$= \frac{V^2}{8\pi\left\{d - t\left(1 - \frac{1}{K}\right)\right\}},$$

that is, the energy is *greater* on the introduction of the slab since the denominator is less than $8\pi d$.

120. Other Capacities.—These are merely added here for purposes of reference; the brief proofs of the formulae will be readily understood by readers with a knowledge of the Calculus, and may be omitted by others.

(1) The capacity of a **cylinder** of length l cm., and radius r cm., is given by

$$C = \frac{l}{2\log_e\frac{l}{r}},$$

the medium being air. If r is very small compared with l the expression becomes very small; hence, in experiments, the capacity of a long thin connecting wire is neglected. *The formula applies to a long thin telegraph wire far removed from the earth.*

Proof.—Consider a thin annulus width dx. The charge on it is $2\pi r\rho\,dx$, where ρ is the surface density. The potential at a point distant x along the axis due to this is $\frac{2\pi r\rho}{\sqrt{r^2 + x^2}}dx$; hence

$$V = 2\int_0^{\frac{l}{2}}\frac{2\pi r\rho}{\sqrt{r^2 + x^2}}dx = 4\pi r\rho\left\{\log_e\left(\frac{l}{2} + \sqrt{r^2 + \tfrac{1}{4}l^2}\right) - \log_e r\right\}$$

or if r be small compared with l we have

$$V = 4\pi r\rho\log_e\frac{l}{r}.$$

The charge $Q = 2\pi r\rho l$; hence the capacity $= Q/V$ is $l/2\log_e\frac{l}{r}$.

(2) The capacity of **two long thin parallel cylinders,** each of radius r and distance d apart, is given by

$$C = \frac{1}{4 \log_e \frac{d}{r}}$$

per unit length, the medium being air. *The formula applies to two parallel telegraph wires far removed from the earth.*

Proof.—Assume the cylinders to have densities $+\rho$, $-\rho$. The intensity at distance x from one of them, due to unit length, is $2Q/x$, *i.e.* $\frac{4\pi r\rho}{x}$, and at the same point the intensity due to the other is $\frac{4\pi r\rho}{d-x}$. Since

$$F = -\frac{dV}{dx},\quad dV = -Fdx;$$

hence V the potential difference between the cylinders is

$$V = -\int_{x=r}^{x=d-r} Fdx \qquad = \left[4\pi r\rho \log_e \frac{x}{d-x}\right]_r^{d-r}$$

$$= 8\pi r\rho \log_e \frac{d}{r} \text{ (approx.)}.$$

The charge Q per unit length is $2\pi r\rho$; hence capacity per unit length $= Q/V$ is $1/4 \log_e \frac{d}{r}$.

(3) The capacity of a **long thin cylinder, parallel to a conducting plane,** and at a distance h from it, is given by

$$C = \frac{1}{2 \log_e \frac{2h}{r}}$$

per unit length, the medium being air. *The formula applies to a telegraph wire at a height h above the ground.*

Proof.—This is readily derived from the preceding by imagining a conducting plane half way between the cylinders, d being now $= 2h$. V now becomes $4\pi r\rho \log_e 2h/r$ and C becomes $1/2 \log_e \frac{2h}{r}$.

(4) The capacity of an isolated thin circular disc of radius r cm. can be proved to be (in air)

$$C = \frac{2r}{\pi}.$$

The mathematical student should establish this formula.

Exercises VII.

Section A.

(1) Explain the action of a condenser and give definitions of (*a*) the capacity of a condenser, (*b*) the specific inductive capacity of a dielectric.

(2) Develop expressions for (*a*) the capacity of a spherical condenser, (*b*) the capacity of a parallel plate condenser, (*c*) the capacity of a concentric cable.

(3) Prove that if C_1, C_2, C_3, C_4 be the individual capacities of four condensers arranged in series and C the joint capacity,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}.$$

Section B.

(1) The thickness of the air layer between the two coatings of a spherical air conductor is 2 cm. The condenser has the same capacity as a sphere of 120 cm. diameter. Find the radii of its surfaces.

(2) A, B, and C are three Leyden jars, equal in all respects. A is charged, made to share its charge with B, and afterwards to share the remainder with C—both B and C being previously without charge. The three jars are now separately discharged. Compare the quantity of heat resulting from each discharge with what would have been produced by the discharge of A before any sharing of its charge.

(3) Give the theory of the Leyden jar and show how to charge it.

A quantity of electricity (5 units) is conducted into the interior of a Leyden jar of surface (2 units); and a quantity of electricity (6 units) is conducted into the interior of a similar jar of surface (3 units). Compare the heat developed by discharging each.

(4) A Leyden jar consists of two concentric spherical surfaces of 5 and 6 cm. respectively, the intervening space being filled with air. The outer sphere is uninsulated, the inner is charged with 20 units of electricity. How much work is done when the inner sphere is put to earth? (B.E.)

(5) A sphere of radius 40 millimetres (mm.) is surrounded by a concentric sphere of radius 42 mm., the space between the two being filled with air. What is the relation between the capacity of this system and that of another similar system in which the radii of the spheres are 50 and 52 mm. respectively, and the space between them is filled with paraffin of specific inductive capacity 2·5? (B.E.

(6) Three equal similar Leyden jars are connected (a) in series, (b) abreast, and in each case the set of jars is charged as fully as can be by the same machine. What proportion does the heat produced, by completely discharging all the jars in the first case, bear to the heat produced by discharging them in the second case? (B.E.)

(7) How would you combine four condensers, each having a capacity of 1 microfarad, so as to produce a capacity of 0·75 microfarad? (C.G.)

(8) Two submarine cables of equal length have conductors whose diameters are 80 and 100 mm., the diameters of the guttapercha coverings being 120 and 180 mm. Determine the relative capacities of the two cables,

$$\log 2 = \cdot 30103, \qquad \log 3 = \cdot 47712. \qquad \text{(C.G.)}$$

Section C.

(1) At what distance should the plates, 3 cm. in diameter, of an air condenser be placed in order to have the same capacity as a sphere 100 cm. in diameter? (Inter. B.Sc.)

(2) A condenser A has plates of area 1000, and dielectric of thickness 4; another condenser B has plates, area 800, and the same dielectric of thickness 5. Compare the charges and energies in A and B, when they are connected, A to a source of potential 4, and B to a source of potential 5. (Inter. B.Sc.)

(3) What is meant by the specific inductive capacity of a substance? Compare the attraction between two parallel plates maintained at given potentials, (i) when the space between the plates is filled with air, (ii) when the space is half filled by air and half by a parallel plate of a dielectric whose S.I.C. is 10. (Inter. B.Sc. Hons.)

(4) Describe a method of comparing the capacities of two small condensers. A condenser is made of 2 concentric spheres (radii $= a$ and b, b being the radius of the outer sphere). If the outer sphere is connected to earth and the inner maintained at potential V, find the charge on either of the spheres. (B.Sc.)

(For answer to first part of question see Chap. VIII.)

(5) Deduce an expression for the electrostatic capacity of two coaxial metallic cylinders separated by a layer of air. Investigate the effect of inserting between the cylinders a coaxial cylindrical shell of a dielectric substance of thickness less than that of the layer of air. (B.Sc. Hons.)

CHAPTER VIII.

ELECTROSTATICS.—INSTRUMENTS AND MEASUREMENTS.

121. The Torsion Balance.—This instrument, due to Coulomb, has already been described in Chapter III. As employed in statical electricity, the suspended magnet is replaced by a light lever, *ab* (Fig. 231), of shellac or other light insulating material, furnished at one extremity, *b*, with a gilt pith ball. Through the aperture in the cover a second equal gilt pith ball, *c*, attached to the end of a glass rod, *g*, can be introduced, and the apparatus is adjusted so that the balls, *b* and *c*, are in contact when the wire is without torsion, *b* being opposite the zero of the scale. The ball *c* is removed and charged; on being replaced it shares its charge with *b*, mutual repulsion ensues, and, as a result, *b* is repelled until the couple due to the torsion balances that due to the repulsion between the balls. To absorb moisture, and thus improve the insulation, a small vessel, *V*, containing calcium chloride or pumice stone soaked in sulphuric acid, is placed in the bottom of the case.

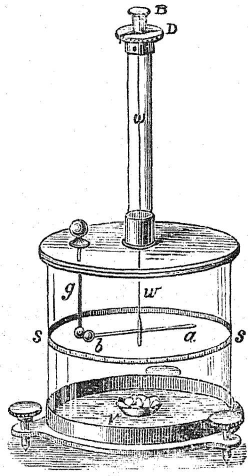

Fig. 231.

A formula for the instrument may be developed, as in

Chapter III., as follows. Let $2q$ be the charge given to c. On inserting it in the case the charge is equally shared between c and b, so that the charge on each is q, and b is repelled. Let the torsion head be now turned through an angle, β, in the opposite direction, so as to reduce the angle through which b is repelled to α. Let F denote the force between the balls at this distance (Fig. 232). The twist on the wire is $(\alpha + \beta)$; hence

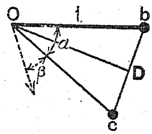

Fig. 232.

$$\text{Couple due to torsion} \propto (\alpha + \beta) = C(\alpha + \beta),$$

where C is a constant for the wire (couple corresponding to unit twist).

Again—

$$\text{Couple due to repulsion} = F \times OD = Fl \cos\frac{\alpha}{2},$$

and, since these balance—

$$Fl \cos\frac{\alpha}{2} = C(\alpha + \beta),$$

$$\therefore Fd^2 = \frac{C(\alpha+\beta)}{l\cos\frac{\alpha}{2}} d^2 = \frac{C(\alpha+\beta)}{l\cos\frac{\alpha}{2}} \times \left(2l\sin\frac{\alpha}{2}\right)^2$$

$$= 4lC(\alpha + \beta)\sin\frac{\alpha}{2}\tan\frac{\alpha}{2},$$

or, since l and C are constants, putting $4lC = \angle =$ a constant,

$$Fd^2 = \angle(\alpha+\beta)\sin\frac{\alpha}{2}\tan\frac{\alpha}{2} \ldots\ldots\ldots\ldots (1)$$

Further, since $F = q^2/d^2$,

$$q^2 = \angle(\alpha+\beta)\sin\frac{\alpha}{2}\tan\frac{\alpha}{2} \ldots\ldots\ldots\ldots (2)$$

Experiments with the Torsion Balance.

Exp. 1. *To verify the Law of Inverse Squares by means of the Torsion Balance Formula.*—If the law be true that $F \propto 1/d^2$, then the product, Fd^2, when the charges are kept constant and the dis-

tance between them is varied, should be constant. Hence, from (1) above, the expression $(\alpha + \beta) \sin\frac{\alpha}{2} \tan\frac{\alpha}{2}$ should be constant under these circumstances; the experiment is, therefore, carried out as follows. Charge c, allow it to share its charge with b, turn the torsion head in the opposite direction to the deflection, and take readings of the angles α and β. Turn the torsion head through a further angle and observe the new values of α and β. Repeat, obtaining a series of corresponding values of α and β, and verify that for each pair of values

$$(\alpha + \beta) \sin\frac{\alpha}{2} \tan\frac{\alpha}{2} = \text{a constant.}$$

If α be small, the approximate relation

$$(\alpha + \beta)\, \alpha^2 = \text{a constant}$$

may be used $\left(\text{for } \frac{\alpha}{2} \text{ may be substituted for } \sin\frac{\alpha}{2} \text{ and } \tan\frac{\alpha}{2}\right)$.

Exp. 2. *To compare two Charges by means of the Torsion Balance.*—Let the first charge Q_1 $(= 2q_1)$ be given to c. Insert c so that the charge is shared with b and the latter repelled. Turn the torsion head through an angle β_1, so as to reduce the deflection to α.

Discharge b and c, let the second charge Q_2 $(= 2q_2)$ be given to c, and insert the latter so that b is repelled. Turn the torsion head through an angle β_2 so as to bring the deflection *to the same value* α as above. Then

$$\frac{q_1^2}{q_2^2} = \frac{\angle(\alpha + \beta_1) \sin\frac{\alpha}{2} \tan\frac{\alpha}{2}}{\angle(\alpha + \beta_2) \sin\frac{\alpha}{2} \tan\frac{\alpha}{2}} = \frac{\alpha + \beta_1}{\alpha + \beta_2},$$

$$\therefore \frac{Q_1}{Q_2} = \frac{q_1}{q_2} = \sqrt{\frac{\alpha + \beta_1}{\alpha + \beta_2}}.$$

Exp. 3. *To verify the Law of Inverse Squares by Coulomb's Method.*—If the angle of deflection be small it may be assumed that the *distance*, **bc** (Fig. 232), is reduced to one half when *the angle* is reduced to one half. Coulomb's method was, therefore, as follows: On inserting c the ball b was repelled through an angle of 36°. The twist on the wire was, therefore, **36°**, and this balanced the repulsion. The torsion head was then turned through 126° in the opposite direction to reduce the deflection to 18°. The twist on the wire was, therefore, (126° + 18°), *i.e.* **144°**, and this balanced the repulsion. Now, 144 = 4 × 36; thus the force of repulsion was increased fourfold, which verifies the law, if we assume the distance was halved.

Exp. 4. *To verify by Coulomb's method that the force is proportional to the product of the charges.*—On inserting c the ball b was repelled through a certain angle θ°, so that the twist on the wire (θ°) balanced the repulsion. The ball c was removed, its charge reduced to one-half by allowing it to touch an equal ball, and it was again inserted (without touching b). The deflection was less, and the torsion head was then turned through a certain angle a°, *in the same direction*, until the angular distance between b and c was again θ°. The twist on the wire was now $(\theta^\circ - a^\circ)$, and this balanced the repulsion. $(\theta^\circ - a^\circ)$ was found to be one-half of θ°, so that the repulsion was halved when the charge on c was halved.

Examples. (1) *In an experiment with the torsion balance the lever was deflected through* 10°. *Find the torsion necessary to reduce this deflection to* 5°, *supposing the charges on the balls to remain constant.*

If the deflection be reduced from 10° to 5°, that is, halved, the force exerted between the charged balls will be quadrupled, and, therefore, the torsion on the wire must be quadrupled. Hence if the *torsion head* be turned through β° before the deflection is reduced to 5°, the twist on the wire will be $(\beta^\circ + 5)^\circ$, and we must have

$$\beta^\circ + 5 = 4 \times 10 = 40 = \text{torsion},$$

or

$$\beta^\circ = 40 - 5 = 35.$$

Thus the torsion head must be turned through 35°, and the torsion on the wire is 40°.

(2) *In an experiment with the torsion balance the lever is deflected through* 90°. *Find the torsion necessary to reduce this deflection to* 60°.

[In this example the deflections are too large to admit of the application of the method of the preceding example, and the following more exact method must be adopted.]

With the usual notation we have—

$$\text{Case (1)—}F_1 d_1^2 = \angle(a_1 + \beta_1)\sin\frac{a_1}{2}\tan\frac{a_1}{2};$$

$$\text{Case (2)—}F_2 d_2^2 = \angle(a_2 + \beta_2)\sin\frac{a_2}{2}\tan\frac{a_2}{2};$$

and, since we are to assume the truth of the inverse square law, $F_1 d_1^2 = F_2 d_2^2$, *i.e.*

$$(a_2 + \beta_2)\sin\frac{a_2}{2}\tan\frac{a_2}{2} = (a_1 + \beta_1)\sin\frac{a_1}{2}\tan\frac{a_1}{2}.$$

But from the problem $a_1 = 90^\circ$, $\beta_1 = 0^\circ$, $a_2 = 60$,

$$\therefore (60^\circ + \beta_2)\sin 30^\circ \tan 30^\circ = 90^\circ \times \sin 45^\circ \tan 45^\circ$$

$$(60^\circ + \beta_2) = 90^\circ \times \frac{\sin 45^\circ \tan 45^\circ}{\sin 30^\circ \tan 30^\circ} = 90^\circ \times \frac{\frac{1}{\sqrt{2}} \times 1}{\frac{1}{2} \times \frac{1}{\sqrt{3}}}$$

i.e. $(60° + \beta_2) = 90\sqrt{6} = \mathbf{220{\cdot}5°}$,

$$\therefore\ \beta_2 = 220{\cdot}5° - 60° = \mathbf{160{\cdot}5°}.$$

Thus the torsion head must be turned through 160·5°, and the torsion on the wire is 160·5° + 60°, *i.e.* 220·5°.

[If this question be worked by the method of Ex. 1 we have—

$$\frac{90}{\beta + 60} = \frac{60^2}{90^2} = \frac{4}{9},$$

$$\therefore \text{Torsion} = \beta + 60 = \tfrac{810}{4} = 202{\cdot}5°$$

and $\beta = 202{\cdot}5° - 60° = 142{\cdot}5°$.]

122. Cavendish Proof of the Law of Inverse Squares.—It has been indicated both experimentally and theoretically (Chapters V. and VI.) that the potential is *uniform*, and, therefore, the electric force *zero*, inside a charged hollow conductor, and this is really an indirect proof of the law of inverse squares.

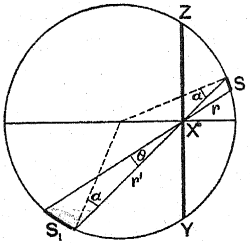

Fig. 233.

Consider a sphere uniformly charged positively, and take any point, X, within it (Fig. 233). Through X draw lines forming two small cones, the apex of each being therefore at X, the small solid angles at X being θ, and let the two small areas intercepted on the sphere be S and S_1 respectively. Consider now a right section of the cone at S. Its area is $r^2\theta$ and (since the angle between two straight lines is equal to the angle between their perpendiculars) it makes an angle α (Fig. 233) with S; thus $r^2\theta = S\cos\alpha$, $\therefore\ S = \dfrac{r^2\theta}{\cos\alpha}$, and, similarly, $S_1 = \dfrac{r_1{}^2\theta}{\cos\alpha}$.

If ρ be the surface density, the charge on S will be $S\rho$, *i.e.* $\dfrac{r^2\rho\,\theta}{\cos\alpha}$, and the charge on S_1 will be $\dfrac{r_1{}^2\rho\theta}{\cos\alpha}$. Now if the force varies inversely as the pth power of the distance the force at X due to S will be $\dfrac{r^2\rho\theta}{r^p\cos\alpha}$, and the force at X due to S_1

will be $\dfrac{r_1^2\rho\theta}{r_1^p \cos a}$, and these will be in the same straight line but in opposite directions. The following cases arise:—

Case 1. If $p = 2$ these become $\dfrac{\rho\theta}{\cos a}$ in each case; thus the field at X, due to the charges on S and S_1, is zero, and, since the whole sphere can be divided into cones in this way, the total field at X, due to the charged sphere, is zero. This is so in practice; hence we infer that $p = 2$.

Case 2. If $p > 2$ these become $\dfrac{\rho\theta}{r^{p-2}\cos a}$ and $\dfrac{\rho\theta}{r_1^{p-2}\cos a}$ respectively. Since $r_1 > r$, the former expression is greater than the latter; hence, taking the whole sphere, the force due to the part of it on the right of the plane, ZXY, will be greater than the force due to the part of it on the left of ZXY, and there will be a resultant force at X acting towards the left, *i.e.* towards the centre. This is not so in practice.

Case 3. Similarly, if $p < 2$ it can be shown that there is a resultant force at X acting towards the right, *i.e.* away from the centre, which is contrary to experiment.

Thus, from the fact that the force inside such a charged sphere is everywhere zero, we deduce that the law of inverse squares is true.

Cavendish, and later Maxwell, placed a sphere, B, inside another, A. A was charged positively, joined to B for a moment by a wire, then the two were disconnected. From the preceding B would have no charge if $p = 2$, it would be positive if $p > 2$, and negative if $p < 2$. In all cases B was without charge; hence $p = 2$. The same fact has been proved in other ways.

123. The Kelvin Quadrant Electrometer.—The *Quadrant Electrometer* consists essentially of four quadrantal boxes (such as might be obtained by cutting a shallow cylindrical brass box into four quadrants), and a light needle of aluminium foil, suspended by a fine silver wire, so that it hangs inside these boxes. Fig. 234 shows

three of the quadrants, *a*, *b*, *c*, and the needle, *n*, the fourth quadrant, *d*, being removed to show the needle.

The four quadrants are mounted horizontally on insulating glass stems, and the suspension wire of the needle is attached, as shown in the figure, to a suspension head, *S*. The principle of action of the instrument is simple. The needle, *n*, is charged to a comparatively high constant potential, by connecting it to the inner coating of a charged Leyden jar: thus charged, it is attracted by the neutral brass quadrants; but if it lie, as in Fig. 235, symmetrically along one of the lines of separation of the quadrants, the forces of attraction are in equilibrium, and the needle is not displaced. If, however, the four quadrants are divided into two pairs by connecting *opposite* quadrants (*a* to *c* and *b* to *d*), the needle may be deflected by charging these pairs to unequal potentials.

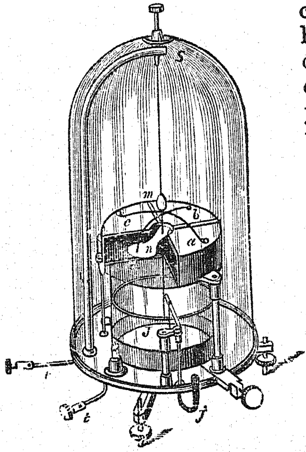

Fig. 234.

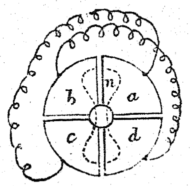

Fig. 235.

For example, if the needle be charged positively, and the quadrants *a* and *c* positively, and *b* and *d* also positively but to a lower potential than *a* and *c*, then, since the capacity of each pair of quadrants is the same, the charge on *b* and *d* is less than on *a* and *c*, and therefore the force of repulsion between *b* and *d* and the needle is less than between *a* and *c* and the needle. As a consequence, the needle is deflected towards *b* and *d* by a couple equal to

the difference between the couples due to the charges on b, d, and a, c. This difference depends ultimately on the potential of the needle and the difference of potential between b, d, and a, c, and therefore *the deflection depends upon the potential of the needle and the difference of potential between the pairs of quadrants.*

For convenience the Leyden jar, with which the needle is connected, is arranged in the base of the instrument. It consists of a glass vessel, J (Fig. 234), coated outside with tinfoil, and, when in use, half filled with strong sulphuric acid. The acid being a conductor serves as the inner coating of the jar, and also helps to secure good insulation by absorbing all moisture. The needle is connected to this inner coating by means of the piece of platinum wire to which it is attached, and which is long enough to dip into the acid.

The opposite quadrants are connected by short pieces of fine copper wire, and the brass rods t, t, which are carried outside the glass case covering the instrument, are connected to two *adjacent* quadrants, and thus serve to make contact with the two pairs of quadrants; they are called the *terminals* of the instrument. Another similar brass rod, j, communicates, by means of a short length of platinum wire suspended from it, with the acid in the jar and its terminal thus represents the knob of the jar.

As in the torsion balance, the couple deflecting the needle is, in the position of equilibrium, balanced by the opposing couple due to the torsion on the wire, which, to secure the necessary sensitiveness, must be very fine. As the deflection is, in general, very small, the lamp and scale method, explained in Chapter III., is employed.

The elementary theory of the action of the Quadrant Electrometer may be given as follows. Let V, v_1, and v_2 denote the potentials of the needle and the two pairs of quadrants. Let θ be the deflection, and T the corresponding couple exerted on the needle by its torsion fibre or bifilar suspension. It will be seen that the quadrant-needle system constitutes a double condenser, each pair of quadrants forming a condenser with the part of the needle that lies within them.

Imagine the deflection increased from θ to $\theta + \alpha$, α being a very small angle. Then we may assume, for an ideally symmetrical arrangement, that the capacity of one condenser is increased and that of the other decreased by a definite equal amount proportional to α.

Let c denote the change of capacity of each condenser for unit angular displacement of the needle. Then, for the displacement α in a direction causing an increase of capacity in the condenser formed by the needle and the quadrants at potential v_2, the change in the energy of the condenser system is given by

$$\tfrac{1}{2} c\alpha (V - v_2)^2 - \tfrac{1}{2} c\alpha (V - v_1)^2$$

or

$$\tfrac{1}{2} c\alpha \{(V - v_2)^2 - (V - v_1)^2\}$$

or

$$c\alpha \left\{ V - \frac{v_1 + v_2}{2} \right\} \{v_1 - v_2\}.$$

This change of energy* is equal to the work done against the couple T during the extra displacement α. Hence we get

$$T\alpha = c\alpha \left\{ V - \frac{v_1 + v_2}{2} \right\} (v_1 - v_2),$$

that is,

$$T = c\left(V - \frac{v_1 + v_2}{2}\right)(v_1 - v_2).$$

But T is the torsion moment for a twist θ, and is equal or nearly equal to $\kappa\theta$, where κ is a constant depending on the wire or other suspension. Hence we have

$$\kappa\theta = c\left(V - \frac{v_1 + v_2}{2}\right)(v_1 - v_2),$$

or

$$\theta = \frac{c}{\kappa}\left(V - \frac{v_1 + v_2}{2}\right)(v_1 - v_2). \quad \ldots\ldots\ldots\ldots \quad (1)$$

* This change of energy, it will be noted, is an *increase* of energy. In a case of this kind, where the displacement takes place at constant potential, there is an increase of energy equal to the work done in the displacement. The battery or generator, which must be supposed connected with the system in order to adjust the several charges so as to maintain all the potentials constant, therefore supplies energy equal to twice the work done in the displacement. If the system is left to itself and the displacement takes place at constant charges the work done is the same as before, but is now, by the principle of conservation of energy, equal to the loss of energy by the system. See Art. 95.

If V be very large compared with v_2 and v_1, then $\left(V - \frac{v_1 + v_2}{2}\right)$ is approximately equal to V, and we have as a rough approximation

$$\theta = \frac{cV}{\kappa}(v_1 - v_2). \quad \text{.................} \quad (2)$$

That is, the deflection, θ, is directly proportional to $(v_1 - v_2)$, the difference between the potentials of the two pairs of quadrants.

It should be noted that if the potential of one pair of quadrants be the same as that of the needle, for example, if $v_1 = V$, we get (from (1))

$$\theta = \frac{c}{\kappa}\left(\frac{V - v_2}{2}\right)(V - v_2),$$

or

$$\theta = \frac{c}{2\kappa}(V - v_2)^2. \quad \text{......................} \quad (3)$$

In the relation (2) given above θ is of the same sign as $(v_1 - v_2)$, but in the result just obtained θ does not change sign with $(V - v_2)$, being proportional to $(V - v_2)^2$.

If one pair of quadrants be earthed so that we may write $v_2 = 0$, then the formula given above reduces to

$$\theta = \frac{cV}{\kappa} v_1 \text{ (from (2))} \quad \text{.................} \quad (4)$$

and

$$\theta = \frac{c}{2\kappa} V^2 {}^* \text{ (from (3))}. \quad \text{..............} \quad (5)$$

Usually c is small, and κ comparatively large even for a thin wire, so that if θ is to be of easily measurable magnitude when v_1 is small, V must evidently be large. If, however, the potential to be measured is fairly large the arrangement giving the second formula which involves V^2 usually gives a satisfactory value for θ. This arrangement can also be used when V is alternating in value.

* This formula cannot be obtained from the one above it directly, for the latter is obtained on the assumption that v_1 is small compared with V.

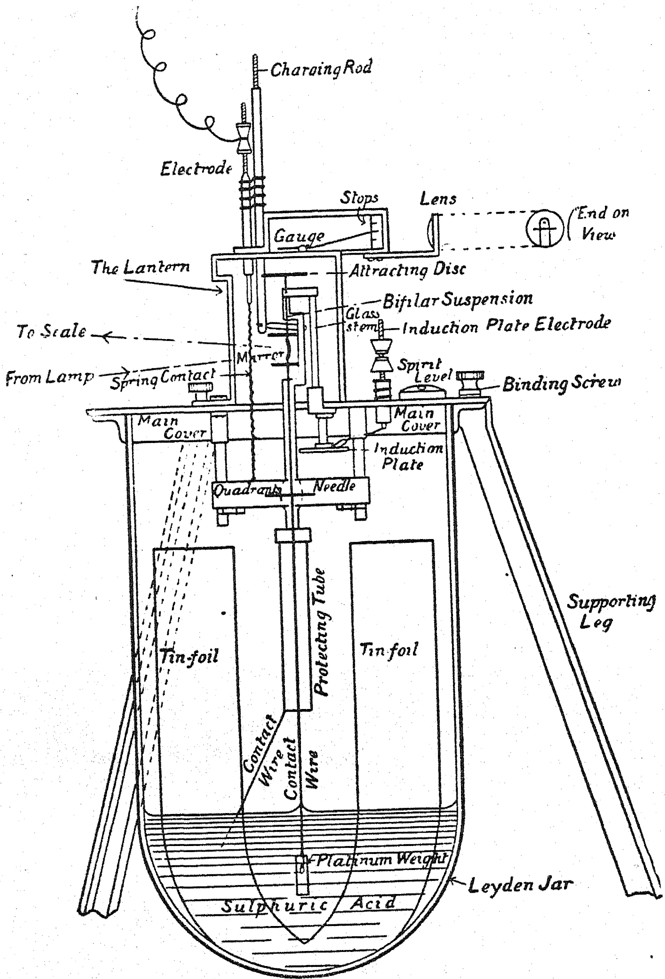
Charging Rod
Electrode
Stops
Lens
End on View
Gauge
The Lantern
Attracting Disc
Bifilar Suspension
Glass stem
Induction Plate Electrode
To Scale
Mirror
From Lamp
Spring Contact
Spirit Level
Binding Screw
Main Cover
Main Cover
Induction Plate
Quadrants
Needle
Tin-foil
Protecting Tube
Tin-foil
Supporting Leg
Contact Wire
Contact Wire
Platinum Weight
Leyden Jar
Sulphuric Acid

Fig. 236.

To summarise important points in connection with the preceding—If the needle be at a constant high potential *the deflection is proportional to the potential difference between the two pairs of quadrants*, and if one pair be earthed (zero potential), and the other pair connected to an electrified body, *the deflection will be proportional to the common potential of the body and the quadrants to which it is joined;* in both these cases, *i.e.* when the needle and the two pairs of quadrants are all at different potentials, the instrument is said to be used **heterostatically.** If the needle and one pair of quadrants be connected together, and at a much higher potential than the other pair, *the deflection is proportional to the square of the potential difference between the two pairs of quadrants;* in this case the instrument is said to be used **idiostatically.**

Fig. 236 depicts a modern form of Kelvin Quadrant Electrometer.

124. The Dolezalek Quadrant Electrometer.—This is of more slender construction and much more sensitive than the preceding, and is shown in Fig. 237. The needle of the instrument is of silvered paper and is suspended by a quartz fibre which is made conducting by dipping it into a solution of calcium chloride. The quadrants are made of brass and are well insulated by being supported on amber pillars. Access to the needle is rendered easy by mounting two adjacent quadrants on a brass piece so that they can be swung to one side and returned again to the working position as desired. There is no Leyden jar, the needle being charged by a battery to about 80 volts; the spot of light moves about a metre on a scale at a metre distance when the potential difference between the quadrants is 1 volt. The instrument is contained in a brass case provided with a window for the light to pass to and from the mirror. The working capacity of the instrument is about 50 electrostatic units. It is extensively used in measurements connected with radio-activity, etc.

125. The Attracted Disc or Absolute Electrometer.—The *absolute* measurement of difference of potential can be effected by the attracted disc electrometer, the principle of which consists in balancing the attraction between two parallel discs at different potentials, and at a

known distance apart, by a weight or other force which can be expressed in absolute units of force.

It has been shown that in the case of a charged disc the density is not uniform, but is greatest round the edge;

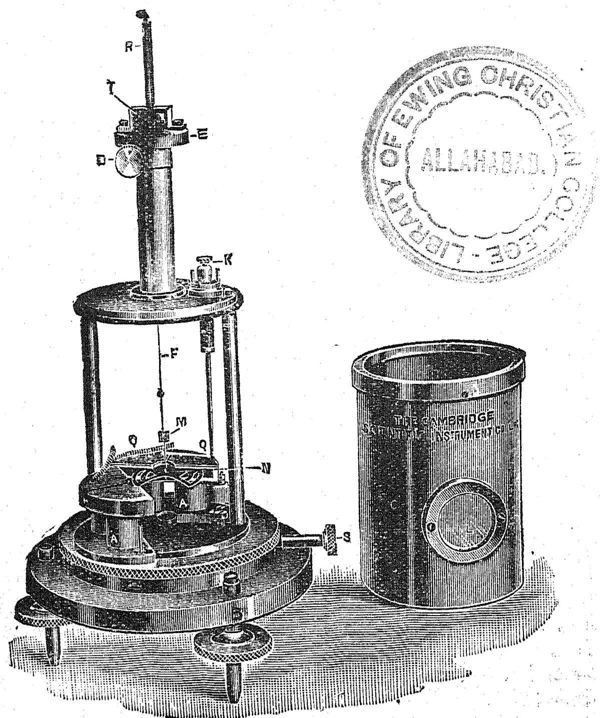

Fig. 237.

this defect in the early forms of this instrument is remedied in Kelvin's pattern by the employment of a "guard-ring." Imagine a small circular plate to be cut centrally out of a larger plate and to be slightly reduced in size so that it fits

freely without contact into the ring left in cutting it out of the larger plate. If this plate and ring be connected by a fine wire and the arrangement charged, the ring will prevent increase of surface density at the edge of the disc when the ring and disc are in the same plane so that the surface density of the disc will be uniform under these circumstances.

The principle of Kelvin's electrometer will be gathered from Fig. 238. The attracted disc, C, is surrounded by a guard-ring, $B\,B$, forming the bottom of a metal box, $A\,A$, which protects the disc from external influences. The disc is suspended from the top of the box by a spring, and its normal position is slightly above the plane of the guard-ring bottom, $B\,B$. The distance between the disc and the lower plate, $P\,P$, can be adjusted so that the attraction on the disc, when there is a potential difference between it and the plate $P\,P$, is just sufficient to bring it exactly into the plane of the guard-ring $B\,B$, and as the force in dynes necessary to do this is readily found by direct experiment the force of attraction on the disc for a known distance between the plates is determined.

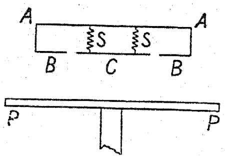

Fig. 238.

Before describing how the instrument is used, it will be advisable to revise briefly how the force of the attraction between two plates at different potentials can be expressed in terms of their difference of potential. In the arrangement of Fig. 238, let the plate C be charged to potential V, and let the surface density of the charge be denoted by σ. If $P\,P$ be now connected to earth it remains at zero potential, but a charge of opposite sign to that on C is induced on it, and the surface density of this charge will be denoted by $-\sigma$. The difference of potential between the plates is therefore V, that is, V units of work would be done in conveying unit quantity of electricity from $P\,P$ to C. Hence if f denote the *electric force* in the space between the plates, and d the distance between the plates, we have $fd = V$, for the field of force between the plates is

uniform. But the electric force between the plates is given by $4\pi\sigma$, for a positive unit of electricity would be repelled downwards by C with a force $2\pi\sigma$, and it would be attracted downwards by PP with a force $2\pi\sigma$. Hence the total force which it would experience would be $4\pi\sigma$ downwards, and giving this value to f we get $4\pi\sigma d = V$, or $\sigma = V/4\pi d$.

Now each unit of electricity on C is attracted downwards by PP with a force $2\pi\sigma$, and if S denote the area of C the charge on it is $S\sigma$, and the total force with which PP attracts it down is $S\sigma \times 2\pi\sigma = 2\pi S\sigma^2$. If this force be denoted by F we have

$$F = 2\pi S\sigma^2.$$

But

$$\sigma = V/4\pi d.$$

Therefore

$$F = \frac{2\pi S V^2}{16\,\pi^2 d^2}$$

$$= \frac{S\,V^2}{8\pi d^2},$$

and

$$V = \sqrt{\frac{8\pi d^2 F}{S}}$$

$$= d\sqrt{\frac{8\pi F}{S}}.$$

That is, if F is constant, the difference of potential between the plates is proportional to their distance apart.

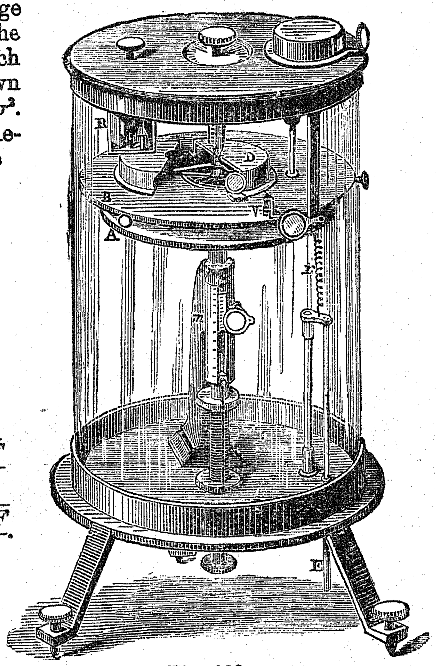

Fig. 239.

One form of Kelvin's attracted disc electrometer is shown in Fig. 239. The guard-ring box and disc cover, arranged

as in Fig. 238, is shown at D. The disc, C, is inside D in the plane of the guard plate B and the sensitiveness of its adjustment can be controlled by means of the screw shown in the centre of the cover of the instrument. In order to maintain the disc and guard-ring at the constant potential V the system is connected to the inner coating of a Leyden jar formed by the glass case of the instrument.

The potential of this coating is maintained by the action of a small replenisher (Art. 131), shown at R, and a gauge

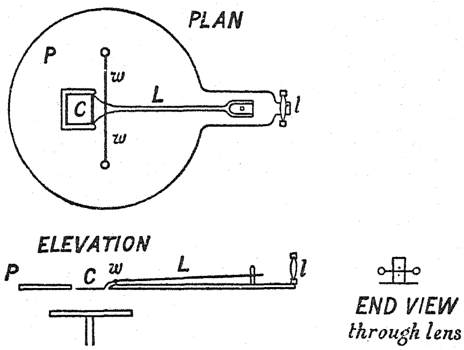

Fig. 240.

shown at G indicates the constancy of the potential. This gauge is itself a small attracted disc arrangement. A small disc or plate, c, Fig. 240, of aluminium, is carried at one end of the lever, L, and fits into the plate, PP, the guard-ring.

The lever L is attached rigidly to the wire, ww, in such a way that the torsion of the wire makes the normal position of the plate, c, to be just a little above the plane of its guard-ring. At the other end the lever L forks, and a fine hair fixed across the two prongs of the fork moves with the lever up and down before a white scale with two small black dots on it. These dots are so placed that when the hair lies exactly between them the disc c is in the plane of its guard-ring.

A lower plate, below and parallel to c and its guard-ring, is arranged at a *fixed* distance from it. If this lower plate is earthed or connected to the outside of the case of the instrument, and the disc and guard-ring of the guage to the inner coating of the Leyden jar, then, since the distance between the lower plate and the disc is fixed, the force of attraction necessary to bring the disc into the plane of the guard-ring is also fixed, and corresponds to a definite constant potential for the inner coating of the jar. The attainment of this potential is indicated by the gauge hair index. Hence, by the use of the replenisher, and by keeping watch on the indication of the gauge, the potential of the jar and everything connected to it may be maintained at a definite constant value for any length of time. A similar gauge is shown in Fig. 236, attached to the Kelvin Quadrant Electrometer.

The position of the attracted disc relative to its guard-plate, B, is indicated by an index, shown at V, similar to that used in the gauge, and to increase the accuracy of observation a lens is fitted to each index in such a way as to allow the observer to view a clear magnified image of the fine hair and the dots on the index scale. The rod carrying the lower plate of the instrument, A, is movable vertically in a sliding socket, and its motion can be accurately measured by means of the micrometer arrangement shown at m in the figure. The terminal for making connection, through the wire r, with the lower plate, A, is shown at E.

Exp. *To measure the potential of a charged conductor by the attracted disc electrometer.*

The disc and the guard-ring are maintained at a constant potential, and the lower plate is first connected to earth and adjusted until the disc is in the plane of the ring. The reading of the micrometer screw attached to the pillar supporting the lower plate is then taken, and the connection between the plate and the earth broken.

The plate is next connected to the conductor whose potential has to be measured, and again adjusted for the equilibrium of the disc. The reading of the micrometer-screw is again taken, and the *difference* of the two readings noted; this difference gives the distance through which the lower plate has been moved in effecting the second adjustment.

Now, if V denote the constant potential at which the disc and guard-ring are maintained, and v the potential to be measured, then we have, since the potential of the lower plate for the first adjustment is zero,

$$V = d_1 \sqrt{\frac{8\pi F}{S}}, \tag{1}$$

where d_1 denotes the distance between the plates when the first adjustment is made. Similarly we have

$$V - v = d_2 \sqrt{\frac{8\pi F}{S}}, \tag{2}$$

where d_2 denotes the distance between the plates when the second adjustment is made. Hence, subtracting (2) from (1), we get

$$v = (d_1 - d_2) \sqrt{\frac{8\pi F}{S}},$$

where v denotes the required potential, and $(d_1 - d_2)$ is accurately given by the micrometer screw.

The factor $\sqrt{\frac{8\pi F}{S}}$ may be determined once for all as a constant of the instrument by finding F and S. Imagine all parts at zero potential and C (Fig. 238) above the level of B. Suppose M grammes must be placed on C in order to bring it to the level of B, then the force required to do this is Mg dynes, *i.e.* $F = Mg$ dynes.

In determining S, the width of the gap between the plate and its guard-ring should be considered. The gap is very narrow and it may be assumed from symmetry that the distribution of the lines of force between plates in the region of the gap is as indicated in Fig. 241, where B and C denote the guard-ring and plate with the gap between. From this it appears that the total charge on C is equal to the charge on a portion of the lower plate equal in area to the plate C *plus* half the area of the gap between the guard-ring and the plate. That is, the total charge on the plate is $(S + a)\,\sigma$, where $2a$ denotes the area of the gap.

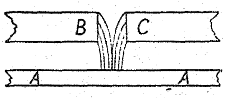

Fig. 241.

Example. *In an attracted disc electrometer the area of the disc* C (Fig. 238) *was* 100 *square centimetres and* 2 *grammes were required to bring it down to the level of the guard-ring* B. *In a subsequent experiment, when* C *was connected to a charged conductor and the lower plate earthed, the distance between the plates was* ·5 *centimetre when* C *was on a level with the guard-ring. Find the potential and charge on the disc* C ($g = 980$).

The force required to bring C down to the level of the guard-ring is evidently (2×980) dynes $= F$.

Since the lower plate is at zero potential, if V be the potential of C

$$V = d\sqrt{\frac{8\pi F}{S}} = \cdot 5\sqrt{\frac{8\pi \times 2 \times 980}{100}} = \mathbf{11 \cdot 10 \text{ e.s. units.}}$$

Again, if ρ be the density of the charge on C, the total charge on C is $S\rho$; but $F = 2\pi S\rho^2$, hence

$$Q = S\rho = S\sqrt{\frac{F}{2\pi S}} = \sqrt{\frac{SF}{2\pi}} = \sqrt{\frac{100 \times 2 \times 980}{2\pi}},$$

$$\text{i.e. } Q = 10\sqrt{\frac{980}{\pi}} = \mathbf{176 \cdot 6 \text{ e.s. units.}}$$

126. Electrostatic Voltmeters.—Commercial instruments for the measurement of potential difference in practical units (volts) are termed *voltmeters*; one type, due to Lord Kelvin and based on electrostatic principles, is shown in Fig. 242. It consists of two fixed brass plates, each of the shape of a double quadrant, parallel to, and in conducting communication with, each other, but insulated from other parts of the apparatus. Between these is a light aluminium vane capable of swinging about a horizontal axis in a vertical plane midway between the fixed brass plates. The brass plates in fact correspond to the quadrants, and the vane corresponds to the needle, of the quadrant electrometer. The upper part of the vane carries a pointer which moves over the scale of the instrument, and the lower part of the vane carries a projection and knife edge to which controlling weights are attached. The fixed and moving systems are connected to two separate terminals.

On connecting the two terminals to two points at different potentials the plates and vane become charged to these potentials and the electrostatic attraction tends to draw the vane completely within the fixed plates; this attraction is opposed by the particular weight attached to the lower end of the vane, and the latter will clearly come to rest when these two opposing influences are equal and balance each other. The scale is graduated so that the deflection gives the potential in volts, each division representing a certain number of volts according to the controlling weight employed.

It will be evident from Art. 123 that in this instrument the deflection varies as the *square* of the potential difference between the two systems, and, as will be seen later,

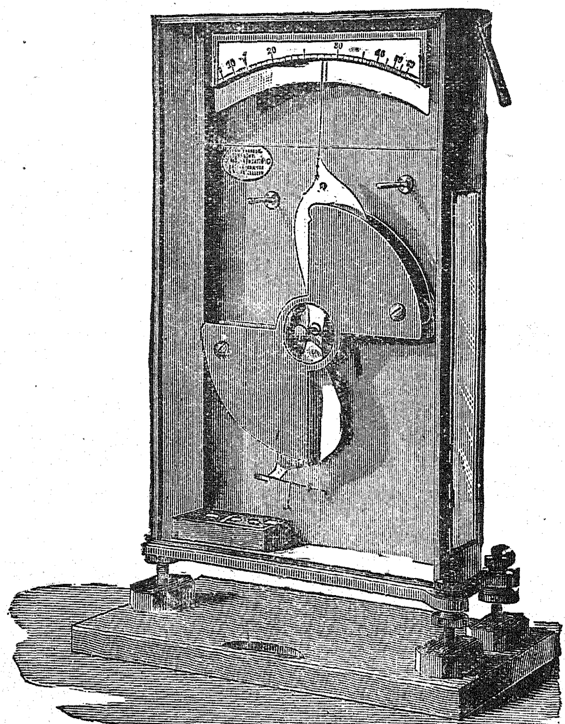

Fig. 242.

such an instrument can also be used for rapidly alternating potentials, for the deflection will be the same and in the same direction, whether the vane is at a higher or lower potential than the fixed plates.

Fig. 243 diagrammatically represents **Kelvin's Multicellular Voltmeter** for use with smaller potential differences than the preceding. The movable part consists of a number of light aluminium vanes, V, fixed on a vertical spindle, S, and suspended from the top of the instrument. The fixed part consists of a number of cells or boxes, formed by a series of triangular brass plates, Q, fixed to two brass supports, B, B. The top of the moving system carries a pointer which passes over the scale of the instrument. The lower end of the spindle carries a disc, D, which, moving in oil, retards undue oscillation of the moving system and renders the instrument dead beat. Two plates, R, R, fixed to the base of the instrument limit the movements of the system V. The controlling influence, when the movable and fixed systems are connected to two points at different potentials, is the torsion of the suspending wire W.

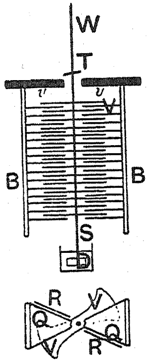

Fig. 243.

127. Comparison of Capacities.—Most laboratory methods for the comparison and determination of capacities are based on principles dealt with in current electricity, and these methods are therefore described in Chapter XVI. In this section only those methods involving principles already dealt with are discussed.

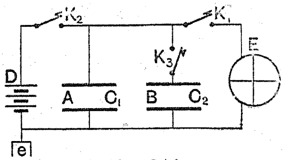

Fig. 244.

Exp. 1. *To compare the capacities of two condensers.*—Connect up the apparatus as indicated diagrammatically in Fig. 244, where A and B are the two condensers of capacities C_1 and C_2 respectively, D is a convenient battery, E a quadrant electrometer (used heterostatically), and K_1, K_2, K_3 well insulated keys; it will be noted that one pair of quadrants, one terminal of the battery, and one plate of each condenser are earthed at e.

Close K_2, thus charging A to a certain potential. Next close K_1, and note the deflection, d_1, indicated by the electrometer; d_1 is proportional to the potential, say V_1.

Open K_2, thus breaking the battery circuit. Close K_3, thus allowing A to share its charge with B, the two taking up a common potential V_2, lower than V_1. Note the deflection, d_2, indicated by the electrometer; d_2 is proportional to V_2.

If Q denote the charge given to A in the first case, then, neglecting the capacity of the electrometer, $Q = C_1 V_1$. Similarly, in the second case, $Q = (C_1 + C_2) V_2$,

i.e.
$$C_1 V_1 = (C_1 + C_2) V_2$$
$$\frac{C_1}{C_1 + C_2} = \frac{V_2}{V_1} = \frac{d_2}{d_1},$$
$$\therefore \frac{C_1}{C_2} = \frac{d_2}{d_1 - d_2}.$$

Evidently the specific inductive capacity K of any solid or liquid dielectric may be found by constructing two suitable condensers exactly alike, except that one, B, has air for dielectric, and the other, A, the given substance, and comparing their capacities as above; in this case the ratio $C_1 : C_2$ measures the specific inductive capacity of the given dielectric.

Exp. 2. *To determine the capacity of a condenser.*—In Exp. 1, if A be a standard condenser of known capacity, C_1, the capacity C_2 of a second condenser, B, may be actually measured, for in the algebraic relation given C_2 would be the only unknown.

In the above it has been assumed that the capacity of the electrometer is negligible. If this be now introduced, we have, denoting it by c,
$$Q = (C_1 + c) V_1 \quad \text{and} \quad Q = (C_1 + C_2 + c) V_2,$$
$$\therefore (C_1 + c) V_1 = (C_1 + C_2 + c) V_2,$$
$$\therefore C_2 = \frac{d_1 - d_2}{d_2}(C_1 + c),$$
in which C_1 and c are supposed known, and d_1 and d_2 are the observed deflections.

The capacity c of the electrometer may be simply determined as follows. Let two spherical conductors of capacities C_1 and C_2, measured by their radii, be charged to the same potential, and their potentials compared when joined successively to the quadrants of the electrometer. Let c denote the capacity of the quadrants of the instrument, V the common initial potential of the two conductors, and V_1 and V_2 the potentials assumed by these conductors when connected with the quadrants; then, if d_1 and d_2 denote the deflections corresponding to these potentials, we have
$$V_1 (C_1 + c) = V C_1$$
and
$$V_2 (C_2 + c) = V C_2,$$

that is,
$$\frac{V_1(C_1+c)}{V_2(C_2+c)} = \frac{C_1}{C_2}.$$

But
$$\frac{V_1}{V_2} = \frac{d_1}{d_2},$$

and therefore
$$\frac{d_1}{d_2} \cdot \frac{C_1+c}{C_2+c} = \frac{C_1}{C_2},$$

or
$$c = \frac{C_1 C_2 (d_2 - d_1)}{d_1 C_2 - d_2 C_1},$$

in which C_1 and C_2 are numerically equal to the radii and c is therefore determined.

Exp. 3. *To determine the capacity of a cable.*—This is merely a repetition of Exp. 2 above, but is introduced to illustrate the technical application of the subject. The arrangement will be understood from Fig. 245, where C is the cable under test and EV is an electrostatic voltmeter. One end, a, of the cable is left "free" and insulated. The switch S is first put on stud c, thus charging the standard condenser A, and the voltmeter EV, to the potential of the positive pole of B, and the reading V_1 volts of EV is noted. The switch S is then moved to d so that the charge is shared with the cable, and the common potential V_2 volts, indicated by EV, is noted.

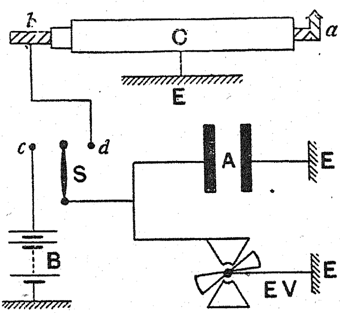

Fig. 245.

Now if Q coulombs be the charge from the battery, C farads the capacity of A, C_x farads the capacity of C, F_1 farads the capacity of the voltmeter at V_1 volts, and F_2 farads its capacity at V_2 volts; then, in the first case, $Q = (C + F_1)V_1$, and, in the second case $Q = (C + C_x + F_2)V_2$. Hence

$$(C + F_1)V_1 = (C + C_x + F_2)V_2,$$

$$\therefore\ C_x = \frac{C(V_1 - V_2) + F_1V_1 - F_2V_2}{V_2}.$$

F_1 and F_2 may be taken from data supplied with the instrument. If F_1 and F_2 be neglected—

$$C_x = \frac{V_1 - V_2}{V_2}C.$$

128. Measurement of Specific Inductive Capacity or Dielectric Constant.—As in Art. 127, only those methods based on principles already dealt with will be given at this stage.

FARADAY'S EXPERIMENTS.—Faraday constructed two similar spherical condensers, each of which consisted, as shown in Fig. 246, of an inner spherical brass ball and an outer spherical shell, made in two halves, so that the space between might be readily filled with any given dielectric. To the inner sphere was attached a thin brass rod carrying a small brass knob, and this rod, insulated by a plug of shellac, passed through the outer shell and formed the pole by which the condenser could be charged—that is, it performed the same function as the knob of a Leyden jar. In one of these condensers air was the dielectric, and into the other different substances, whose specific inductive capacities were to be determined, were successively introduced. In each case the capacities of the two condensers were compared by the method given in Art. 127, and the specific inductive capacity of the substance used as dielectric determined.

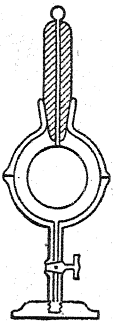

Fig. 246.

Thus, in an actual experiment, the *lower half* of one of the condensers was filled with sulphur, and the capacities of this and the equal air condenser were compared. If C_s denote the capacity of the condenser with the sulphur, and C_a the capacity of the air condenser, Faraday found that $C_s = 1{\cdot}6\,C_a$. But since only half the sulphur condenser is filled with sulphur, and the capacity of this condenser, when the sulphur is not there, is C_a, we have

$$C_s = K\frac{C_a}{2} + \frac{C_a}{2} = \frac{K+1}{2}C_a,$$

i.e. $$1{\cdot}6\,C_a = \frac{K+1}{2}C_a, \quad \therefore \frac{K+1}{2} = 1{\cdot}6,$$

i.e. $$K = 2{\cdot}2.$$

Hopkinson's Experiments.—The principle of Hopkinson's method for solids will be gathered from Fig. 247*a*. In the figure B is a battery, the middle point of which is earthed (zero potential), in which case, as will be seen later, if $+V$ be the potential of the positive pole the negative pole will have the equal but opposite potential $-V$. S is a sliding condenser consisting of two coaxial cylinders, the inner one capable of sliding to and fro so as to vary the amount of it inside the outer one, thus varying the capacity. G is a guard-ring condenser, the movable plate of which is earthed; E is an electrometer, and other earth connections are as indicated.

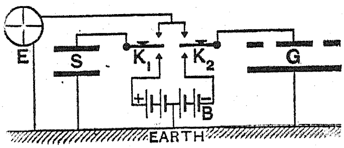

Fig. 247*a*.

On bringing the keys, K_1 and K_2, into contact with the lower studs, S is charged to a positive potential and G to an equal negative potential. Since $Q = CV$, if the capacities of S and G are equal their charges will be equal but of opposite sign. Hence on raising K_1 and K_2 to the upper studs the charges will neutralise and the electrometer will not be deflected. The sliding condenser S is adjusted, until on performing these operations there is no deflection, in which case the capacities of S and G are equal.

A slab of the solid dielectric, whose specific inductive capacity is required, is next placed on the movable plate of G, thus increasing the capacity of this condenser. S is kept fixed and the movable plate of G is lowered until, on repeating the experiment, there is again no deflection. Now

$$\text{Capacity of } G \text{ in Case } 1 = \frac{A}{4\pi d},$$

where A is the effective area of G, and d the distance between the plates. Again—

$$\text{Capacity of } G \text{ in Case } 2 = \frac{A}{4\pi\left\{(d - t + x) + \dfrac{t}{K}\right\}},$$

where t is the thickness of the slab of specific inductive capacity K and x the distance the movable plate is lowered (see Chapter VI.).

Now these capacities are equal, both being equal to that of S, *i.e.*

$$\frac{A}{4\pi\left\{(d-t+x)+\frac{t}{K}\right\}}=\frac{A}{4\pi d},$$

$$d-t+x+\frac{t}{K}=d,$$

$$K=\frac{t}{t-x}.$$

In dealing with liquids Hopkinson used a special cylindrical condenser, a section of which is shown in Fig. 247b; this takes the place of the guard-ring condenser above. With air as dielectric in the special condenser, the sliding condenser S is adjusted for no deflection and the position of the sliding tube of S is noted. The special condenser is then filled with the liquid, S again adjusted for no deflection and the position of the sliding tube of S again noted. The capacity of the special condenser is, in each case, equal to the capacity of S, and these are

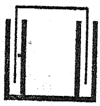

Fig. 247b.

$$\textit{Case 1. } C_1=\frac{l_1}{2\log_e\frac{b}{a}}. \qquad \textit{Case 2. } C_2=\frac{l_2}{2\log_e\frac{b}{a}}.$$

$$\therefore\ K=\frac{C_2}{C_1}=\frac{l_2}{l_1}. \quad \text{(Chapter VI.)}$$

In practice S is graduated so that its capacity is known for any number of divisions of the inner tube inside the outer tube.

Silow's Experiments.—Fig. 248a indicates the apparatus used by Silow in dealing with liquids. A cylindrical glass vessel has four vertical strips of tinfoil A, B, C, D attached to the inside. One pair of strips AC are connected together and earthed, and the other pair BD are

connected and maintained at a constant potential. The needle E consists of a horizontal arm carrying two curved pieces of platinum F, F; the needle is suspended and is also earthed. It has been shown that *when the potentials are kept constant* the force between two conductors is *directly* proportional to the specific inductive capacity of the medium (Art. 84); hence the experiment consists in observing the deflection of the needle (1) when the vessel contains air, (2) when the air is replaced by a given liquid; the ratio of the second deflection to the first will be the value of K required.

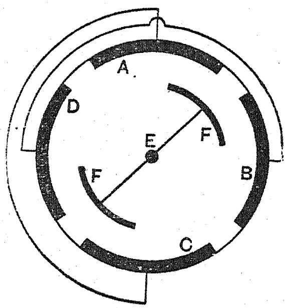

Fig. 248*a*.

Another method is to suspend the needle by a wire from a torsion head, and to note the angle the top of the wire must be turned through to reduce the deflection to zero in the two cases mentioned. If α_1 and α_2 be the required angles, then α_2/α_1 is the ratio of the capacities, and therefore the specific inductive capacity of the liquid.

Arons and Cohn's Experiments.—Cohn and Arons employed Silow's apparatus, and a quadrant electrometer

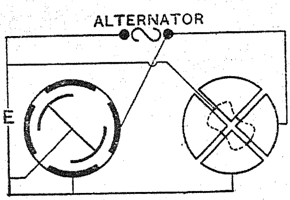

Fig. 248*b*.

connected as shown in Fig. 248*b* and used idiostatically; an alternating potential difference was employed. With

air in both pieces of apparatus, if θ_1 and θ_2 be the deflections, then (Art. 123)—

$$\theta_1 \propto c_1 (V - v)^2 \quad \text{(Quadrant)}$$
$$\theta_2 \propto c_2 (V - v)^2 \quad \text{(Silow)},$$
$$\therefore \frac{c_1}{c_2} = \frac{\theta_1}{\theta_2}.$$

The liquid is now introduced into Silow's apparatus and the deflections θ_3 and θ_4 are noted; then

$$\frac{c_1}{Kc_2} = \frac{\theta_3}{\theta_4},$$

i.e.

$$K = \frac{\theta_1}{\theta_2} \times \frac{\theta_4}{\theta_3},$$

where K is the specific inductive capacity of the liquid.

Boltzmann's Experiments on Gases.—The condenser consisted of two metal plates P and Q (Fig. 249), surrounded by an enclosure of brass, B, the whole being contained in a glass receiver. The whole could be exhausted or filled with any desired gas. Connections between the condenser plates P, Q, and the battery, electrometer, and earth were made by wires insulated from the walls of the chambers by passing through shellac plugs. A battery of 300 Daniell's cells was employed to charge the condenser.

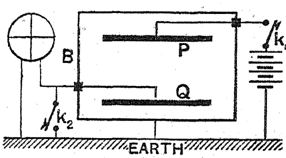

Fig. 249.

With the enclosure exhausted, and k_1, k_2 closed, the plate P is charged to a potential 300 V, where V is the potential due to one cell. Q is at zero potential.

The keys are opened and the gas admitted. If K be its specific inductive capacity the potential of P falls to $\frac{300V}{K}$. Q is still at zero potential.

P is again joined to the battery by closing k_1, an additional charge passes to P raising its potential again to

$300\ V$. Q now acquires a potential due to this extra charge which has passed to P and a deflection a is obtained on the electrometer. a depends on the amount by which the potential of P has altered; hence, we may write

$$a \propto \left(300\ V - \frac{300\ V}{K}\right) \propto 300\ V\left(1 - \frac{1}{K}\right).$$

An additional cell is added, thus altering the potential of P by an amount V. If β be the *change* in the deflection

$$\beta \propto V,$$

$$\therefore\ \frac{a}{\beta} = \frac{300\ V\left(1 - \frac{1}{K}\right)}{V} = 300\left(1 - \frac{1}{K}\right),$$

i e.
$$K = \frac{300\ \beta}{300\ \beta - a}.$$

BOLTZMANN'S EXPERIMENTS ON CRYSTALLINE SULPHUR. —In Art. 102 it has been shown that in the case of a *conducting* sphere in a uniform field F_1 the intensity at an external point is changed as if there were an electric doublet at the centre of moment F_1R^3, where R is the radius of the sphere, *i.e.*

$$\text{Doublet in case of Conducting Sphere} = F_1R^3.$$

Again, in Art. 104 it has been shown that in the case of a *dielectric* sphere in a uniform field F_1, the moment of the equivalent doublet is $(F_1 - F_2)\,R^3$ where F_2 is the field inside the sphere and R the radius. It is also shown in Art. 104 that in this case $F_2 = 3F_1/(K + 2)$: substituting this in the expression $(F_1 - F_2)\,R^3$ we get:—

$$\text{Doublet in case of Dielectric Sphere} = \frac{K-1}{K+2}F_1R^3,$$

i.e. it is $\dfrac{K-1}{K+2}$ of the doublet in the case of the equal conducting sphere. Thus the action of the dielectric sphere on any external charge will be $\dfrac{K-1}{K+2}$ of the action

of an equal conducting sphere on the charge. Conversely the action of an external charge on the dielectric sphere will be $\frac{K-1}{K+2}$ of the action on the equal conducting sphere.

Boltzmann applied this to the determination of the dielectric constant of crystalline sulphur. A small conducting sphere was suspended by silk threads from one end of a torsion arm. An electrified sphere was then brought into the vicinity and the pull ascertained. The conducting sphere was then replaced by an equal sulphur sphere and the experiment was repeated. From the ratio of the pulls on the conducting sphere and on the sulphur sphere the value of K for sulphur was found. The sulphur sphere was arranged so that the three axes of the crystal were in turn pointing towards the charged sphere and the values of K parallel to the three axes were found.

GENERAL—*Rosa* determined the dielectric constant of liquids by measuring the attraction between two plates situated first in air and then in the liquids. One plate was fixed and the other suspended from a torsion arm, the torsion measuring the attraction. A battery was used to charge the plates, but by means of a commutator the charges on the plates were reversed 2,000 and more times per minute. As the potential difference between the plates is the same in air and in the liquid, the charges on each are K times greater in the liquid, and therefore the force of attraction is K times greater in the liquid (see Example 4, Art. 84). Rosa compared the forces in air and in the liquid and determined K.

Nernst in experimenting on liquids constructed a suitable condenser to contain the liquid, and used a Wheatstone Bridge (Art. 224) with an alternating current.

Drude employed electric waves and measured the wave length δ_1 in air and δ_2 in the liquid, calculating K from the relation $K = \delta^2{}_1/\delta^2{}_2$; his method is described in Chapter XXII. In the case of gases it has been found that the change in specific inductive capacity is proportional to the change in pressure; taking K for a vacuum as unity,

VALUES OF K (AIR = 1).

Substance.	K.	Authority.
Ebonite	3·15	Boltzmann
Paraffin	2·32	,,
Mica	6·64	Klemencic
Glass-light flint ...	6·72	Hopkinson
,, -dense flint ...	7·37	,,
,, -extra dense flint	9·90	,,
,, -crown	6·96	,,
Turpentine	2·23	,,
Olive Oil	3·16	,,
Water	76	Cohn and Arons
,, ...	80·6	Drude
,, at 15° C.... ...	80	Fleming and Dewar
,, at − 185° C. ...	2·44	,, ,,
Air (Vacuum = 1) ...	1·00059	Boltzmann

the specific inductive capacity of a gas at pressure $P\,mm$ is given by

$$K = 1 + Z\frac{P}{760},$$

where Z is a constant.

The effect of temperature on specific inductive capacity has been investigated by various experimenters. *Cassie* found that for carbon bisulphide, glycerine and paraffin oil there is a slight *decrease* with rise in temperature, for glass and ebonite a more pronounced *increase* with rise in temperature. In the case of water *Heerwagen* gives the following formula—

$$K_t = 80{\cdot}878 - {\cdot}362\,(t - 17^\circ).$$

Dewar and Fleming's experiments at very low temperatures gave the specific inductive capacity of ice at − 200° C. as 2·43, rising to 70·8 at − 7·5° C. The specific inductive capacities of a few selected substances are indicated above.

Exercises VIII.

Section A.

(1) Describe the Torsion Balance and Cavendish Proofs of the Law of Inverse Squares.

(2) In an experiment with the torsion balance the lever is deflected through 80°. Find the angle the torsion head must be turned through to reduce the deflection to 45°.

(3) Establish the working formulae for the Quadrant and the Attracted Disc Electrometers.

(4) Describe Hopkinson's experiments on the specific inductive capacities of solids and liquids.

Section B.

(1) Two Leyden jars are exactly alike, except that in one the tinfoil coatings are separated by glass, and in the other by ebonite. A charge of electricity is given to the glass jar, and the potential of its inner coating is measured. The charge is then shared between the two jars, and the potential falls to 0·6 of its former value. If the specific inductive capacity of ebonite be 2, what is that of the glass? (B.E.)

(2) An insulated plate, 10 cm. in diameter, is charged with electricity and supported horizontally at a distance of 1 mm. below a similar plate suspended from a balance and connected to earth. If the attraction is balanced by the weight of one decigram, find the charge on the plate. (g = 980 C.G.S.) (B.E.)

(3) Describe some form of quadrant electrometer, and explain how it may be used to compare the E.M.F's. of two batteries. (B.E.)

(4) Explain what is meant by difference of potential, and describe some method by which it can be measured in absolute units. (B.E.)

(5) The terminals of a condenser, with mica as the dielectric, are connected to a quadrant electrometer, and the condenser is charged so that the scale deflection is 90. When a second condenser of the same dimensions as the first, but having paraffin-wax as the dielectric, is connected in parallel with the first, the deflection falls to 30 divisions. Compare the dielectric constants of mica and paraffin-wax. (B.E.)

(6) If one pair of quadrants of a quadrant electrometer is connected with the earth, and if a constant *charge* is given to the other pair of quadrants, the deflections of the electrometer will be a maximum when the potential of the needle has a certain value. any

increase of the potential beyond this value diminishing the deflection; explain this result. (B.E. Hons.)

(7) How would you determine the specific inductive capacity of a solid substance, being given a slab of the material in question? (B.E. Hons.)

Section C.

(1) How would you compare the capacity of a Leyden jar with that of a standard condenser by some one method? Describe the precautions that must be taken to secure an accurate result. (Inter. B.Sc.)

(2) Define potential, capacity, and specific inductive capacity. Describe any instrument which may be advantageously employed for the measurement of differences of potential. (Inter. B.Sc.)

(3) Describe some form of quadrant electrometer and deduce a formula for use with it. (B.Sc.)

(4) Describe some form of absolute electrometer and give the theory of its action. (B.Sc. Hons.)

CHAPTER IX.

ELECTROSTATICS.—INDUCTION MACHINES AND ATMOSPHERIC ELECTRICITY.

129. Introductory. The Electrophorus.—In the present branch of our subject any mechanical device for the *rapid production of electric charges* may be termed an electrical machine, but an important feature in connection with the more modern machines dealt with in this chapter is their power of *developing large potential differences*.

The older types, such as the "cylinder" and "plate" machines, depended essentially on *friction*, but these are now practically obsolete; in the newer forms *influence* or *induction* is the main factor on which the action depends.

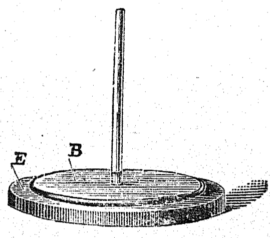

Fig. 250.

An instrument in which an *initial small frictional charge* is caused to produce, *by induction*, a large number of additional small charges, is known as the **electrophorus.** As usually constructed, it consists (Fig. 250) of a disc of ebonite, *E*, and a plate of brass, *B*, of slightly smaller diameter, to which is attached an insulating handle of shellac-varnished glass or ebonite. When in use the lower side of *E* must be earthed, either by resting it direct on the table or by fixing it to a brass or tinfoil base, called the *sole*, which in turn rests on the table.

The ebonite disc is excited by rubbing it with fur, so that it acquires a negative charge (Fig. 251), and a negative potential is established in the air field above. The brass plate is then lowered on the disc, during which operation it acquires the negative potential of the field and inductive displacement ensues, its lower surface becoming positively charged and its upper surface negatively charged. The brass plate is next earthed by momentarily touching it with the finger, the result being that electricity flows into it until its potential becomes zero and the plate exhibits only a positive charge. On removing the plate from the disc, the former attains a positive potential due to its acquired positive charge; this positive charge may be communicated to another body, and the whole series of operations may be repeated several times without again exciting the ebonite disc.

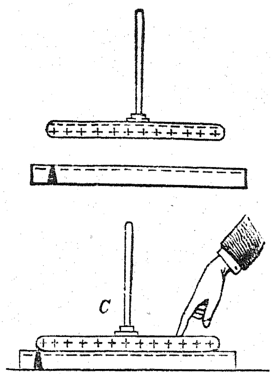

Fig. 251.

It has been mentioned that the lower surface of the ebonite disc must be earthed, preferably by fitting it to a metal *sole*, which rests on the table when the instrument is in use, and the importance of this earthing will be readily understood. The negative charge developed by friction on the upper surface of the disc produces by induction an equal positive charge on the walls and other earth-connected bodies in the vicinity. By far the greater portion of this induced positive charge will, however, be developed on the metal sole, and the attraction between this and the negative charge on the disc draws the latter charge partly within the disc, thus reducing the tendency of the ebonite to lose its charge.

Further, it has been stated that when the brass plate is earthed by touching it with the finger, electricity flows

into it until its potential is zero, and it is important to notice that this electricity mainly comes from the earth-connected sole, i.e. *the charge taken away when the plate is removed is mainly derived from the sole.*

Thus, suppose the charge on the ebonite disc be -20, and let $+18$ denote the induced charge on the sole, and therefore $+2$ the induced charge on the walls, etc., of the room. On placing the plate on the disc and earthing the former, the whole inductive action of the disc is concentrated on the sole and plate, but the latter will contain the greater portion of the induced charge since it is nearer to the inducing charge. Hence the distribution of the induced charge may now be represented by, say, $+17$ on the brass plate and $+3$ on the sole, *i.e.* $+15$ units have passed *from the sole to the plate* when the latter was earthed. On removing the plate the charge $+17$ is removed with it, and the original distribution is again attained, *i.e.* electricity passes *into the sole* until the charge there becomes $+18$, and $+2$ appears on the walls, etc., as before.

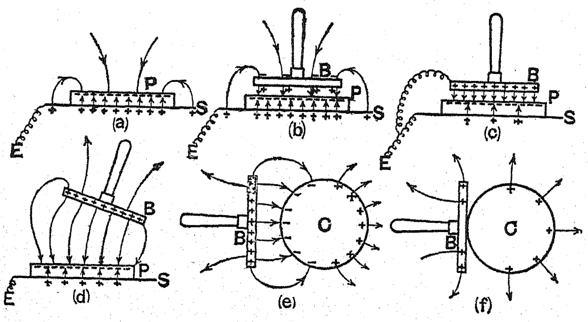

Fig. 252.

Fig. 252 depicts the tubes of induction in the process of charging an electrophorus, and then allowing the brass plate to share its charge with an insulated conductor C; the reader will readily grasp the details from the figure.

It should be noticed that when the brass plate is laid on the ebonite disc it does not receive a charge by conduction, but is acted on inductively. This is due to the fact that, neither surface being truly plane, the points of actual contact are very few, and the ebonite, being an insulator, gives up only an inappreciable fraction of its charge, viz. that spread over the small area of actual and intimate contact. Thus the charging of the plate does not practically diminish the charge on the disc, and a theoretically unlimited number of charges can be obtained on the metal plate from the single frictional charge on the ebonite disc; leakage, of course, interferes with this.

To do away with the necessity of touching the plate with the finger each time it is charged, a brass pin connected with the sole is sometimes let into the ebonite disc in the way shown in Fig. 251. The brass plate, when laid on the ebonite, makes contact with the point of the pin, and is thus connected to earth as required.

As previously indicated, the electrical *energy* possessed by the plate is equivalent to the work done in overcoming the mutual attraction between the negative charge on the disc and the positive charge on the plate when the latter is removed from the former.

130. Principle of Action of an Induction Machine.—A careful consideration of the following experiment will enable the reader to grasp the principles involved in the action of induction machines.

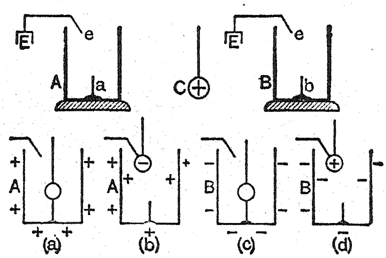

Fig. 253.

Exp. Let A and B (Fig. 253) be two metal cans (which may be called *inductors* or *armatures*), fixed on insulating stands and placed some distance apart. Although not essential in this experiment, let the cans be provided with springs, a, b (called the *contact springs*), projecting from their inner surfaces. Two springs, e, e, are earth-connected and fixed as indicated, *i.e.* one projecting into each can. Finally, C is a metal ball (called the *carrier*) attached to an insulating rod.

Give the carrier C, say, a small positive charge, and then perform the following operations :—

(1) Lower the carrier C into the inductor A, allowing it to touch the contact spring a: it gives up the whole of its charge, which passes mainly to the outside of A, i.e. *A has now a small positive charge* (Fig. 253 (a)).

(2) Raise C and allow it to touch the spring e. Since C is at a positive potential (Art. 75) electricity will flow out of it when it touches e until its potential becomes zero, i.e. *C acquires a small induced negative charge* on contact with e (Fig. 253 (b)).

(3) Lower the carrier C into the inductor B, allowing it to touch the contact spring b: as before, C becomes discharged, the negative charge mainly appearing on the outside of B, i.e. *B has now a small negative charge* (Fig. 253 (c)).

(4) Raise C and allow it to touch the spring e. Since C is at a negative potential (Art. 75) electricity will flow into it when it touches e until its potential becomes zero, i.e. *C acquires a small induced positive charge* on contact with e (Fig. 253 (d)).

(5) Again lower C into A, allowing it to touch a. The charge again leaves C and passes to A, i.e. *A has now a greater positive charge.*

(6) Repeat operation (2). C acquires, as before, an induced negative charge on touching e, and, since the charge on A is greater, this induced negative charge on C is greater.

(7) Again lower C into B, allowing it to touch b. C again becomes discharged, and *B acquires a greater negative charge.*

(8) Repeat operation (4). C again acquires an induced positive charge on touching e, and, since the charge on B is greater, this induced positive charge on C is greater.

These operations may be repeated several times, *the inductors A and B acquiring larger and still larger charges of opposite sign, out of the initial small charge, on the compound interest principle.* The accumulative action will, in fact, go on until A and B are charged as highly as the circumstances of their insulation will permit, *i.e.* to such a potential that their loss of charge, owing to imperfect insulation and air leakage, is as great as their gain from the carrier C. If the inductors A and B were each fitted with a metal rod terminating in a ball, and if arrangements were such that the balls were fixed at a short distance apart, a spark would pass between them when the difference of potential became sufficiently great to overcome the resistance of the intervening air; the balls

would, in fact, correspond to the "poles" of the modern machines, although, in most cases, this arrangement of the poles is somewhat modified in practice.

131. The Kelvin Replenisher.—The construction and action of this instrument will be understood from Figs. 254 and 255, which are lettered to correspond with the previous experiment. The inductors *A* and *B* are two portions of a metal cylinder. Each inductor is insulated and fitted with a contact spring (*a* and *b* in Fig. 255). The springs *e*, *e*, projecting into *A* and *B*, are connected by the strip of brass, *M*, which runs round the ebonite base on

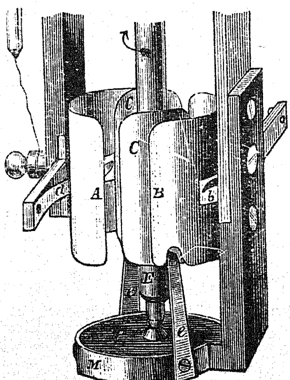

Fig. 254.

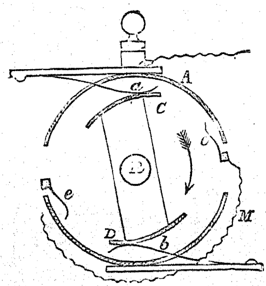

Fig. 255.

which the spindle for rotating the carriers *C*, *D*, rests; *M* need not be insulated. There are two carriers, *C*, *D*, also portions of a metal cylinder, which are fixed to an insulating bar and capable of rotation about the axis *R*.

Imagine *A* to have a small positive charge and that the spindle is turned in the direction of the arrow until *C* is in contact with the spring *e* on the right and *D* with the spring *e* on the left (Fig. 255); *C* becomes negatively charged and *D* positively charged. When *C* comes into

contact with b, D also comes into contact with a, with the result that C imparts its negative charge to B and D its positive charge to A. When C and D again touch the springs e, e, both inductors act so as to charge D negatively and C positively. When D reaches b and C reaches a, D imparts its negative charge to B and C its positive charge to A, and the actions are repeated as rotation continues. Thus the rotation results in A acquiring a strong positive charge and B a strong negative one. The *energy* of the collected charges has its equivalent in the work done in separating the oppositely charged carriers and inductors.

The instrument was not intended to produce large charges however, but for use with the Kelvin electrometer to maintain the Leyden jar at a constant potential—one of the inductors (say A) is connected to the inner coating of the jar and the other to earth, and when the potential of the jar shows signs of falling a few turns of the replenisher quickly restore it to its original value, which is indicated by a suitably constructed gauge (Art. 123).

132. The Kelvin Water Dropper.—This apparatus is represented in Fig. 256. It consists of four insulated metallic cylinders A, A', B, B', connected in pairs as indicated, the lower ones B, B' being fitted with funnels C, C'; D, D' are the fine nozzles of a metal pipe E, earth-connected, from which water is permitted to issue in drops.

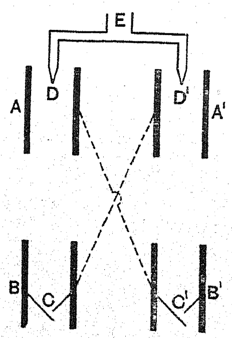

Fig. 256.

Let A (and therefore B') be given a slight positive charge and imagine a drop of water just issuing from the nozzle D. Being earth-connected and within a hollow positively charged conductor, the drop acquires a negative charge which it gives up to B (and therefore A') when it strikes the funnel C; this is repeated by each drop issuing from the nozzle D.

Consider now a drop of water just leaving D'. Since A'

is negative the drop acquires a positive charge which it gives up to B' (and therefore A) when it strikes the funnel C'; this is repeated by each drop issuing from the nozzle D'.

Thus the positive charge on one system (AB') and the negative charge on the other (BA') tend to increase until leakage interferes, or B and B' become so strongly charged that the drops are repelled and do not enter.

This repulsion between a drop and the like charged cylinder which it is approaching reduces the velocity of the drop, and the consequent decrease in the kinetic energy of the drop is a measure of the electric energy acquired by the cylinder.

Further, it can be shown mathematically that if V be the potential of the system A, V' the potential of the system A', C the capacity of a drop, N the number of drops per second and L the coefficient of leakage, then $V - V'$ will *increase* as long as NC is greater than L.

133. The Wimshurst Induction Machines.—Of the modern machines—the Holtz, Voss, Varley, Wimshurst—the best and most frequently used in this country is that of Wimshurst; Fig. 257 depicts the actual machine, while Fig. 258 will serve to explain the action.

The machine consists essentially of two vulcanite or shellac-varnished glass plates capable of rotation in opposite directions about a horizontal axis. On the sides of the plates remote from each other are fixed a number of metal sectors; in Fig. 258 the sectors on the front plate are represented by the inner broken circle, those on the back plate by the outer and thinner broken circle. Standing at each side is a U-shaped conductor, each carrying on the inside two rows of sharp points (called the combs) facing the plates; these are connected to the "dischargers" or "poles" of the machine X, Y, and to the inside coatings of two condensers K, K, the object of the latter being to increase the capacity and to render the accumulation of heavy charges possible (Art. 105). Lying across the front plate, at an angle of about 45° with the combs, is a metal rod BC, terminating in wire brushes, which graze the sectors as the plate rotates. Across the back plate and, roughly speaking, at right angles to BC is a second metal

rod *EH*, terminating in brushes; for convenience this is drawn outside in the figure.

Disregarding for a moment the "earthing" of the rods *BC* and *EH*, and imagining the sector *A* to have a slight positive charge given to it, the action of the machine may be described as follows:—*A* acts inductively on the conductor *BC*, causing the sector *B* to acquire a negative charge

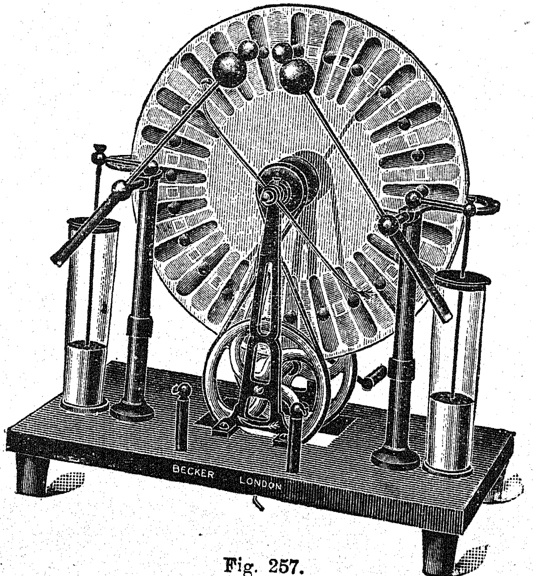

Fig. 257.

and *C* a positive one. Concentrating our attention on the front plate, *B* moves to the right with its negative charge, and on reaching *F* acts inductively on *EH*, making the sector *E* on the back plate positive and *H* negative Leaving *F*, the front sector moves into the comb on the right, where it again acts inductively, causing the points to become positive and the discharger *Y* negative. The density at the points is so great that particles of dust are

attracted, charged, and repelled; the air is also "ionised" (Chap. XXIII), the negative ions drawn to the points and the positive ions repelled; thus the comb loses its charge, leaving the discharger negative, and the positive repelled carriers coming in contact with the negative sector neutralise it, so that it comes out uncharged. When the front sector reaches *C* it acquires a positive charge, when it

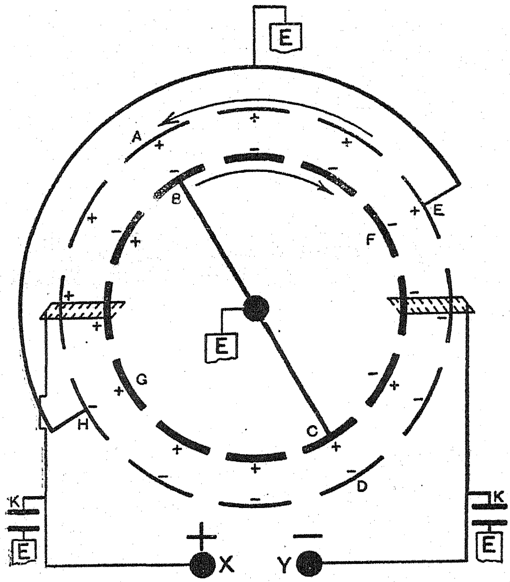

Fig. 258.

reaches *G* it acts inductively on the back plate, causing *H* to become negative and *E* positive, and when it enters the comb on the left, actions similar to those already explained take place, the result being that the discharger *X* becomes positive, and the sector comes out uncharged. At *B* the sector becomes negative and the action is repeated.

Turning to the back plate, the sector *E* moves to the

left with its positive charge. At A it acts inductively on the front plate, making the sector at B negative and that at C positive; it then enters the comb, with the result that X acquires a further positive charge. On reaching H the sector in question becomes negative, at D it acts inductively on CB, making C positive and B negative; it then enters the comb, with the result that Y acquires a further negative charge and the action is repeated. Thus the rotation of both plates results in X acquiring a strong positive charge and Y a strong negative one.

In the above we have assumed that a small charge is given to A; in practice the small difference of potential between different parts is usually sufficient to start the action.

Compound machines consisting of four, six, eight, and more plates are also constructed.

134. A few Details of the Discharge from an Induction Machine.—When the gradient of potential in the insulating medium (air) is raised sufficiently high by the approach of the positive and negative poles of the machine to each other, the medium breaks down under the stress, and the energy of the field is liberated by *disruptive discharge* between the poles. The appearance presented may be either that of a single or branched line of light (straight, curved, or zig-zag) from one pole to the other, when it is called a "**spark,**" or that of a brush-like glow, with branches diverging from a stem springing from one of the conductors, when it is called a "**brush.**"

The discharge takes the spark or brush form according as the quantity of electricity to be discharged is large or small. When the quantity is large and the distance small the spark is short, straight, and intense.

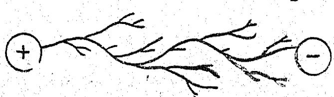

Fig. 259.

As the distance increases the spark line ceases to be straight, and takes a branching form similar to that shown in Fig. 259. It should be noted that in a spark of this kind the tips of the branches point from the

positive to the negative pole. With a limited quantity of electricity the discharge takes the *brush* form. A bright luminous brush of a light violet colour branches from the positive terminal in the way shown in Fig. 260. The negative terminal is covered with a soft luminous glow showing here and there small bright star-like points where any small irregularities break the smoothness of its surface. Fig. 261 shows, according to Faraday, the difference between

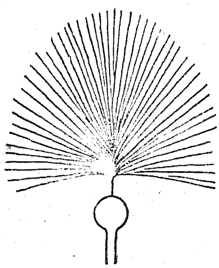

Fig. 260.

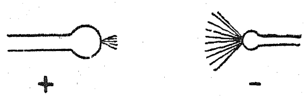

Fig. 261.

the brush from a positive and negative pole.

The brush discharge is accompanied by a sharp hissing sound readily distinguished from the sharp crackle of spark discharge. When it takes place between two terminals, such as the poles of a Wimshurst machine, which are supplied with electricity at a uniform rate, the difference of potential between the terminals remains practically constant during the discharge. In the case of a spark under the same conditions, the difference of potential rises to a maximum at the instant of discharge and at once falls to zero to rise again to a maximum for the next spark.

When a Wimshurst machine is worked without the Leyden jars the discharge between the poles is, for anything but short distances, of the brush form, but when the jars are used the capacity of the terminals is increased, and a sufficient quantity accumulates when the maximum difference of potential is reached to give a spark discharge. Experiments with a Wimshurst machine show that, for a given distance between the poles, the difference of potential necessary to produce brush discharge between terminals of low capacity is practically the same as that necessary to give spark discharge between terminals of large capacity.

The nature of the discharge is also affected by the shape of the opposing surfaces; rounded surfaces tend to produce sparks and pointed ones brushes.

The distance which separates the two conductors when the discharge takes place is called their "striking distance," and for a given pair of conductors similarly placed it is found to be very nearly proportional to their potential difference, but for differently shaped pairs (or even for the same pair with differently shaped portions of their surfaces facing one another) it depends also upon the shape. Thus for a pair of spheres the striking distance is about one inch per 100,000 volts, while for a point facing a flat plate it is much greater, viz. about one inch per 20,000 volts; and for a pair of points greater still. There appears to be no practical difference so far as striking distance is concerned between the "spark" and the "brush," that is, taking a given pair of conductors in a given relative position and at a given difference of potential, and causing them gradually to approach, we do not get a brush at one distance and a spark at another, but we get *either a spark or a brush* at a definite distance.

It should be carefully noted that the term "striking distance" refers to the *starting* of the discharge and not to its *maintenance*. For example, suppose we have two spheres which by means of a machine are kept at a potential difference of 10,000 volts; then in order to start a spark between them they must be brought to within $\frac{1}{10}$ inch (the striking distance), but after the spark has once started they may be gradually separated to a much greater distance and it will still continue.

The numbers given above suppose the discharge to take place in air at the ordinary pressure. For solid (and liquid) dielectrics the striking distance is *ceteris paribus* much less owing to their greater mechanical rigidity. For the same pressure the striking distance is under similar conditions much the same in all gases, but when the pressure is diminished the striking distance increases up to a certain point, and afterwards diminishes until when the exhaustion is very complete no discharge occurs however great the difference of potential. For high but

not extreme exhaustion the appearance is neither that of the spark or brush, but of delicate striae, and is known as the electric "glow."

Maxwell defined **the "electric strength" of a gas as the greatest "electromotive intensity," or "potential difference per unit length," which the gas can bear before spark discharge takes place.** It is found, however, that the electromotive intensity does not depend upon the gas only, but is influenced more or less by the material, shape, sign, and distance apart of the electrodes, the nature of their surface, and the character of the intervening field. The electric strength of a gas is not therefore a characteristic specific property of the gas.

The relation between spark length and potential difference (and therefore between spark length and electric strength or electromotive intensity or P.D. *per unit length*) has been the subject of much research by Kelvin, Baille, Liebig, and others. Liebig's results, given in the table below, for air at atmospheric pressure are typical.

POTENTIAL DIFFERENCE AND SPARK LENGTH.

[*C.G.S. Electrostatic Units and Centimetres* (1 *C.G.S. e.s unit* = 300 *volts*).]

Spark Length in Centimetres.	Potential Difference.	Electro-motive Intensity.	Spark Length in Centimetres.	Potential Difference.	Electro-motive Intensity.
·0066	2·630	398·5	·2398	30·662	127·7
·0105	3·357	319·7	·2800	35·196	125·7
·0143	4·017	280·9	·3245	39·816	122·7
·0194	4·573	235·7	·3920	47·001	119·9
·0245	5·057	206·4	·4715	55·165	117·0
·0348	7·190	206·6	·5588	63·703	114·0
·0438	8·863	195·5	·6226	69·980	112·4
·0604	10·866	179·9	·7405	82·195	111·0
·0841	13·548	161·1	·8830	95·540	108·2
·0903	13·816	153·0	·9576	102·463	107·0
·1000	15·000	150·0	1·0672	110·775	103·8
·1520	20·946	137·8	1·1440	117·489	102·7
·1860	24·775	132·2			

The curves shown in Fig. 262 exhibit these results graphically. The nearly straight line shows the relation between spark length and potential difference, and the other curve that between electromotive intensity (P.D. per unit length) and spark length.

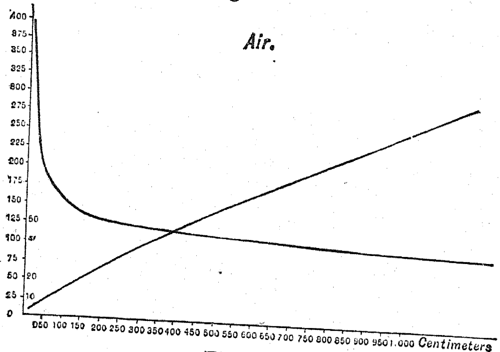

Fig. 262.

It will be seen from these results that the electromotive intensity is much greater for short than for long sparks; this was first discovered by Kelvin in 1860.

It has been shown from the data of Baille's and Liebig's experiments that the relation between spark length and potential difference may, for sparks more than 2 mm. long, be given by the relation

$$V = a + bl,$$

where V is the potential difference, l the spark length, and a and b constants. Baille's results for air gave $a = 4{\cdot}997$ and $b = 99{\cdot}593$, with l in centimetres and V in electrostatic units.

There is evidence from experiments conducted by Mr. Peace at the Cavendish Laboratory that the potential difference decreases as the spark length decreases down to a *minimum* value for a very short length and then *increases* with further decrease of the spark length. The curve shown in Fig. 263 indicates generally the nature of the relation between spark length and potential difference. At atmospheric pressure the minimum potential difference

occurs at such a small value of the spark length that accurate measures of the relation in the neighbourhood of this critical value cannot well be made. At pressures below atmospheric pressure, however, the minimum potential difference corresponds to longer spark lengths and the existence of a minimum value can be clearly established.

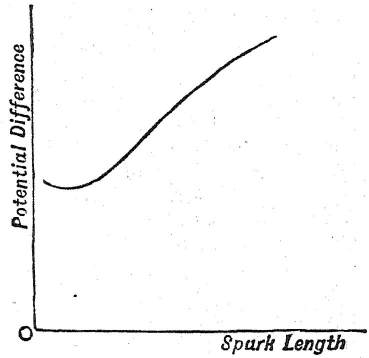

Fig. 263.

Baille has published results giving the potential difference and spark length for discharge between two equal spherical electrodes of different diameters. These results, some of which are given in the following table, show that the potential difference for a given spark length varies with the diameter of the spherical electrodes, increasing as the diameter decreases, but reaching a maximum value for a certain diameter which is greater the greater the spark length.

POTENTIAL DIFFERENCES (*C.G.S. e.s. units*): PRESSURE 760 MM., TEMPERATURE 15° TO 20° C.

Spark length in cm.	Planes.	Spheres 6 cm. in diameter.	Spheres 3 cm. in diameter.	Spheres 1 cm. in diameter.	Spheres ·6 cm. in diameter.	Spheres ·35 cm. in diameter.	Spheres ·1 cm. in diameter.
·10	14·70	14·78	14·99	15·25	15·53	16·04	**16·10**
·15	20·20	20·31	20·47	21·28	21·24	**21·87**	19·58
·25	30·38	30·99	31·33	32·10	**32·33**	31·96	23·11
·35	40·45	41·45	41·47	**42·48**	42·16	39·39	25·34
·40	44·80	45·00	45·00	**45·50**	44·80	41·07	26·58
·45	49·63	50·33	49·63	**52·04**	48·42	43·29	28·49
·50	54·35	**55·06**	54·96	54·66	53·25	47·21	30·00
·70	74·09	**75·40**	73·79	72·28	64·22	56·47	32·92
1·00	105·49	**112·94**	104·69	83·05	72·38	59·49	36·24

The nature of the influence of the shape of the electrodes which determine the nature of the intervening field is shown by the curves given in Fig. 264. These curves, taken from a paper by De la Rue and Müller, show the variation of striking distance with potential difference for different forms of electrodes. The cells by which the potential difference was measured had an electromotive force (Art. 153) of 1·03 volts each.

The relation between spark length and potential difference is, in general character, the same in all gases, but the quantitative details vary with the gas.

The influence of gas pressure on the relation between spark length and potential difference is of great

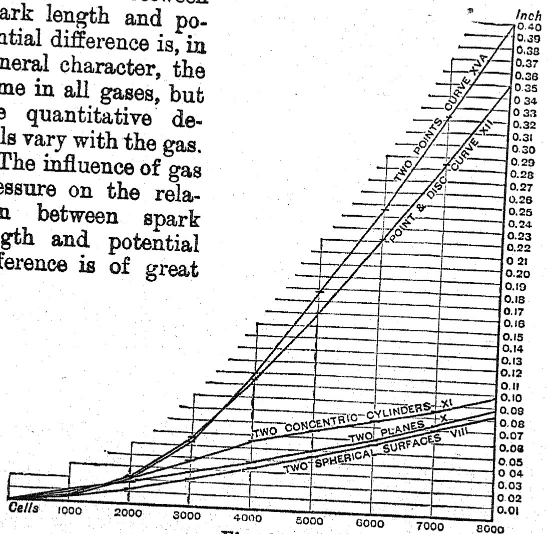

Fig. 264.

interest. The general result of change of pressure is that, for a given spark length, the potential difference decreases as the pressure decreases, down to a minimum value, and then increases. The **critical pressure** at which the potential difference attains its minimum value is lower the greater the spark length, but the minimum potential difference, although it increases slightly as the spark length increases, is roughly constant at a little more than

300 volts. The following table gives numerical results for short sparks in air.

Spark Length.	Minimum Potential Difference.	Critical Pressure.
·00100 cm.	326 volts.	250 mm.
·00254 cm.	330 volts.	150 mm.
·00508 cm.	333 volts.	110 mm.
·01016 cm.	354 volts.	55 mm.
·02032 cm.	370 volts.	35 mm.

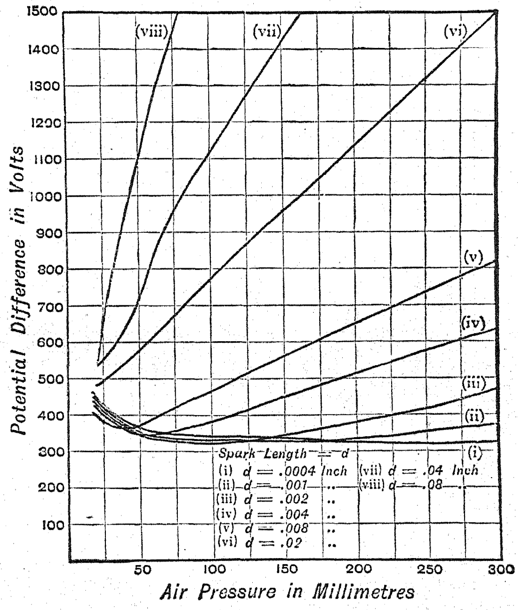

Fig. 265.

Fig. 265 exhibits graphically the results of experiments, made by Mr. Peace at the Cavendish Laboratory, on the

relation between potential difference and gas pressure at given spark lengths. From these results Mr. Peace also gives curves showing the relation between potential difference and spark length at different pressures.

These curves are given in Fig. 266 and show clearly that at each pressure there is a critical spark length at which

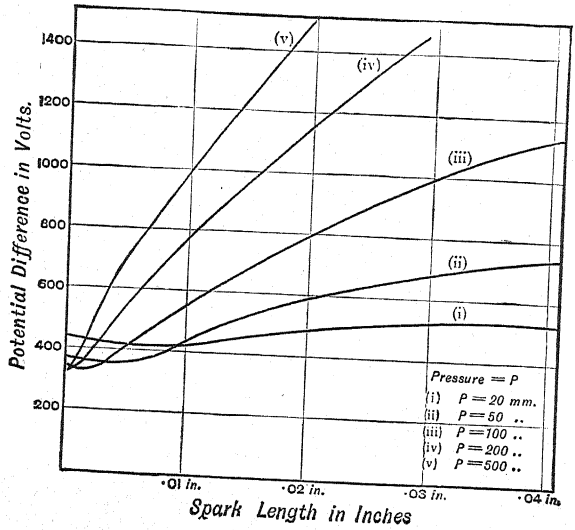

Fig. 266.

the potential difference attains a minimum value, and that this critical spark length increases with *decrease* of pressure.

According to Paschen, "*if the product of the gas pressure and spark length is kept constant, the potential difference required to produce the spark is constant*"; this is known as **Paschen's Law.**

Experiment shows that there is a certain interval of time, known as the "lag," between the application of a potential difference and the passage of the spark, and,

further, that this lag is reduced and sparking accelerated by illuminating the negative electrode with ultra-violet light. These phenomena may be explained by assuming the presence of a few *negatively charged ions* (Chapter XXIII.) in the air between the electrodes. When the high potential difference is attained these ions acquire a high velocity, and, colliding with other molecules, produce further ionisation, so that, as explained in Chapter XXIII., the air attains increased conductivity and the spark passes. Ultra-violet light assists ionisation and resulting conductivity, and therefore assists sparking and reduces "lag."

The colour of a spark is due chiefly to the volatilisation of particles of the metal terminals in the path of the spark, but it is also influenced by the medium itself and by particles of foreign matter therein.

When a disruptive discharge of any form is examined by means of a rapidly rotating mirror it is found to be of an *oscillatory* character. Thus the image of a short straight spark seen in a revolving mirror is a number of short straight parallel lines of light separated by narrow intervals corresponding to the period of oscillation of the discharge.

Modern research and theory on the discharge of electricity through gases are dealt with in Chapter XXIII.

When the discharge takes place *through a conductor* it is accompanied by all the effects produced by an electric current in the conductor. The heat developed may be made to produce fusion or volatilisation of the metal of the conductor. A magnetic field is developed round the conductor, and a magnetic needle is deflected or a piece of iron may be magnetised. Liquids may be decomposed. When the conductor is of bad conducting material, offering a high resistance to the discharge, violent mechanical effects are often produced. When the discharge takes place through the human body as a conductor, the physiological effects which accompany the *shock* are found to depend on the energy of the discharge, that is, upon the quantity discharged as well as upon the difference of potential, and also to an important degree upon the time rate of discharge. The larger the quantity and the

potential difference, and the shorter the time of discharge, the greater the shock.

135. Lichtenberg's Figures.—The difference between discharge from a positive and from a negative electrode is illustrated by the figures obtained when the discharge from the electrode spreads over a plate of insulating material covered with a badly conducting powder. Thus, if we cover a plate with a finely powdered mixture of red lead and sulphur and take the discharge from a positively charged electrode to the plate, the sulphur, which by friction with the red lead is negatively electrified, is attracted to the lines of the positive discharge over the plate, and marks out a pattern similar to that shown in Fig. 267. Similarly the lines of a negative discharge over the plate are indicated by the positively charged red lead, and a pattern of the form shown in Fig. 268 is obtained.

If a plate of glass or other insulating material is covered with a fine powder, such as lycopodium powder, and a discharge passed over the surface, between two pointed electrodes in contact with it, the powder arranges itself in beautifully branched moss-like figures, showing a distinct difference between the positive and negative centres. Fig. 269, due to Joly, shows this effect.

136. Potential at a Point in the Air.—It has been found by direct measurement that the potential at a point in the open air is always different from the potential of the earth, and usually higher than it. A satisfactory method of measuring this potential is by means of a *water dropper*. A metal cistern, fitted with a tap having a very fine nozzle, is filled with water, carefully insulated, and fixed in position so that the end of the nozzle is at the point in the air at which the potential is to be measured. Water is allowed to drop rapidly from the nozzle, and owing to the inductive action on the drops as they are detached from the nozzle, the cistern gradually becomes charged up to the potential at the point of the nozzle. If the cistern is connected to an absolute electrometer, or to one pair of quadrants of a quadrant electrometer, the potential it attains when equilibrium is set up can be directly measured, and this gives the potential of the air at the point selected.

A more modern method is to employ a wire tipped with radium or other radio-active substance; this ionises the air, and the charge on the end of the wire is neutralised by the oppositely charged ions (Chapter XXIII.).

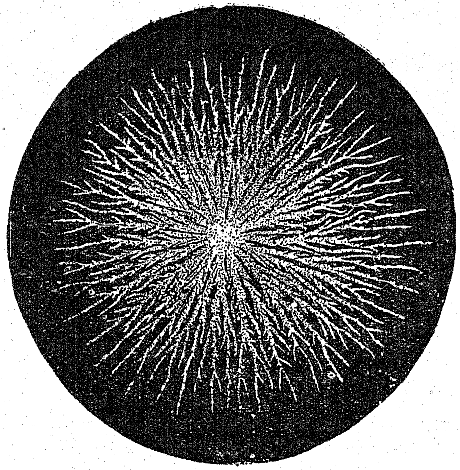

Fig. 267.

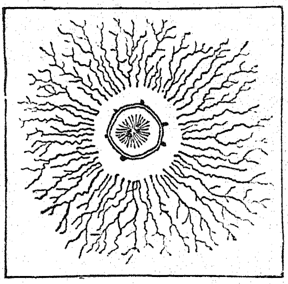

Fig. 268.

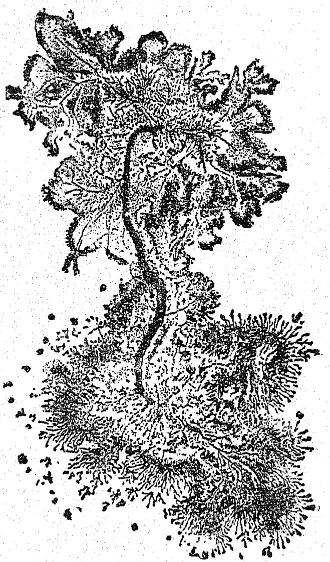

Fig. 269

The potential at a point in the air is, in fine weather, always positive, and increases with height above the ground. The rate of increase with height is very variable. Measurements made by Lord Kelvin in Scotland gave results varying from 20 to 40 volts per foot, but the rate of increase may be much greater or much less than this. During wet and changeable weather the potential at a point in the air may be negative, and is always very variable in value.

The electrification of the air varies, not only with the state of the weather, but, under settled conditions of weather, varies with the season and the hour of the day. The electrification is stronger in winter than in summer, and the diurnal variation is associated with the variations of temperature, the electrification being a maximum at the times of greatest variation of temperature, and a minimum during the hours of constant temperature.

The equipotential surfaces in the air are usually planes parallel to the surface of the earth. Inequalities on the earth's surface influence the form of the lower equipotential surfaces; but the irregularities due to this cause disappear at a comparatively low height.

The potential at any point in the air near the earth's surface may be considered as due to the electrification of the air or the electrification of the earth. The increase of positive potential with increase of height is, for example, consistent either with positive electrification of the air or negative electrification of the earth. The fact that a water dropper gives the same indication in the open as when enclosed in wire netting, or in an enclosure of perforated zinc, where it is free from external influence, supports the theory that the mass of the air is electrified.

137. Thunderstorms.—The phenomena of a thunderstorm are supposed to result from the intense electrification of the clouds. It is probable that an electrically charged cloud is made up of a very large number of isolated charged drops of water, and is thus charged throughout its mass and not on its surface only. It is conceivable that the condensation of water vapour in

positively charged air may give rise to a cloud of positively charged drops of water. As the cloud grows these drops unite and form larger drops, and in doing so the potential of each drop must evidently rise. For, if eight small drops unite to form one larger one, the charge on this drop will be eight times that of one of the smaller drops, but its radius will be only twice as great, and, therefore, the potential of each drop will be four times as great and the surface density twice as great as before coalescence.

The potential of a heavy cloud made up of comparatively large droplets may in this way rise to a very high value. Different clouds formed under widely different atmospheric conditions may thus become positively or negatively charged to very high potentials, and in an assemblage of such clouds disruptive discharge between the clouds, or between the clouds and the earth, may give rise to all the phenomena of a thunderstorm.

138. Lightning.—Lightning is disruptive spark discharge on a large scale between clouds charged to widely different potentials or between a charged cloud and the earth or an earth-connected object. The discharge may be simple or oscillatory in character. It is intensely bright, and is usually of the **forked** or branching character similar to that shown in Fig. 259. **Sheet** lightning is probably reflection in the clouds of an ordinary discharge at too great a distance for the thunder to be heard, or it may be due to partial brush-like discharges between adjacent clouds. **Globe** lightning, if it really exists as described, has not been explained It is said to consist of balls of fire which move slowly along, and ultimately burst with a loud explosion. The physiological effects of lightning are generally so intense as to produce death or temporary paralysis.

The sound accompanying the discharge, known as thunder, is due to the sudden and violent disturbance of the air along the lines of discharge, and its nature depends on the nature of the path. When the path is short and straight the thunder **clap** is produced, whilst a long and zig-zag path results in the thunder **rattle.** The **rumbling** or **rolling** is due to echoes among the clouds.

139. Lightning Conductors.—The lightning conductor still in common use as a protection from lightning was suggested by Franklin over a hundred years ago. It consists of an iron or copper rod or flat strip of about one-quarter of a square inch in section. It runs from the top to the bottom of the building to be protected. At its upper end it is sharp-pointed and, in order to resist the action of the atmosphere, the point-piece may be of copper, thickly gilt at the point, or of platinum. At its lower end it should be in good connection with the earth, and to secure this it should be attached to a large earth-plate sunk in the earth to a depth sufficient to be always in wet soil.

Under certain conditions the action of Franklin's conductor may be such as to prevent the lightning flash altogether or to preserve the building from damage even when the flash does take place. Thus consider a positively charged cloud acting inductively on the earth and all earth-connected bodies beneath it. As previously explained in connection with the "action of points" (Art. 73), a stream of negatively charged particles of air, etc., proceeds from the point, thus slowly neutralising the positively charged cloud and preventing the flash altogether. If the flash is not prevented in this way, it will tend to pass between the cloud and the conductor, and, choosing the path of least resistance, will proceed down the conductor to the earth without damage to the building.

Other conditions may arise, however, which render the Franklin rod of little service; the theory underlying these will be better understood later, but the following experiment of Sir Oliver Lodge will, at this stage, illustrate the points in question.

Exp. Two Leyden jars are connected in series between the terminals of an influence machine, the two outer coatings being connected, as shown in Fig. 270, by a continuous conductor, and a spark gap $a\,b$ arranged in parallel. When the machine is worked it is found that every time a spark passes between the terminals of the machine, a spark also passes in the gap $a\,b$, even when the length of this gap is greater than that between the machine terminals. This shows that an instantaneous difference of potential is set up between the two points, and in such cases a spark between

the points is not necessarily prevented by the points being in metallic connection, that is, the discharge does not necessarily follow the path of least electrical resistance. As will be explained later, *when the current is one of very rapid alternation* the opposition to its passage between any two points on a conductor *is not determined by the resistance* of the conductor but by (Chapter XVII.) what is called its *inductance* and by the fact that, for very rapid alternations, the current is confined to a very thin surface layer or skin of the conductor.

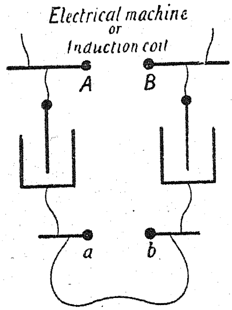

Fig. 270.

In the above experiment the sparks at AB could be prevented by joining the terminals by a conducting wire, whereas those at ab take place notwithstanding the presence of the conductor joining the points. The difference of potential at AB gradually rises to the sparking value, then falls to zero and rises again. At ab the difference of potential is normally zero, but when a spark passes at AB it rises to the sparking value too rapidly for the conductor connecting the points to prevent the spark. When a lightning discharge tends to take place between a cloud and the earth under the conditions which obtain at AB, it is usually possible to prevent it by a lightning conductor; but when the conditions are similar to those which obtain at ab, an ordinary lightning conductor is of no service.

If, for example, a positively charged cloud, A, Fig. 271, hangs over the earth B, and if the charge on A increases, or the distance between A and B decreases, the difference of potential between A and B may rise to sparking value, and ultimately a spark may pass between the cloud and the earth under the same conditions as between the terminals A, B of the machine. A metal conductor L, well connected to earth, would prevent this discharge by inductive action, or if discharge took place

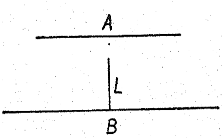

Fig. 271.

between A and the top of the conductor the discharge would be carried by the conductor without the damage that would result to a tree or building of badly conducting material which might in the absence of the conductor be struck by the discharge.

Suppose, however, that a positively charged cloud, A, Fig. 272, hangs over the earth with an uncharged cloud, a, between it and the earth, and that a long conductor, L, serves to equalise potential between a and the earth. Then, a lightning flash or spark may pass between A and a, and when it does pass the electrical equilibrium between a and B is suddenly disturbed, and a difference of potential sufficient to cause a flash to pass between a and the earth may be suddenly set up. A flash produced in this way resembles, in the conditions of its production, that which takes place at $a b$ in the experiment of Fig. 270, and may strike through the air or through any non-conducting mass between a and the earth, in spite of the presence of the conductor L. In fact lightning is oscillatory of high frequency so that the *impedance of the conductor* (Ch. XX.) may become very large and the lightning may therefore spring from it to neighbouring bodies.

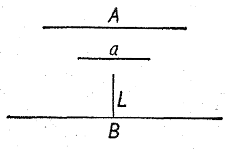

Fig. 272.

The general conclusions as to lightning conductors seem to be—

(*a*) They should be of iron ribbon as free from joints and sharp bends as possible. Several such conductors should be used, so that they may be taken to *all* high points. They should be insulated from the walls and good deep wet earth connections should be made independent of gas or water pipes. The conductors may be connected by barbed wire run round the eaves and ridges.

(*b*) All external metal work such as spouts, etc., should be connected together and to earth, but not to the lightning conductor except under ground.

(*c*) For an ordinary house a cheap yet satisfactory method is to run common galvanised iron telegraph wire up all the corners, along all the ridges and eaves, and over all the chimneys, taking these wires down to the earth at several places, and at each place burying a load of coke round the wire.

The principle is to surround the building with a network of conductors and to give the lightning many easy paths to earth.

140. Causes of Atmospheric Electricity.—The whole question of atmospheric electricity, its origin, the constitution of thunder-clouds, the causes of their electrification and other details are, at present, to a large extent, matters of speculation rather than of exact knowledge. Modern ideas on the subject cannot be clearly dealt with at this stage, but will appear in later chapters.

One theory attributes atmospheric electricity to the processes of evaporation, vegetation, and combustion. This theory supposes that the water vapour arising from water on the surface of the earth carries a positive charge with it, leaving the water and the earth negatively charged; the processes of vegetation and combustion are also supposed to carry into the air a positive electrification. Experiments indicate, however, that these operations are insufficient to account for all the observed phenomena, and even the work of various experimenters is inconsistent in this respect.

It has been known for some time that the *splashing of liquids* results in electrical separation. Thus, if water falls on a metal plate the air around becomes negatively electrified, the spray positively electrified; but salt water produces an opposite result, *i.e.* the air becomes positively charged, and the spray negatively charged. Further, Dr. Simpson has recently shown that when pure water is broken into drops by means of an air jet electrical separation ensues, the water exhibiting a positive charge. Such experiments have led to the suggestion that water in the atmosphere may be broken up by air currents, etc., and the electrification partly accounted for in this way.

Elster and Geitel have put forward the idea that the electrification of the air is due to ultra-violet rays from the sun, such rays having been shown to have the property of dissipating a negative charge from dry ice and other substances.

Other suggestions emphasise the point that the air is *ionised* (Chapter XXIII.) by ultra-violet light, radio-active emanations, etc.; such ionisation, as will be seen later, accounting for its conductivity. Further, it has been shown that the process of condensation necessitates some nucleus, *e.g.* a dust particle; and it is now known that the positive and negative ions in an ionised gas perform this function, the negative ions being superior to the positive ions in this respect. Condensation occurring in this way will thus produce electrical separation, the negative being brought down as rain charged negatively; the latter is more or less supported by experiment.

The reader will follow these points better in later chapters; but the whole perplexing subject of atmospheric electricity is still in a state of flux.

141. The Aurora.—The Aurora is a luminous effect visible in polar regions; in northern regions it is called the *Aurora Borealis* or *Northern Lights*, and in southern regions it is called the *Aurora Australis*.

The Aurora is probably an electric discharge (similar to discharge in low vacuum) in the upper regions of rarefied air. The appearance is very varied. It frequently consists of an arch of pale light with a characteristic quivering appearance, and may consist largely of streamers of a light rose colour radiating from a polar centre, but the streamers vary in colour and intensity. The spectrum shows a characteristic yellowish green line, probably due to *krypton*, one of the rare inert gases in the atmosphere; and, since the lines of other constituents of the atmosphere have also been detected, the facts support the view that the phenomenon is due to the passage of an electric discharge through the atmosphere. The percentage composition of the upper rarer regions of the atmosphere is, of course, probably quite different from that of lower regions.

Franklin put forward the theory that the discharge was caused by differences of potential between the cold air near the poles and the currents of warmer air and vapour from the equator; but a more recent theory, due to Arrhenius, suggests that the phenomena are due to streams of electrons discharged from the sun, which, under the influence of the earth's magnetic field, tend to rotate round the lines of force, and to move towards the polar regions. The Aurora is certainly closely associated with magnetic storms, and is most frequent during periods of maximum sunspots.

Exercises IX.

Section B.

(1) Why does a sharp point attached to an electrical machine prevent a high potential being obtained while a knob has no such effect? Describe some practical application of this action of points. (B.E.)

(2) Describe some apparatus by which an indefinitely large quantity of electricity can be obtained by means of electrostatic induction from a minute initial charge. (B.E.)

(3) Two equal soap bubbles, equally and similarly electrified, coalesce into a single larger bubble. If the potential of each bubble while at a distance from the other was P, what is the potential of the bubble formed by their union? (B.E.)

Section C.

(1) Briefly enumerate the different kinds of electric discharge and describe their differences. (Inter. B.Sc.)

(2) Give a brief sketch of some method by which the electrical potential of the air may be measured. (B.Sc.)

(3) In using the quadrant electrometer it is usual to keep the needle at a constant potential. Describe (*a*) the arrangement employed for indicating the constancy of the potential, (*b*) the arrangement for raising or lowering to the required value. (B.Sc.)

ANSWERS.

Exercises II.

B.—(1) (*a*) 37·5 lb. (*b*) 16; 686; 810 dynes.

(2) (*a*) $Q = 19{\cdot}62$ C.G.S. units. (*b*) $R = 30{\cdot}9015$ C.G.S. units.

(*c*) 24·623 cm. (3) 7 : 11. (4) $t = 2\pi\sqrt{\dfrac{K\sqrt{2}}{MH}}$.

(6) 370·3. (7) ·0075 dyne.

C.—(1) 1157.

(2) If r = distance between the centres, $H = 8\sqrt{5}\,\dfrac{M}{r^3}$; the direction is inclined at $\tan^{-1}\frac{1}{2}$ to the line joining the centres.

(3) 160 ergs. (5) 3·8. (6) 1 : 3·82.

Exercises III.

B.—(1) 64 : 81; 36. (2) 13·56 min. (3) 64; 125.

(4) 30°. (5) 205,000. (6) $H = {\cdot}177$; $M = 795$. (7) 24.

C.—(1) $(150\sqrt{2} + 45)$ degrees; $(150\sqrt{3} + 60)$ degrees.

Exercises IV.

C.—(5) 23′ nearly (taking field intensity = ·45).

Exercises VI.

B.—(1) 1 dyne.

(2) 1 dyne; parallel to the line joining the two charges.

(3) $3\sqrt{3}\,\dfrac{Q}{4d^2}$ dynes, where $Q =$ magnitude of each charge and $d =$ side of hexagonal base.

(4) $8\sqrt{2}$; $8\left(1 + \dfrac{\sqrt{5}}{5}\right)$. (5) 5.

(7) 4·9. (8) $\dfrac{1}{6\pi \times 10^7}$ cm.

(10) Charges on $A = O$ and $98\frac{2}{3}$; charges on $B = 98\frac{2}{3}$ and $1\frac{1}{3}$; Force on $A = 197$ dynes.

(11) 6·$\dot{7}$. (12) 1·57 seconds.

(13) 33; 5. (14) 15.

C.—(2) $\frac{1}{16}$; $\frac{1}{4}$; $\frac{1}{16}$.

(4) 375 C.G.S. units; $333\frac{1}{3}$ C.G.S. units. (7) 95·58.

Exercises VII.

B.—(1) 10 cm.; 12 cm. (2) 1 : 16. (3) 25 : 24.

(4) $6\frac{2}{3}$ ergs. (5) 84 : 325. (6) 1 : 9.

(7) Three in parallel with one in series. (8) 1 : 1·4.

C.—(1) ·01125. (2) 5 : 4; equal.

(3) 121 : 400. (4) $V \cdot \dfrac{ab}{b - a}$.

Exercises VIII.

B.—(1) 3. (2) 35. (5) 2 : 1.

Exercises IX.

B.—(3) $\dfrac{2P}{\sqrt[3]{2}}$.

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